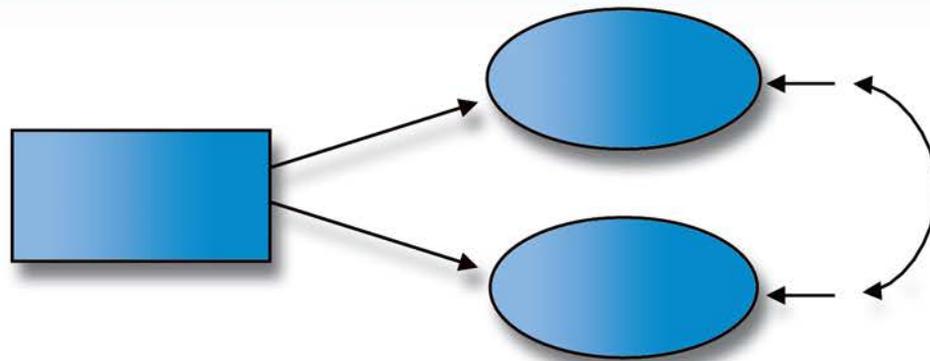


# Multilevel and Longitudinal Modeling with IBM SPSS

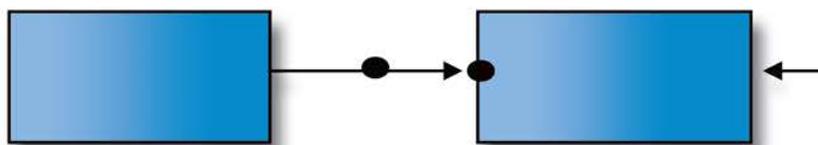
Second Edition

QUANTITATIVE METHODOLOGY SERIES



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WITHIN



Ronald H. Heck | Scott L. Thomas | Lynn N. Tabata

# Multilevel and Longitudinal Modeling with IBM SPSS

This book demonstrates how to use multilevel- and longitudinal-modeling techniques available in the IBM SPSS mixed-effects program (MIXED). Annotated screen shots provide readers with a step-by-step understanding of each technique and navigating the program. Readers learn how to set up, run, and interpret a variety of models. Diagnostic tools, data management issues, and related graphics are introduced throughout. Annotated syntax is also available for those who prefer this approach. Extended examples illustrate the logic of model development to show readers the rationale of the research questions and the steps around which the analyses are structured. The data used in the text and syntax examples are available at [www.routledge.com/9780415817110](http://www.routledge.com/9780415817110).

## Highlights of the new edition include the following:

- Updated throughout to reflect IBM SPSS Version 21.
- Further coverage of growth trajectories, coding time-related variables, covariance structures, individual change, and longitudinal experimental designs.
- Extended discussion of other types of research designs for examining change (e.g., regression discontinuity, quasi-experimental) over time.
- New examples specifying multiple latent constructs and parallel growth processes.
- Discussion of alternatives for dealing with missing data and the use of sample weights within multilevel data structures.

The book opens with the conceptual and methodological issues associated with multilevel and longitudinal modeling, followed by a discussion of SPSS data management techniques that facilitate working with multilevel, longitudinal, and cross-classified data sets. Chapters 3 and 4 introduce the basics of multilevel modeling: developing a multilevel model, interpreting output, and trouble-shooting common programming and modeling problems. Models for investigating individual and organizational change are presented in Chapters 5 and 6, followed by models with multivariate outcomes in Chapter 7. Chapter 8 provides an illustration of multilevel models with cross-classified data structures. The book concludes with ways to expand on the various multilevel- and longitudinal-modeling techniques and issues when conducting multilevel analyses.

Ideal as a supplementary text for graduate courses on multilevel and longitudinal modeling, multivariate statistics, and research design taught in education, psychology, business, and sociology, this book's practical approach also appeals to researchers in these fields. The book provides an excellent supplement to Heck and Thomas's *An Introduction to Multilevel Modeling Techniques*

(2nd ed.); however, it can also be used with any multilevel- and/or longitudinal-modeling book or as a stand-alone text.

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# Multilevel and Longitudinal Modeling with IBM SPSS

Second Edition

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# Preface

Multilevel modeling has become a mainstream data analysis tool over the past decade, emerging from a somewhat niche technique in the late 1980s to a technique now figuring prominently in a range of social and behavioral science disciplines. As the approach gained popularity over the 1990s, specialty software programs began to appear addressing the needs of an ever-widening group of users. Eventually, mainstream statistics packages such as SAS, SPSS, and Stata began to include routines for multilevel modeling in their programs.

Although some devotees of the various mainstream packages began making use of this new multilevel-modeling functionality, progress toward carefully documenting these routines was slow, thereby hindering meaningful access to the average user. In the meantime, the specialty software packages were becoming increasingly refined and accessible, offering users a growing number of generalizations of the traditional multilevel model. In some ways, the software proved to be both driving and limiting the development of the field.

The various approaches to multilevel modeling represented in these packages have in some ways made it difficult for a clear lingua franca to emerge and have long challenged those interested in teaching these techniques. In addition to the considerable expense of purchasing the better-documented specialty software programs, there is also the additional challenge of mastering the new programming logic, syntax, and file structure unique to each program. Our second edition demonstrates how to use the multilevel- and longitudinal-modeling techniques available in IBM SPSS (Version 21). We have devoted most of our energy to providing both new and seasoned multilevel analysts with a set of concepts and programming skills to enable the development, specification, and testing of a broad range of multilevel models using a statistical program that is standard in many graduate programs and organizations around the world.

Drawing on our own teaching and workshop experience with graduate students and researchers from diverse fields (e.g., education, management, sociology, psychology, and public health) and our work explicating the multilevel approach (Heck & Thomas, *An Introduction to Multilevel Modeling Techniques*, 2nd ed., 2009), we have chosen to adopt a workbook format here. Our intent is to help readers set up, run, and interpret a variety of different types of multilevel and longitudinal models using the linear mixed-effects models (MIXED) procedure in SPSS. The routine enables users to fit linear mixed-effects models with continuous outcomes. Readers interested in estimating single-level and multilevel models with categorical outcomes in IBM SPSS might consult our recently completed workbook on that topic (Heck, Thomas, & Tabata, *Multilevel Modeling of Categorical Outcomes Using IBM SPSS*, 2012). In that volume, we provide a conceptual treatment of single-level and multilevel models for categorical outcomes (i.e., dichotomous, multinomial, ordinal, and count) and then walk readers in a step-by-step fashion through data management, model conceptualization, and model specification issues related to these types of quantitative models.

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## New to This Edition

In the second edition of *Multilevel and Longitudinal Modeling with IBM SPSS*, we have broadened our presentation to consider a number of different types of research designs and model specifications than can be readily adapted to multilevel analyses. These more complex models are included in Chapters 5–7, after we present our basic multilevel-modeling material in Chapters 1–4. Highlights include the following:

- A significant reworking of Chapter 5 to reduce coverage of repeated measures ANOVA and provide more coverage of defining various types of growth trajectories using MIXED.

Discussion and annotated illustrated instruction is provided for alternative ways of coding the time-related variables and specifying various covariance structures. The chapter presents the basic two-level approach for examining individual change and includes a new example illustrating a simple longitudinal experimental design with treatment and control groups.

- In Chapter 6, extending the individual-change model to organizational settings and to other types of research designs, more specifically, illustrating the applicability of MIXED for conducting regression-discontinuity analyses and for specifying a quasi-experimental design to investigate the effects of policy implementation over time. In this latter example, we develop a piecewise growth model, which provides a means of examining two or more different growth trends within one model.
- In Chapter 7, introducing the multilevel multivariate approach in MIXED with expanded examples including defining multiple latent constructs and parallel growth processes.
- Updating our earlier coverage of missing data and the use of sample weights by offering some considerations of different alternatives for dealing with these issues within multilevel data structures. As these issues can be problematic when working with multilevel models in IBM SPSS, we offer some guidance regarding the program's limitations and possible options for analyzing multilevel data in each case.

Similar to the first edition, we continue to offer multiple examples of several types of multilevel models with continuous outcomes, carefully showing how to set up each model and how to interpret the output. Most chapters feature one or more extended examples illustrating the logic of model development. These examples show readers the context and rationale of the research questions and the steps around which the analyses are structured. Annotated screen shots from SPSS are provided to help readers navigate the software program and facilitate learning the various techniques developed sequentially in each chapter.

We also provide an introduction to data management, diagnostic tools, and relevant graphics. Readers can work with the various examples developed in each chapter by using the corresponding data and syntax files on the Web site at <http://www.routledge.com/9780415817110>.

The workbook begins with an introductory chapter highlighting several relevant conceptual and methodological issues associated with defining and investigating multilevel and longitudinal models, followed by a chapter highlighting available IBM SPSS data management techniques that we have found facilitate working with various combinations of multilevel, longitudinal, and cross-classified data sets. In our experience, the first challenge in undertaking multilevel analyses is properly organizing the data we wish to examine. In Chapters 3 and 4, we detail the basics of multilevel modeling including how to develop two- and three-level multilevel models, how to interpret relevant output, and how to solve common programming and modeling problems. We develop several basic models for investigating individual and organizational change in Chapters 5 and 6, followed by an introduction to multilevel models with multivariate outcomes in Chapter 7. Chapter 8 illustrates SPSS's facility for examining models with cross-classified data structures, a type of hierarchical structure that greatly expands the possibilities for following subjects through multiple organizational units or subunits over time. We conclude with thoughts about ways to expand on the various multilevel- and longitudinal-modeling techniques introduced and issues to keep in mind in conducting multilevel analyses.

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## Intended Audience

We hope the workbook is a useful guide to readers' efforts to learn more about the basics of multilevel and longitudinal modeling and the expanded range of research problems that can be addressed through their application. Ideal as a supplementary text for graduate-level courses on multilevel, longitudinal, and latent variable modeling; multivariate statistics; and/or advanced quantitative techniques taught in departments of psychology, business, education, health, and

sociology, we hope the workbook's practical approach will also appeal to researchers in these fields. We believe the workbook provides an excellent supplement to our other workbook, *Multilevel Modeling of Categorical Outcomes Using IBM SPSS*, and our earlier volume, *An Introduction to Multilevel Modeling Techniques* (2nd ed.); however, it can also be used with any multilevel- and/or longitudinal-modeling book or as a stand-alone text.

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**Ronald H. Heck**  
**Scott L. Thomas**  
**Lynn N. Tabata**

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# CHAPTER 1

## Introduction to Multilevel Modeling with IBM SPSS

Social science research presents an opportunity to study phenomena that are multilevel, or hierarchical, in nature. Examples might be college students nested in institutions within states or elementary-school-aged students nested in classrooms within schools. Attempting to understand individuals' behavior or attitudes in the absence of group contexts known to influence those behaviors or attitudes can severely handicap one's ability to explicate the underlying processes of interest. People within particular organizations may share certain properties including socialization patterns, traditions, attitudes, and goals. The interactions between individuals and their social groups within various settings therefore lend themselves to numerous investigations.

For studying the relationships between individuals and their social groupings, multilevel modeling is an attractive approach because it allows the incorporation of substantive theory about individual and group processes into the clustered sampling schemes of many research studies (e.g., repeated measures designs or multistage stratified samples) or into the hierarchical data structures found in many existing data sets. Consider, as a different example, the examination of individual-level change within specific social or organizational settings. This becomes a problem of understanding change over time where a series of repeated measurements (Level 1) are nested within individuals (Level 2) who may be associated with particular groups (Level 3) that are also undergoing change over time. We could extend the analysis of such change trajectories to include situations where individual change processes accelerate or decelerate after the introduction of group-level treatments or where there are parallel change processes occurring (e.g., where people are changing in two or more domains simultaneously).

Multilevel modeling is fast becoming the standard analytic approach for examining data in many fields (e.g., sociology, education, psychology, and management) due to its applicability to a broad range of research designs and data structures (e.g., nested, cross-classified, cross-sectional, and longitudinal data). It is referred to by a variety of different names including random-coefficient models, mixed-effect models, multilevel regression models, hierarchical linear models, and multilevel covariance structure (or structural equation) models. This diversity of names is an artifact of the statistical theory underlying multilevel models, theory that has developed out of methodological work in several different fields, and this has led to differences in the manner in which the methods (and software) are used in various fields. At the core of these types of procedures, however, is an interest in the decomposition of variance in outcomes across several hierarchical levels and the explanation of this variance with variables specified at corresponding levels.

Despite a growing recognition of the promise and importance of this approach, multilevel-modeling procedures have not yet been fully integrated into research and statistics texts used in typical first- or second-year graduate courses. Two major obstacles are responsible for this reality. First, no standard language has emerged from this work. Second, until recently, the specification of multilevel models has generally required special software programs. With respect to the first

obstacle, despite the varied ways in which multilevel models are conceptualized, notated, and specified, they focus on the relationships between variables within and across multiple levels of a data hierarchy. Such potentially complex sets of empirical relationships must be explained by multilevel theories, a conceptual limitation that has not kept equal pace with advances in quantitative modeling incorporated into various computer software packages (Hox, 2002). Theories for dealing with the complexity of the multilevel features of organizations, for example, have been somewhat limited historically in terms of defining direct, mediating, or moderating relationships both within and between organizational layers (MacKinnon, 2008).

Available software, the second obstacle, has been largely an artificial barrier resulting from the increasing use of multilevel modeling in many areas of the social, behavioral, and health sciences. Simply put, our interpretation is that there have been several camps of scholars who have popularized these methods through the development of their own software and approach. Three of the best known of these packages are HLM (Raudenbush, Bryk, Cheong, & Congdon, 2004), MLwiN (Goldstein, 2003), and Mplus (Muthén & Muthén, 1998–2006). Although the mainstream emergence and acceptance of these methods is in large part due to the work and activity of these researchers, other more widely used statistical programs have implemented routines over the years that enable the development and specification of a wide variety of multilevel models. IBM SPSS, SAS, and Stata all have such routines (see Albright & Marinova, 2010, for an overview of each package's offering).

Despite the widespread availability of these packages, there is little available to help the researcher align the concepts and methods popularized through the specialty programs with the terminology and conceptualization used within these mainstream statistical packages. Important exceptions to this can be found in Rabe-Hesketh and Skondral's (2008) *Multilevel and Longitudinal Modeling Using Stata* (2nd ed.) and in Singer and Willett's (2003) *Applied Longitudinal Data Analysis*, which provides detailed setups for examples in their book using procedures in these widely available statistical packages. Also noteworthy are the resources available through the University of California at Los Angeles' Academic Technology Services Statistical Computing Web site (<http://www.ats.ucla.edu/stat/>) that have used a wide variety of statistical packages and developed annotated output for a number of multilevel texts. Few resources, however, provide a start-to-finish overview of how to actually carry out a multilevel analysis. This is the relative void that we hope to address.

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## Our Intent

IBM SPSS MIXED is a very flexible modeling routine that can be used to estimate a wide variety of multilevel, multilevel cross-classified, and multilevel repeated measures designs with continuous and categorical outcomes. This makes the program quite useful in estimating a wide variety of models with single or multivariate outcomes from numerous sampling distributions (e.g., normal, binomial, multinomial, and Poisson). The term *mixed* model implies the existence of data in which individual observations on an outcome are distributed (or vary at random) across identifiable groups. The variance of the random effect indicates its distribution in the population and therefore describes the degree of heterogeneity present (Hedeker, 2005).

Our intent is to help you conceptualize, set up, estimate, and interpret the output from a variety of different types of multilevel and longitudinal models with continuous outcomes using the linear mixed-effects (MIXED) procedure available in IBM SPSS. In this second edition, we have also attempted to broaden our presentation to consider a number of different types of research designs than can be readily adapted to multilevel analyses. Some of these include a typical treatment intervention, an example of a regression-discontinuity design, a parallel growth model, and a piecewise growth model. These more complex models are included in Chapters 5–7, after we present our basic multilevel-modeling material in Chapters 1–4. We assume readers have had at least an introductory statistics course (e.g., descriptive statistics, analysis of variance, and multiple

regression) prior to using this workbook. It is intended for use in a second course (e.g., multivariate statistics) or a third course (e.g., introduction to multilevel and longitudinal models). In our own series of courses, we have used the materials presented in this workbook in multivariate statistics courses (i.e., after presenting multivariate analysis of variance [MANOVA], exploratory factor analysis, and repeated measures analyses) and in beginning multilevel-modeling courses.

Mixed, or multilevel, models are differentiated from more familiar linear models (e.g., ANOVA and multiple regression) through their capability of examining correlated data and unequal variances. Such data are commonly encountered where subjects are nested in groups within a data hierarchy or as repeated measures nested within individuals. Where designs are unbalanced (i.e., having different numbers of participants within groups), estimation procedures available in multilevel-modeling routines yield asymptotically efficient estimates of a model's structural parameters and covariance components. We run the examples in this workbook using IBM SPSS Version 21 and Windows 7 Professional, both in the 64-bit version. Users running the models with earlier versions of MIXED or with IBM SPSS/SPSS for Mac or Windows (e.g., XP and Vista) may notice slight differences between their screens and our text screen shots, as well as slight differences in output appearance (and perhaps even estimates).

For our second edition, there are a few updates regarding multilevel-modeling procedures available in SPSS to note. Most important, at the time we finished our first edition (April 2010), a major limitation of the MIXED routine was that dependent variables had to be continuous. This precluded many situations where researchers might be interested in applying multilevel analytic procedures to various types of categorical (e.g., dichotomous, ordinal, and count) outcomes. Since then, the MIXED routine has been expanded to include procedures for modeling several different types of multilevel models with categorical outcomes (which are referred to as generalized linear mixed models). The routine is referred to as GENLINMIXED in IBM SPSS terminology. This capability begins with Version 19, which was introduced in Fall 2010 and is refined in Versions 20 and 21 (introduced in Fall 2012). The inclusion of this new categorical multilevel-modeling capability in MIXED prompted us to develop a companion workbook focused on estimating single-level and multilevel models with categorical outcomes (Heck, Thomas, & Tabata, 2012). We refer readers to that volume for an applied treatment of single-level and multilevel models with categorical outcomes using IBM SPSS.

A second issue concerns the SPSS program's features to deal with missing data. For examining models with missing data, there are several different approaches that can be used to estimate model parameters in the presence of missing data. Some of the traditional approaches include listwise deletion (i.e., eliminating any case with at least one missing value), pairwise deletion (e.g., eliminating pairs of cases when missing data are present, as in calculating a correlation), mean substitution, and various regression-based approaches. These approaches will lead to biased parameter estimation under most conditions that analysts encounter with real data. Two newer approaches are full information maximum likelihood (FIML) estimation with the partially complete data included in the analysis and multiple imputation (MI) of plausible values (Peugh & Enders, 2004). FIML estimation with missing data present is not presently available in IBM SPSS for typical multilevel models where there is a single data row for each subject. An individual with any missing data will be dropped from the analysis. For situations where the data are vertically arranged (i.e., as in a growth model), however, the information on the *Y* outcome will only be dropped for that occasion or occasions where the data are missing.

IBM SPSS has a missing data module that can be used to impute values for missing data; however, users should check whether or not it is included in the particular SPSS package they purchase or have available to use by licensing agreement. In some cases, the missing data module may have to be purchased as an add-on program. It is important to note, however, that MI routines are typically designed for single-level analyses of data with missing observations. It is important for the analyst to be aware of how missing data may affect the analysis. The MIXED routine in SPSS does a good job of accounting for the clustering effects present in the data (i.e.,

where individuals within a unit share some common characteristics) but does not necessarily address the patterns of missing data that may be present in various types of hierarchical data structures. Given these concerns, we suggest that users consider carefully the nature of their missing data and devise an appropriate strategy for dealing with data shortcomings. We spend some time later in this chapter discussing how to develop a strategy to deal with missing data.

A final issue concerns the program's limited capability to incorporate sample weights for multilevel data. Currently, the basic IBM SPSS software does not support techniques for the adjustment of design effects, although there is an add-on module that can be used for conducting *single-level* analyses of continuous and (some) categorical outcomes from complex survey designs. As Kish (1987) notes, traditional methods for statistical analysis are based on simple random sampling, which assumes independence of observations. This assumption seldom holds with most data sets that are encountered. Sample weights used in most secondary data sets have been devised to deal with more complex sampling designs than simple random sampling, but they treat the clustering of individuals within higher level units as "noise" to be filtered out, rather than as the focus of the analysis. For example, two-stage sampling involves first selecting a random sample of units (e.g., universities) from a population of universities having the individuals one is interested in studying. At the second step, a random sample of individuals (e.g., students) is drawn from within the sampled institutions. The unit selection process will influence the characteristics of the student sample.

At present, sample weights at two (or more) levels cannot be incorporated into multilevel analyses using MIXED. In the last section of the chapter, we update our earlier coverage of sample weights, offering some considerations of different weighting schemes that help guide the use of sample weights at different levels in the analysis. This is an area in which we feel more attention needs to be focused in subsequent versions of IBM SPSS and other software programs designed to analyze multilevel data structures. Population estimates can be affected by the various sample-weighting options in each software program and their underlying assumptions. We attempt to offer some clear guidance in the interim.

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## Overview of Topics

In this chapter, we provide a number of conceptual and methodological issues associated with multilevel modeling, which foreshadows our further development of these issues in subsequent chapters. In Chapter 2, we present several issues to deal with in structuring and working with various types of multilevel, longitudinal, and cross-classified data sets. Chapter 3 develops the basics of two-level multilevel regression models. Chapter 4 extends this general two-level model to three-level, cross-sectional models. Chapter 5 presents the basic two-level approach for examining individual change. In this edition, we have also included a new example illustrating a simple experimental design with treatment and control groups. Chapter 6 extends the individual-change model to organizational settings and to other types of research designs, more specifically, its applicability for conducting regression-discontinuity analyses and piecewise growth models. Chapter 7 introduces the multilevel multivariate approach in IBM SPSS. In this second edition, we have expanded the examples presented to include multiple latent outcome variables and parallel growth models. Chapter 8 presents examples of two-level and a three-level cross-classified data structures. Finally, Chapter 9 provides a short synthesis of our presentation.

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## Analysis of Multilevel Data Structures

We begin with the principle that theoretical frameworks are essential guides to empirical investigation and suggest that quantitative analysis concerns the translation (or operationalization) of abstract theories into concrete models. Our statistical models represent a set of proposed

theoretical relations that are thought to exist in a population—a set of theoretical relationships that account for relationships actually observed in the sample data from that population (Singer & Willett, 2003). Decisions about analysis therefore are located within the researcher’s methodological framework that begins with research questions, designs, data structures, and methods of analysis (Raudenbush, 1988). These decisions about how to conceptualize and conduct a study are critical to the credibility of one’s results and the study’s overall contribution to the relevant knowledge base.

Clearly there is also a place for less theoretically bound exploratory work, and we believe that these models can provide a powerful platform for this purpose. The multilevel framework opens up a range of new possibilities for exploratory analyses. With opportunities to examine relationships at multiple levels of analysis, to specify cross-level relationships between higher and lower levels of a hierarchical data structure, to include multiple group membership, and to incorporate a time dimension, the potential for exploratory work is great. That opportunity also presents a risk, however. With multiple levels within which one could test for specific relationships and a wide range of potential interactions, there is much to explore, and the models can easily bog down. This can yield questionable and perhaps even nonsensical results. Multilevel models are very data demanding. Adequate sample sizes are needed at each level to ensure sufficient power to detect effects, and, as a result, the models can very quickly become quite complicated, difficult to estimate, and even more difficult to interpret. Even in a simple two-level model, one might allow the intercept and multiple slopes to vary across groups, while sets of variables from the higher level are used to explain the variances in this set of intercepts and slopes. Correct model specification in a single-level framework is one thing; correct specification with a multilevel context is quite another (Goldstein, 1995). It is for this reason that we place an emphasis on the importance of sound conceptual frameworks to guide multilevel model development, even if it is largely exploratory in nature.

One’s choice of analytic strategy and model specification is therefore critical to whether the research questions can be appropriately answered. More complete modeling formulations may suggest inferences based on relationships in the sample data that are not revealed in more simplistic models. At the same time, however, better modeling formulations may lead to fewer findings of substance than have often been claimed in studies that employ more simplistic analytic methods (Pedhazur & Schmelkin, 1991). We feel it is important to draw a clear distinction between concerns over model specification limited to the exclusion or inclusion of theoretically relevant variables and model specification related to the mathematical explication of the relationships among these variables. Although theory should guide both considerations, the former concern deals with the availability of the relevant variables in the data set, while the latter deals with the structure of the data itself and the choice of modeling approach used to exploit theoretically important relationships presumed to exist in the population (Heck & Thomas, 2009). This is important to keep in mind because compromise is often necessary, since the complexity of proposed theoretical models (e.g., multiple levels and multiple random intercepts and slopes) can at times overwhelm the capability of the statistical software to estimate the model. We recall one example in Chapter 7 where simply changing the structure of the covariance matrix of random effects in a three-level model from a more simplified structure to one that might be considered as optimal resulted in lengthening the time it took to estimate the model by approximately two hours. At the end of that, the model estimation procedure failed to reach a viable solution that converged. In short, this is a case where our data and model specification failed to meet the demands of the propositions we attempted to test.

Besides the choices we make about methods of analysis, our inferences are also affected in practice by potential biases in our sample (e.g., size, sampling variation, and missing data). Another of our guiding principles is that when making decisions about analytic methods, the responsible researcher should consider approaches that are likely to take full advantage of the features of particular data structures and the goals of the overall research.

Multilevel data sets are distinguished from single-level data sets by the nesting of individual observations within higher level groups. In single-level data sets, participants are typically selected through simple random sampling. Each individual is assumed to have an equal chance of selection, and, at least in theory, the participants do not belong to any groups that might influence their responses. For example, individuals can be differentiated by variables such as gender, religious affiliation, and participation in a treatment or control group. However, in practice, in single-level analyses individual variation within and between a large number of these types of subgroups cannot be considered simultaneously—for example, cell means consisting of differences due to gender, religious affiliation, an intervention, their various interactions, and perhaps even a number of separate workplace settings. The number of groups created will quickly overwhelm the capacity of the analytic technique. In contrast, in multilevel (or nested) data sets, the groupings of participants may result from the overall sampling scheme used to select participants in a large study (e.g., neighborhoods may be selected first, and then individuals are selected second), or the social grouping of participants (i.e., their degree of common experiences due to closeness in space or time) is the primary focus of the theory and conceptual model proposed in the study (Kreft & de Leeuw, 1998). For example, we might be interested in whether differences in organizations' productivity can be explained by their employee and management value and decision-making structures.

We refer to the lowest level of a data hierarchy (Level 1) as the *microlevel*, with all successive levels in the data structure referred to as *macrolevels*. The relationships among variables observed for the microlevel often refer to individuals within a number of macrolevel groups or contexts (Kreft & de Leeuw, 1998). Within a contextual model, therefore, one could envision successive levels extending well beyond an organization (e.g., region, state, nation, etc.). Each of these successive groupings might exert effects on organizational productivity. Organizational outcomes may be influenced by combinations of variables related to the backgrounds and attitudes of individuals (demographics, experience, education, work-related skills, attitudes, etc.), the processes of organizational work (e.g., leadership, decision making, professional development, organizational values, resource allocation, etc.), the context of the organization, or the cross-level interactions of these variables within the structure of the organization (e.g., size, management arrangements within its clustered groupings, etc.). We summarize some of these possible relationships within and between levels in Figure 1.1.

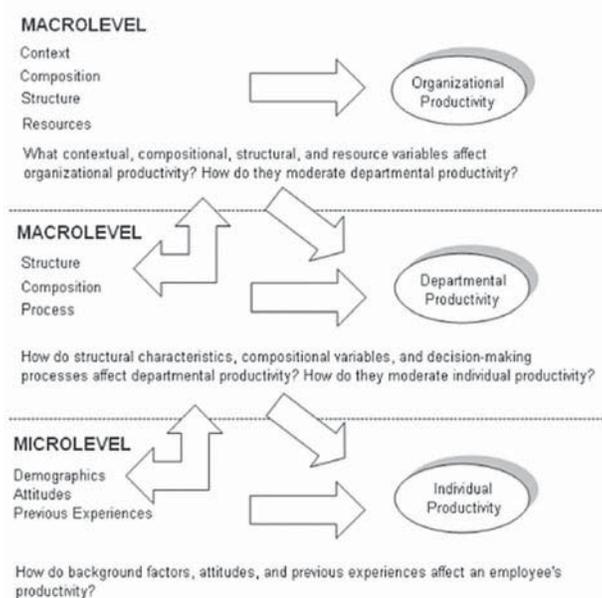


FIGURE 1.1 Defining variables in a multilevel model.

In the past, analytic strategies for dealing with the complexity of the multilevel, or contextual, features of organizations were somewhat limited. Researchers did not always consider the implications of the assumptions they make about measuring variables at their natural levels, or about moving them from one level to another level through aggregation or disaggregation. These possible choices are summarized in Figure 1.1 with two-headed arrows. *Aggregation*, for example, meant that the productivity level of individuals within departments or organizations would be combined at a higher level of the data hierarchy (e.g., the organizational unit). The comparison is then made between organizations' mean productivity outcomes. Failing to acknowledge the potential variability present among individuals within groups, however, can bias estimates of parameters between such units, a situation Robinson (1950) called the ecological fallacy. Averaging estimates over groups also drastically reduces the statistical power to detect proposed relationships by ignoring the number of individuals in a study (Tabachnick, 2008).

In contrast, *disaggregation* refers to moving a variable conceptualized at a higher level to a lower level of the data hierarchy. For example, in a different analysis we might have productivity measured at the organizational level but also have information about employee job satisfaction, motivation, and intention to leave the workplace. In this type of analysis, we might intend to analyze the data at the individual-employee level to see whether employees' perceptions influence their productivity. If we assign all employees the same value on the productivity outcome (and possibly other organizational variables), we attribute properties of the organization to individuals. This can also bias the estimation of model parameters since we are basing analyses of some of the variables on the number of individuals in the study instead of the number of groups.

Examples such as these suggest that analyses conducted at the microlevel or macrolevels may produce different results. Treating individuals as if they were independent of their organizational groupings ignores the complexity inherent in the data and introduces a potentially important source of bias into the analysis. This is because individuals in a group or context tend to be more similar on many important variables (e.g., attitudes, socialization processes, perceptions about the workplace, etc.) compared with individuals in other contexts. With hierarchical data, therefore, a more complex error structure must be added to the model to account for the dependence among observations within groups. Such dependencies violate key assumptions of single-level multiple regression models (e.g., simple random sampling that provides independent errors) and will lead to underestimated variances and standard errors, which, in turn, may lead to erroneous conclusions (Thomas & Heck, 2001). As we have noted, eliminating within-group variability also biases relationships observed between groups. One important contribution of multilevel modeling is that it allows the researcher to avoid the aggregation or disaggregation problem.

Developing a multilevel conceptual framework of relationships to guide the analysis also helps the researcher avoid another potential source of bias in the analysis—that of ignoring the different levels of the explanatory (independent) variables. As the reader may surmise, it is important to develop a conceptual scheme to place the explanatory variables hypothesized to affect individuals and other types of processes in their proper hierarchical locations (Hox, 2002). In a single-level model, we are faced with the problem of whether the outcome variable (or other predictors) should be defined as aggregate, or collective, measures or as individual (or disaggregated) measures. In a multilevel formulation, analysts must consider the implications of how they define variables within the proposed model before actually testing the model. This preliminary process of building a model helps clarify the organizational, or contextual, level to which they rightly belong. Examining variability in an outcome present at each level of the data hierarchy implies different types of research questions that can be asked, as summarized in Figure 1.1. These potential relationships are represented by horizontal arrows in the figure. A second contribution of multilevel modeling, therefore, is that through investigating the variation in outcomes that exists at different levels of the data hierarchy we can develop more refined conceptual models about how explanatory variables at each level contribute to variation in outcomes.

Finally, multilevel procedures also facilitate the investigation of variability in regression coefficients (slopes) across higher order units in the study. Cross-level interaction involves the effects of explanatory variables at a higher level of the data hierarchy on a relationship at a lower level (e.g., employee motivation and productivity). Cross-level interactions are shown in Figure 1.1 with arrows that extend from the macrolevels toward the microlevel. To illustrate, we might ask whether the relative degree of departmental teamwork moderates the relationship between employee motivation and productivity. In the past, mapping these sorts of relations has often been problematic, frequently focusing on single, discrete elements while ignoring the multi-dimensional and interrelated aspects of broader sets of organizational processes that influence subgroups or individuals within organizations.

### Partitioning Variation in an Outcome

Generally, the first step in a multilevel analysis is partitioning the variance (referred to as  $\sigma^2$ ) in an outcome variable into its within- and between-group components. If it turns out that there is little or no variation (perhaps less than 5%) in outcomes between groups, there would be no compelling need for conducting a multilevel analysis. In this case, a simple ordinary least squares (OLS) regression analysis conducted at the microlevel (individual) would be adequate. The proportion of variance that exists between groups compared to the total variation is described by an intraclass correlation ( $\rho$ ), or the proportion of variance explained by the grouping structure in the population. It is defined as

$$\rho = \sigma_b^2 / (\sigma_b^2 + \sigma_w^2), \quad (1.1)$$

where  $\sigma_b^2$  is the between-group variance and  $\sigma_w^2$  is the within-group variance. The intraclass correlation (also referred to as the ICC) can also be interpreted as the expected correlation between any two randomly chosen individuals in the same group (Hox, 2002). If it is substantial, therefore, it suggests that the groups are relatively homogeneous and likely quite different from each other.

There are at least two ways to think about the relative homogeneity of groups. The first is in terms of the potential remedies required for conducting a conventional single-level analysis of data that are hierarchically differentiated. A common example is the multistage type of sampling strategy used in large-scale surveys (where higher units may be sampled first and then individuals within these units). The focus in this type of analysis to remedy possible bias due to multistage sampling is on statistical adjustments to yield unbiased estimates of variances and standard errors. Acknowledging the ICC is important because it changes the error variance in single-level regression analyses. When clusters and nontrivial ICCs are present, the OLS regression assumption of independent errors resulting from simple random sampling will likely be violated. This problem results in a downward bias in the estimation of standard errors (i.e., the errors calculated will be too small).

Let's develop this important point. Because statistical tests of model parameters are based on the ratio of an estimate to its standard error, the underestimation of standard errors will often lead to more findings of significance than would be observed if clustering were considered. Suppose in a single-level analysis we observe that the estimate of the effect of gender on achievement is 4.0 points, and the standard error is estimated as 2.0. This would result in a  $t$ -ratio of 2.0 (i.e., the ratio of the estimate to its standard error). At a commonly adopted significance level of  $p = .05$  and a sample size of 500 individuals, the required  $t$ -ratio would be 2.0. Suppose the same analysis conducted as a two-level analysis (e.g., individuals clustered in schools) results in an estimated standard error of 2.5. Now when we calculate the  $t$ -ratio (4.0/2.5), the result would be 1.6, which would not be significant at  $p = .05$ . Single-level analytic approaches, such as multiple

regression, ignore the clustered nature of individuals within higher level groupings; therefore, in the presence of similarities among individuals within groups, estimated parameters may be biased. Although there are various ways to adjust the estimates produced by these analytic techniques, these remedies do not facilitate the investigation of models that specify presumed effects at different levels of the data hierarchy.

To address this shortcoming, the second way to think about group-level homogeneity in hierarchical data is in terms of the opportunities it presents to specify conceptual models designed to operationalize organizational processes at more than a single level. After determining that an outcome varies across units, the analyst can investigate how various within-group and between-group variables explain variance in the outcome at each successive level. As readers may notice in Figure 1.1, with horizontal arrows we can model variability in intercepts; that is, the predictors at each level will account for variability in the outcome at that level. Moreover, the concept of investigating random slope coefficients across groups is also central to multilevel modeling. As we noted, the cross-level arrows in Figure 1.1 suggest how higher level explanatory variables might explain random variability in slopes at that specific level.

Multilevel modeling, then, also contributes to our understanding of hierarchical data structures by allowing the analyst to estimate structural and variance/covariance parameters (e.g., residual variance in intercepts or slopes at Level 2 and covariance between intercepts and slopes at Level 2) more efficiently and accurately. In a multilevel model, we typically focus on output concerning two types of model parameters. Structural parameters are referred to as the model's *fixed effects*. These include intercept coefficients (e.g., a group mean) or slope coefficients (e.g., the relationship between gender and achievement). The complete set of variances and covariances in model parameters is referred to as its *covariance components* (Singer & Willett, 2003). Specific parameters can be designated as *randomly varying*, which means that the sizes of the estimates are allowed to vary across groups. Investigating these randomly varying intercepts and slopes is at the center of our general multilevel-modeling strategy presented in subsequent chapters.

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## Developing a General Multilevel-Modeling Strategy

In this workbook, we apply a general strategy for examining multilevel models (e.g., Bryk & Raudenbush, 1992; Heck & Thomas, 2009; Hox, 2010; Raudenbush & Bryk, 2002). Multilevel models are useful and necessary only to the extent that the data being analyzed provide sufficient variation at each level. “Sufficiency” of variation is relative and depends as much on theoretical concerns as it does on the structure and quality of data. Multilevel modeling can be used to specify a hierarchical system of regression equations that takes advantage of the clustered data structure (Heck & Thomas, 2009). Multilevel models can be formulated in two ways: (a) by presenting separate equations for each of the levels in a data hierarchy (e.g., employees, workgroups, departments, divisions, corporations, etc.) or (b) by laying out the separate equations and then combining all equations through substitution into a single-model equation.

For readers familiar with HLM (Raudenbush et al., 2004), the software uses separate equations specified at each level to build the multilevel model. This approach results in the need to generate separate data sets at each level first (e.g., individuals, classrooms, schools, etc.), which then are “combined” within the software program to make the final data file (called a multivariate data matrix or .mdm file). The user can neither see nor edit the case-specific contents of this final data set. Most other software packages, such as IBM SPSS MIXED, use single-equation representation (through algebraic substitution), so all analyses can be conducted from within one data set. As we will show in Chapter 2, however, we sometimes need to reorganize the single data set for particular types of analyses (e.g., longitudinal and multivariate analyses).

As Hox (2002) describes, both ways of specifying multilevel models have advantages and disadvantages. The separate-equation approach such as that used in HLM has the advantage of being clear about how the proposed model is built up from Level 1 through successive levels.

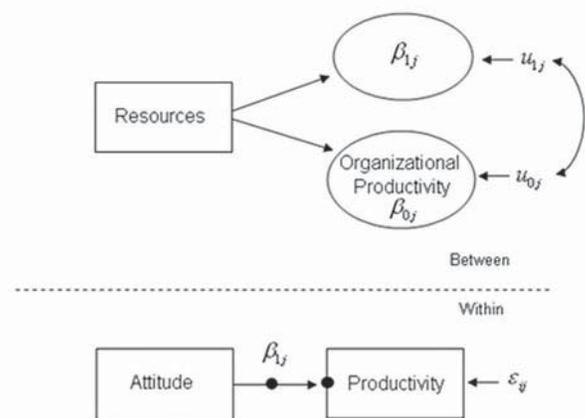
The disadvantage of this approach is that it hides from view the fact that modeling a cross-level interaction on a Level 1 regression slope results in adding an interaction to the overall model (Hox, 2002). The single-equation approach makes the existence of interactions obvious, but it is a little more difficult to see the sets of predictors that may be explaining each of the randomly varying slopes. We will walk readers through this process for one example in the following section. In successive chapters, however, we primarily adopt the approach of laying out separate equations by level for clarity in building models and often leave it to users to recognize that the sets of equations are reduced to single-model equations in MIXED when substitution is applied.

### Illustrating the Steps in Investigating a Proposed Model

We have found that in many instances multilevel investigations unfold as a series of analytic steps. Of course, there may be times when the analyst might change the specific steps, but, in general, we have found that this overall model development strategy works pretty well. In Figure 1.2, we provide an illustration of what a simple two-level model to explore a random intercept describing productivity and a random slope describing the effect of employee attitudes on their productivity might look like. We use this model to provide a conceptual overview of the modeling strategy we adopt in successive chapters.

In a single-level model we could focus on the overall relationship between employee attitudes and their productivity. In the traditional OLS model, we would get a slope expressing the relationship between the two variables and an intercept representing some adjusted value of the outcome. This would be fine if our interests were limited to an overall relationship. But if we were interested in understanding how employee groupings (e.g., workgroups or departments) might moderate that relationship (or even controlling for such a possibility), we would need to incorporate information about the organization or grouping of employees in our sample. Once we shift into thinking about such multilevel relationships, where employees are now nested within groups of some kind, a range of new analytic possibilities emerges.

Within groups we might define a randomly varying intercept (productivity) and randomly varying slope (i.e., the effect of employee attitudes on productivity). The random slope is shown in the figure with a filled dot on the line representing the relationship between attitudes and productivity. The Greek symbol beta ( $\beta$ ), which is often used to describe a slope or regression parameter at Level 1, is above the line. Between groups, we might propose that differences in resource allocation affect levels of organizational productivity. Additionally, we might propose that these variables also moderate the size of the effect of employee attitudes on productivity.



**FIGURE 1.2** Proposed two-level model examining a random intercept and slope.

This type of effect, which is referred to as a cross-level interaction, implies that the magnitude of a relationship observed within groups is dependent on contextual or organizational features defined by higher level units. We therefore formulate a Level 2 model to explain variability in intercepts and variability in slopes (shown as ovals representing *unknowns*, or latent variables, in Figure 1.2) across organizations.

### 1. One-Way ANOVA (No Predictors) Model

A good first step is to examine the extent to which variation in a Level 1 outcome exists *within* Level 2 units relative to its variation *between* Level 2 units. In our example, we want to know if there exists significant variance in productivity across groups—something that would be invisible to us in a single-level model. Little variability within the Level 2 units suggests greater homogeneity among all Level 1 observations than exists among Level 1 observations from different Level 2 groups—that is, such evidence suggests that the nesting of Level 1 observations is not systematically associated with levels of an outcome. The partitioning of outcome variance into Level 1 and Level 2 components without other predictors in the model allows the researcher to test the validity of this assumption and provides important information about the sources of variation in the outcome variable, productivity in our example here.

Notice that in Equation 1.2, we add a subscript for individuals ( $i$ ) and for organizations ( $j$ ). The null model for individual  $i$  in organization  $j$  can be represented as:

$$Y_{ij} = \beta_{0j} + \varepsilon_{ij}, \quad (1.2)$$

where  $\beta_{0j}$  is the mean of productivity (intercept) for the  $j$ th group, and the Greek lowercase letter epsilon ( $\varepsilon_{ij}$ ) represents the errors in estimating individual productivity within groups. In a two-level model, Level 1 fixed effects are typically expressed as unstandardized  $\beta$  coefficients. *Unstandardized* means the coefficients are in their original metrics. The subscript  $j$  indicates that the intercept represents the mean outcome for groups. Individual-level error (referred to as residual in the MIXED output) is also considered a random component.

Between groups, variation in intercepts ( $\beta_{0j}$ ) can be represented as

$$\beta_{0j} = \gamma_{00} + u_{0j}. \quad (1.3)$$

Level 2 fixed-effect coefficients (which are also unstandardized) are generally expressed as the Greek lowercase letter gamma ( $\gamma$ ). Variation in estimating organizational intercepts is represented as  $u_{0j}$ , which is considered a Level 2 random effect. Through substituting Equation 1.3 into Equation 1.2, we arrive at the single-equation model, which can be written as

$$Y_{ij} = \gamma_{00} + u_{0j} + \varepsilon_{ij}. \quad (1.4)$$

The null model therefore provides an estimated mean productivity score for all organizations. It also provides a partitioning of the variance between Level 1 ( $\varepsilon_{ij}$ ) and Level 2 ( $u_{0j}$ ). Altogether, Equation 1.4 suggests that there are three parameters to estimate: the intercept; the between-organization error, or deviation, from the average intercept ( $u_{0j}$ ); and the individual-level residual, or variation in individual scores within organizations ( $\varepsilon_{ij}$ ).

We can confirm this specification by examining the output from the intercept-only model in IBM SPSS. The average intercept is considered a fixed effect, while the between-group variation

TABLE 1.1 Model Dimension<sup>a</sup>

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
Random Effects	Intercept	1	Identity	1	orgid
Residual				1	
Total		2		3	

<sup>a</sup> Dependent variable: productivity.

in average intercepts is referred to as a random effect. The Level 1, or individual-level, error is referred to as residual in the output. We note that the “Number of Levels” column in Table 1.1 refers to whether the predictors are defined as continuous (number of levels = 1) or categorical. If a predictor such as gender is defined as categorical (i.e., as a *factor*) in building the model, the number of levels will be two, which refers to the categories of female and male. If it is defined as a *covariate* (i.e., because it is a dichotomous variable), as in a typical multiple regression model, the number of levels in the table will be one instead of two. Of course, predictors with three or more categories must be defined as factors (or else dummy coded). Information about the model’s parameters in the model dimension table is also useful in examining the baseline (no predictors) model with three estimated parameters against subsequent models with more estimated parameters. It is good practice to always check this output to ensure that the model estimated was the one that the analyst intended.

## 2. Analyze a Level 1 Model with Fixed Predictors

Assuming that sufficient levels of variance in  $Y$  exist at each level, we can investigate a model with only fixed predictors at Level 1. Level 1 predictors are often referred to as  $X$ . For each individual  $i$  in organization  $j$ , a proposed model similar to Equation 1.1 (summarizing the effect of employee attitudes on productivity) can be expressed as

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + \varepsilon_{ij}. \quad (1.5)$$

Equation 1.5 suggests that, within groups,  $X_{ij}$  (employee attitude) is related to productivity levels. Often, the unstandardized within-group predictors ( $\beta_1 X_{ij}$ ) are either grand-mean or group-mean centered to facilitate interpretation of the coefficients. Grand-mean centering indicates that the variable is compared against the mean for the sample. For example, if the sample mean for employee attitude were 4.4 (on a 7-point scale), an individual on the grand mean would have her or his score rescaled to 0. In contrast, group-mean centering implies that the individual’s attitude score is rescaled against the mean for her or his group, with the group mean now equal to 0. We describe in Chapter 2 how grand-mean and group-mean centered predictors can be developed in IBM SPSS.

At Level 2, Equation 1.6 implies that variation in intercepts can be described by an organization-level intercept ( $\gamma_{00}$ ), or grand mean, and a random parameter capturing variation in individual organization means ( $u_{0j}$ ) from the grand mean:

$$\beta_{0j} = \gamma_{00} + u_{0j}. \quad (1.6)$$

**TABLE 1.2 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	attitude	1		1	
Random Effects	Intercept	1	Identity	1	orgid
Residual				1	
Total		3		4	

<sup>a</sup> Dependent variable: productivity.

In the case where we wish to specify the within-group slope as fixed (i.e., it does not vary across organizations), we can express the Level 2 slope equation as follows:

$$\beta_{1j} = \gamma_{10} \quad (1.7)$$

This suggests that the variance component of the slope is fixed at 0. Because there is no random effect ( $u_{1j}$ ) in Equation 1.7, the slope coefficient is therefore fixed to one value for the sample. Through substitution of  $\beta_{0j}$  and  $\beta_{1j}$  into Equation 1.5, the single-equation model can be summarized as the following:

$$Y_{ij} = \gamma_{00} + u_{0j} + \gamma_{10}X_{ij} + \varepsilon_{ij} \quad (1.8)$$

and then reorganized with fixed parameters ( $\gamma$ s) and variance parameters ( $u_{0j} + \varepsilon_{ij}$ ) as follows:

$$Y_{ij} = \gamma_{00} + \gamma_{10} \textit{attitude}_{ij} + u_{0j} + \varepsilon_{ij} \quad (1.9)$$

where we have replaced  $X$  with the individual-level variable name. Equation 1.9 and the following output in Table 1.2 suggest that there are four parameters to estimate. The two fixed effects are the intercept and the Level 1 predictor *attitude*. The third parameter is the randomly varying intercept between groups (referred to as the *random effect* in the table). The fourth parameter is the Level 1 residual variance. With one random effect in the model, the default covariance matrix is an identity covariance matrix (ID). This is because there is only one Level 2 variance in the model (i.e., variability due to differences in outcome means between groups). We note that if other covariance structures are specified (e.g., variance components or diagonal), they will be ignored, which results in the same output.

### 3. Add the Level 2 Explanatory Variables

At the third step, it is often useful to add the between-group predictors of variability in intercepts. Group variables are often referred to as  $W$  (or  $Z$ ). From Figure 1.2, the Level 2 model with resources added will look like the following:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j}, \quad (1.10)$$

**TABLE 1.3 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	resources	1		1	
	attitude	1		1	
Random Effects	Intercept	1	Identity	1	orgid
Residual				1	
Total		4		5	

<sup>a</sup> Dependent variable: productivity.

where  $W_j$  refers to the level of resources in the organization. When we substitute the new Level 2 intercept equation (Eq. 1.10) and the previous fixed slope for attitude (Eq. 1.7) into the Level 1 equation (Eq. 1.5), we obtain the new combined equation:

$$Y_{ij} = \gamma_{00} + \gamma_{01}resources_j + \gamma_{10}attitude_{ij} + u_{0j} + \varepsilon_{ij}, \quad (1.11)$$

where we have included the names of the individual- and group-level predictors for  $W$  and  $X$ , respectively. We find that it is sometimes useful to write in the variable names to provide easy recognition of the predictors in the model. Readers will notice that in a two-level model, after substitution, all estimates of group-level and individual-level predictors are expressed as  $\gamma$  coefficients. Equation 1.11 suggests that there are five parameters to be estimated in this model. Table 1.3 confirms that there are three fixed effects, one random effect (i.e., the intercept), and the residual to be estimated, since the Level 1 slope (attitude) is still considered as fixed at Level 2.

#### 4. Examine Whether a Particular Slope Coefficient Varies Between Groups

We may next assess whether key slopes of interest have a significant variance component between the groups. Our theoretical model (Figure 1.2) proposes that the relationship between employee attitudes and productivity may vary across organizations. Testing random slopes is best accomplished systematically, one variable at a time, since if we test several slopes simultaneously, we are unlikely to achieve a solution that converges (Hox, 2002). As we suggested previously, if the within-unit slope (e.g., attitude-productivity) is defined to be randomly varying across units, the Level 2 slope model can be written as follows:

$$\beta_{1j} = \gamma_{10} + u_{1j}. \quad (1.12)$$

Equation 1.12 suggests variability in slopes can be described by a group-level intercept coefficient ( $\gamma_{10}$ ), often centered on the grand mean or the group mean, and a random parameter capturing variation in individual organization coefficients ( $u_{1j}$ ) from the mean.

Through substitution, the combined model will be

$$Y_{ij} = \gamma_{00} + \gamma_{01}resources_j + \gamma_{10}attitude_{ij} + u_{1j}attitude_{ij} + u_{0j} + \varepsilon_{ij}. \quad (1.13)$$

As we suggested earlier, notice the substitution of  $\beta_{1j}$  in the within-group (Level 1) model (Eq. 1.5) results in the addition of the interaction  $u_{1j}X_{1ij}$  (i.e., where  $X_{1ij}$  is employee attitude) to

**TABLE 1.4 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	resources	1		1	
	attitude	1		1	
Random Effects	Intercept + attitude	2	Diagonal	2	orgid
Residual				1	
Total		5		6	

<sup>a</sup> Dependent variable: productivity.

the single-equation model. This interaction is considered to be a random effect, which is defined as the deviation in slope for cases in group  $j$  multiplied by the Level 1 predictor score ( $X_1$ ) for the  $i$ th case in group  $j$  (Tabachnick, 2008). With a random slope and intercept, we typically first specify a diagonal (DIAG) covariance matrix at Level 2. A diagonal covariance matrix provides an estimate of the variance for each random effect, but the covariance between the two random effects is restricted to be 0. Specifying variance components (VC) will also provide this same diagonal covariance matrix specification. We can confirm in Table 1.4 that there are six parameters to estimate. The three fixed effects are the intercept, the Level 2 predictor (*resources*), and the Level 1 predictor (*attitude*). The two random effects are the variability in intercepts ( $u_{0j}$ ) and the variability in the attitude-productivity slope ( $u_{1j}X_{1ij}$ ). The Level 1 residual variance is the final estimated parameter.

### 5. Adding Cross-Level Interactions to Explain Variation in the Slope

Finally, we would build a Level 2 model to explain variation in the Level 1 randomly varying slope of interest (i.e., assuming the slope has significant variance across groups). Our simplified model in Figure 1.2 suggests that organizational resource levels may moderate the within-unit (e.g., attitude-productivity) slope:

$$\beta_{1j} = \gamma_{10} + \gamma_{11}resources_j + u_{1j}. \quad (1.14)$$

This cross-level interaction term is built during the model specification phase in IBM SPSS.

Substitution of the previous  $\beta_{1j}$  equation and the previous intercept equation (Eq. 1.10) into Equation 1.5 results in the final single-equation model to be estimated:

$$Y_{ij} = \gamma_{00} + \gamma_{01}resources_j + \gamma_{10}attitude_{ij} + \gamma_{11}resources_j * attitude_{ij} + u_{1j}attitude_{ij} + u_{0j} + \varepsilon_{ij}. \quad (1.15)$$

The model represented in Equation 1.15 results in seven parameters to estimate. The four fixed effects are the intercept, resources, attitudes, and the cross-level interaction (*attitude \* resources*). The two random effects at the organizational level (Level 2) are the intercept ( $u_{0j}$ ) and the attitude-productivity slope ( $u_{1j}attitude_{ij}$ ). The final parameter is the individual-level residual ( $\varepsilon_{ij}$ ). We can confirm this from the model dimensions output in Table 1.5.

When investigating the covariance structures associated with random effects, we adopt a similar strategy. One of the advantages of IBM SPSS MIXED is the ease in which users can specify alternative covariance structures for random effects. If this model converges, or reaches a solution without any warnings in the output, we can also try using a completely unstructured (UN)

**TABLE 1.5 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	resources	1		1	
	attitude	1		1	
	attitude * resources	1		1	
Random Effects	Intercept + attitude	2	Diagonal	2	orgid
Residual				1	
Total		6		7	

<sup>a</sup> Dependent variable: productivity.

**TABLE 1.6 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	resources	1		1	
	attitude	1		1	
	attitude * resources	1		1	
Random Effects	Intercept + attitude	2	Unstructured	3	orgid
Residual				1	
Total		6		8	

<sup>a</sup> Dependent variable: productivity.

covariance matrix of random effects. Specifying an unstructured covariance matrix the Level 2 provides an additional covariance term in the estimated model, which represents the covariance between the random intercept and random slope (i.e., there are three random parameters to be estimated in the unstructured covariance matrix in Table 1.6). This additional term in the covariance structure of the model will result in eight total parameters estimated, instead of the seven specified in Equation 1.15. We note, however, that specifying this type of covariance matrix for random effects sometimes results in models that fail to converge.

We can then use various objective tests (e.g., likelihood ratio test) to determine which covariance structure best describes the data. This allows the analyst to compare several models before arriving at a model that provides the closest fit to the observed data. It is generally best to keep models simplified, for example, by including only random effects that are of strong theoretical or empirical interest. Three-level models are more complicated to specify and estimate than two-level models because of the possibility of more cross-level interactions, random effects, and differing sample sizes; however, the strategy for building models is basically the same. We will address these issues more completely in subsequent chapters.

### Syntax Versus IBM SPSS Menu Command Formulation

In IBM SPSS MIXED, we can formulate models using either syntax statements or menu commands. We suspect most users are more familiar with the menu framework since they have likely used that to examine single-level models (e.g., analysis of variance, multiple regression, and factor analysis). We have chosen the menu-command approach, but we also provide examples of syntax

in an appendix for each chapter. Readers should keep in mind that with more complex models, it is often beneficial to use the syntax to formulate and run models for a couple of reasons. First, the syntax statements provide a nice record of a progression of models (if you save the syntax files and label them according to the general modeling steps we have laid out). This is helpful if you close the IBM SPSS program. You can return later and easily start up from where you left off.

Second, we have found that it is easier to reorganize the syntax statements than to rebuild the whole model from the menu commands if you wish to change the order of the variables in your output tables. Generally, we like to organize the output such that the predictors explaining intercepts are organized by levels of the data hierarchy, and predictors that explain variation in slopes (i.e., cross-level interactions) can be similarly organized. In the IBM SPSS Windows format, one can always use the PASTE function (within the IBM SPSS MIXED dialog box) to generate the underlying syntax before actually running the model. The syntax is sent to its own window within IBM SPSS and can be saved for future use. We find this is a good way to check whether the model you are running actually corresponds with the model that you had in mind. Often, we find doing this helps us see where we have forgotten or incorrectly specified something. This gets more important when the multilevel models that one is investigating take a considerable amount of time to produce a solution.

For readers unaccustomed to the syntax approach, following is an example of the syntax generated by IBM SPSS MIXED when we built the model within the menu system to produce the model in Equation 1.15.

```
MIXED
productivity WITH attitude resources
/CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1)
SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
PCONVERGE(0.000001, ABSOLUTE)
/FIXED = resources attitude resources*attitude | SSTYPE(3)
/RANDOM INTERCEPT attitude | SUBJECT(orgid) COVTYPE(DIAG)
/METHOD = REML
/PRINT = SOLUTION TESTCOV.
```

The initial syntax lines provide the dependent variable (productivity) and identify covariates in the analysis (signified by the key word “WITH”). Categorical variables (variables with two or more categories) can be specified with the key word “By,” with the last category serving as the reference group. Note that dichotomous variables (0, 1) can be defined as continuous (covariate) variables as in multiple regression. The criteria command (/CRITERIA) provides information concerning IBM SPSS default values (e.g., confidence intervals for estimates, iteration information, and convergence criteria) for the estimation algorithm. The statements can be excluded if the analyst chooses to maintain the default criteria. We recommend that the analyst exercise caution in reflexively adopting the IBM SPSS defaults as these will sometimes introduce problematic conditions on the models being estimated. Regardless of the type of model being developed, it is always good practice to review the default settings to ensure that they are appropriate for your purposes. Following this, the fixed effects (/FIXED = resources, attitudes, resources\*attitudes) in the model are defined, with the intercept also included as a default, and the random effects (/RANDOM = intercept, attitude). The Level 1 residual is also estimated as a default. The syntax also identifies the Level 2 cluster (SUBJECT) variable (orgid) and the type of covariance matrix used for the random effects (DIAG). If UN is specified for the covariance matrix instead of DIAG, the additional covariance between the intercept and random slope will be estimated. Finally, the syntax includes the default estimation procedure used (/METHOD = REML), which is restricted maximum likelihood, default model convergence criteria, and user-specified output (/PRINT = G SOLUTION TESTCOV). SOLUTION provides the tests of

fixed effects and TESTCOV the tests of random variances and covariances. Predicted values and residuals can be saved using the /SAVE command.

One can easily change the method of estimation (/METHOD), for example, from restricted maximum likelihood (REML) to maximum likelihood (ML) by just typing ML to replace REML in the appropriate syntax line. The order of the fixed effects (FIXED = resources attitude resources\* attitude) in the output can easily be changed by switching the order of the variables in that command line. Keep in mind that you can also add information to the IBM SPSS output tables (by using the Chart Editor) to help readers understand your presentation of output. We will explore many of these options in subsequent chapters.

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## Model Estimation and Other Typical Multilevel-Modeling Issues

We now turn our attention to several important issues related to model estimation (see Heck & Thomas, 2009; Hox, 2010; Marcoulides & Hershberger, 1997; or Raudenbush & Bryk, 2002, for further discussion). Model estimation attempts to determine the extent to which the sample covariance matrix representing our model specification is a good approximation of the true population covariance matrix. This determination is made through a formal test of a null hypothesis that the data are consistent with the model we have proposed. Note that, in general, confirmation of a proposed model relies on a failure to reject the null hypothesis. In contrast to the common use of the null hypothesis that readers may be more familiar with (e.g., rejecting the null hypothesis that two means are the same), here one wishes to accept the null hypothesis that the model cannot be rejected on statistical grounds alone. This implies that the model is a plausible representation of the data (although it is important to emphasize that it may not be the only plausible representation).

Unlike traditional single-level approaches using OLS, IBM SPSS MIXED (and other multilevel-modeling programs) employs maximum likelihood (ML) estimation. This is necessary where individuals are clustered within groups of differing size and, therefore, we have to deal with more than one set of residuals and distinguish between multiple levels (Hox, 2010). While we could certainly estimate a large number of single-level OLS regression models (i.e., one for each group), multilevel modeling is a more efficient approach that proceeds by estimating a single average intercept (or slope) and then a variance component describing the variance in intercepts or slopes across the Level 2 units in the study (Hox, 2010). Differing sample sizes, however, affect the reliability of the individual estimates produced for each unit (i.e., with estimates produced from smaller units being less reliable) and, hence, the overall reliability for that unit within the complete set of Level 2 units. Units where the group estimate is further from the overall grand-mean estimate of units tend to be estimated less reliably than units closer to the grand mean, other factors being equal (Hox, 2010).

In the past, multilevel models were limited by the need to have balanced sample size designs, which were required for using closed-form mathematical formulas to estimate the variance and covariance components (Raudenbush & Bryk, 2002). For designs with unbalanced sample sizes, what is needed is an iterative process that incorporates information about each group to obtain efficient estimates of the model's fixed and random parameters. Most often, maximum likelihood estimation (ML) is used for this purpose. Because of the relative complexity added with models where there are several predictors and potential random parameters, we often start with relatively simple models that estimate fixed effects and then add random components, in steps, one at a time.

Maximum likelihood determines the optimal population values for parameters in a model that maximize the probability or likelihood function—that is, the function that gives the probability of finding the observed sample data given the current parameter estimates (Hox, 2002). This involves an iterative process that determines a set of weights for random parameters in the model that minimizes the negative of the natural logarithm multiplied by the likelihood of the data.

Because the likelihood, or probability, can vary from 0 to 1, minimizing this discrepancy function amounts to maximizing the likelihood of the observed data. Since the process is iterative (i.e., beginning with a set of starting values and proceeding through rounds of parameter adjusting to reach an optimal set of estimates where further adjustment would not improve the estimates), it is not guaranteed to stop. Most programs, therefore, put limits on the number of iterations since most “good” models will converge within a relatively short number of iterations (e.g., perhaps 20–50 iterations). Often, making a change in the proposed model (e.g., eliminating a random slope or a covariance between an intercept and a slope) will result in the model converging. At other times, the small sample size (e.g., available number of Level 2 units) may be the culprit (Hox, 2010) since the data in such cases may not be relatively normally distributed.

If we consider the sample covariance matrix ( $S$ ) to represent the population covariance matrix ( $\Sigma$ ), then the difference between the observed sample matrix  $S$  and the model-implied covariance matrix ( $\hat{S}$ ) should be small if the proposed model fits the data. The evaluation of the difference between these two matrices depends on the estimation method used to solve for the model’s parameters (Marcoulides & Hershberger, 1997). The mathematical relationships implied by the model are solved iteratively until the estimates are optimized. As suggested, the difference between  $S$  and  $\hat{S}$  is described as a discrepancy function—that is, the actual difference in the two estimates based on a likelihood. The greater the difference between these two covariance matrices, the larger the discrepancy in the function becomes (Marcoulides & Hershberger, 1997). ML estimation produces a model deviance statistic, defined as  $-2 \times \log$  likelihood ( $-2LL$ ), where likelihood is the value of the likelihood function at convergence and  $\log$  is the natural logarithm. The deviance is an indicator of how well the model fits the data. Models with lower deviance (i.e., a smaller discrepancy function) fit better than models with higher deviance. Nested models (i.e., where a more specific model is formed from a more general one) can be compared by examining differences in these deviance coefficients under specified conditions (e.g., changes in deviance between models per differences in degrees of freedom).

IBM SPSS MIXED currently offers two estimation choices: full information ML estimation (which we will abbreviate as ML) and REML estimation, which is the default setting. In ML estimation, both regression coefficients and variance components are included in the likelihood function, while in REML, only the variance components are included in the likelihood function (Hox, 2002). REML, therefore, is referred to as a *restricted* solution. One of the shortcomings of ML estimation is that the estimates of variances and covariances depend on the point estimates obtained for the regression coefficients. As there are more parameters in the model and smaller sample sizes, the variance estimates obtained through ML may be too small, which leads to overly liberal hypothesis tests (Raudenbush & Bryk, 2002). In contrast, REML considers the regression coefficients to be unknowns to be estimated, which can lead to better estimates when there are small numbers of groups in the study (Raudenbush & Bryk, 2002). In other words, REML takes into account the loss in degrees of freedom due to the estimation of the  $P + 1$  regression coefficients in the model in order to obtain unbiased estimation of the variance components (Snijders & Bosker, 1999).

To illustrate this concept, in a simple model for estimating the variance in a sample mean, the REML approach amounts to dividing the sum of the squared deviations about the mean by the total sample size minus 1 ( $n - 1$ ) instead of  $n$  (Hox, 2002). This correction in the denominators used to calculate the variance will be greatest when the sample size is small. As described previously, when a series of nested models is to be compared in terms of fixed effects (e.g., regression coefficients), ML should be used because the approach takes into consideration the regression coefficients in solving the likelihood function (Hox, 2002). In later chapters, we show that MIXED also provides other fit indices that can be used to compare various models.

We note in passing that for the GENLINUX program, the available estimation procedure for models with continuous outcomes (i.e., which result from normal versus other types of probability distributions) is REML. This means that currently the GENLINUX estimation

option for continuous outcomes is limited to the REML approach used in MIXED. We therefore recommend estimating multilevel models with continuous outcomes using MIXED, even though they can also be estimated in the GENLINMIXED routine with REML. As mentioned previously, the REML approach would not be optimal when the research focus is on comparing successive models with both fixed regression coefficients and random coefficients.

### Sample Size

Under various sampling conditions (size of sample and normality of data) there has been considerable debate among methodologists about the efficiency of maximum likelihood estimation, given nonnormal features of the data (Goldstein, 1995; Longford, 1993; Morris, 1995). An important conceptual difference between single-level and multilevel approaches is that sample size considerations are quite different. With the multilevel approach, a sufficient sample size is required at each level of analysis. In smaller group samples, the difference in estimation between ML and REML results in a downward bias in variance components estimated with ML compared to REML. With small group samples, therefore, we generally prefer REML estimation. With large sample sizes, there should be no difference in the estimates produced by either method. It is important to keep in mind that under less-than-ideal sampling conditions (e.g., small numbers of groups and convenience samples), it may be difficult to determine whether model results might be replicated in other samples. IBM SPSS applies the Satterthwaite (1946) correction to standard errors, which provides more conservative estimates of standard errors, especially in small groups (e.g., Loh, 1987). We note that it is often more efficient to add higher level units than to add individuals within in groups since this former approach generally reduces that need for sizable samples within the groups and tends to be more efficient in estimating random coefficients.

### Power

Power refers to the ability to detect an effect, should one exist. In the single-level analysis, most researchers know that the significance level ( $\alpha$ ), the effect size (i.e., with larger effects easier to detect), and the sample size are determinants of power. Multilevel models raise a number of additional issues involving power. Issues about power typically concern the appropriate (or minimum) sample size needed for various types of multilevel analyses (e.g., determining whether an intercept or a slope varies across groups). As we suggested previously, one issue refers to the sample size required to ensure that estimates of fixed effects (e.g., at Level 1 and Level 2) and variances are unbiased (i.e., sampling bias). In most multilevel studies, the estimation of Level 2 effects is generally of greater concern as the number of groups available may be limited. As Snijders (2005) shows, when fixed effects are the focus, characteristics of the groups themselves have little bearing on the precision of Level 1 estimates. In general, we prefer adding groups (as opposed to individuals) to reduce parameter bias.

Another issue refers to the minimum sample size required to ensure that an effect would be detected if, in fact, one exists (i.e., power). In addition to these two determinants of power, in multilevel analyses there are at least two other considerations that inform estimates of power: sample size at each level (i.e., the number of individuals  $i$  within each group  $j$ , and the number of  $j$  groups) and the intraclass correlation (see Muthén & Satorra, 1995, for further discussion). With higher ICCs, the power to detect Level 1 effects will be lower (since the groups are more homogeneous), holding sample size constant at all levels. This suggests that the power to detect Level 2 effects is much more sensitive to the number of groups in the sample, as opposed to the number of observations within groups.

As designs become more complex, the need for larger samples at both levels increases. For example, in a given sample of individuals within groups, slopes in some units may be less reliably estimated than intercepts because, while intercepts depend only on the average level (mean)

of a variable within each group, slope estimates depend both on the levels of an outcome and a particular covariate, as well as the variability of their covariance among individuals within each group (Mehta & Neale, 2005). This means that estimating variability in random slopes across units generally requires larger sample sizes for more stable estimates than simply estimating random intercepts. Complications can also arise due to correlations between random effects and the investigation of cross-level interactions (Raudenbush & Bryk, 2002). As this limited discussion of power suggests, a number of considerations must take place to assess potential bias in parameter estimates and power in various types of multilevel designs. Readers can consult Scherbaum and Ferrer (2009), Snijders and Bosker (1999), and Snijders (2005) for further discussion of issues related to power.

### Differences Between Multilevel Software Programs

In preparing this workbook, we compared models estimated in IBM SPSS with models estimated with other multilevel software with which we are familiar (e.g., HLM and Mplus). Different software programs use slightly different algorithms to estimate models, for example, in calculating standard errors, especially in small groups. There are also differences in the means of testing the variance components. For example, IBM SPSS uses a Wald Z test, and HLM uses a chi-square test. In general, however, we have found that the differences between software programs are generally small—that is, output will carry the same substantive interpretation. We provide several examples in Appendix B.

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### Standardized and Unstandardized Coefficients

We present the analyses in subsequent chapters using unstandardized regression coefficients. Unstandardized coefficients provide estimates of changes in the dependent variable associated with a unit change in the explanatory variable (e.g., male versus female; a standard-deviation increase in motivation). Standardizing variables is useful in comparing the size of effects due to several variables measured on different scales within a sample. If the goal is to compare estimates across samples, one should use unstandardized estimates (Hox, 2010). Standardizing estimates is more complicated in multilevel modeling, however, due to the presence of variance components for outcomes at different levels of the data hierarchy. Because standardizing variables depends on their standard deviations, in multilevel modeling the analyst faces the issue of using Level 1 standard deviations and Level 2 standard deviations to compute standardized effects of predictors (Bloom, Hill, Black, & Lipsey, 2008).

There are a number of different ways to approach standardizing variables in multilevel modeling. Some software programs (e.g., Mplus and LISREL) provide a variety of different standardizations. For example, standardizing estimates to the within-group variance implies a focus on within-group relations without concern for how large or small the portion is of the within-group variance to the total variance. Mplus standardizes estimates within each level (which requires the between-group and within-group covariance matrices in order to construct accurate estimates). Other programs (e.g., HLM and SPSS MIXED) currently do not provide standardized solutions.

One can standardize all the indicators in the model first (e.g., by saving standardized estimates of each variable, which can be obtained using the Descriptives command in IBM SPSS). Essentially, this amounts to modeling with all z-scores. This approach succeeds in putting the variables in the same metric, but it does have a few drawbacks. First, it does not completely resolve the issue about the “proper” size of the coefficients in relation to an assumption about decomposing variance in the model. Second, as Hox (2002) notes, this type of linear transformation of the fixed coefficients in a model through standardizing also has the effect of changing estimates of the random part of the model (i.e., the model’s variance components). If one has random slopes

in the model, standardizing the estimates can alter the calculation of the variance components at each level. Standardizing tends to reduce variability at different levels, which is often important in the focus of the multilevel analysis—and maybe even the theory underlying the relationships in a proposed model. Third, in the process of standardizing things, we usually find that  $p$  values can be changed slightly. So for example, something significant in a raw metric may or may not be significant when everything is standardized. Interactions are more challenging to standardize since standardizing before the analysis can change the size of the interaction, the model's variance components, and also significance levels of variables (Hox, 2010; Preacher, 2003).

We have found that various assumptions about standardizing variables (i.e., standardizing with respect to within-group variance only, the between-group variance only, within each level of the data hierarchy, or with respect to total variance) can lead to very different sets of coefficients and interpretations of the results, such that it can be difficult to determine what each standardization means (Heck & Thomas, 2009). It is important to understand the assumptions and possible consequences of providing standardized solutions. Despite these cautions, we do favor providing a standardized solution in many instances since an audience may well understand the effect of a predictor as producing a 0.6 standard deviation ( $SD$ ) increase in the dependent variable. In such cases, however, we recommend examining both unstandardized and standardized results and then comparing the pattern of significant and nonsignificant effects of predictors and variance components for consistency. These choices about the presentation of empirical results need to be considered in terms of both the technical and substantive issues they raise, as well as with respect to the overall goals of the research study.

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## Missing Data

It is often the case that the weakest point of a study is the quality of the data that can be brought to bear on the research problem. Missing data can be a problem in multilevel applications, depending on the sampling design underlying the data set, the extent to which the data are missing at each level, and whether or not the data can be assumed to be missing at random. The more one can find out about why the data are “as they are,” the more one can develop a case about the patterns of missing data, as well as a rationale about why the pattern may or may not matter. In some modeling situations, there may be considerable missing data.

Users should consider data preparation and data analysis as two separate steps. In preparing the data for analysis, it is often useful first to determine the amount of missing data present, as well as the number of missing data patterns (e.g., For which variables do missing values occur? Are there specific patterns of missing data?). It is important to keep in mind that the reality is that there is no real way to get data that are missing back (short of actually following up with subjects in a study). In a sense, then, we are always dealing with the problem of missing information to some extent when we use actual data. The quality of our analysis depends on assumptions we make about the patterns of missing responses present and what is reasonable to conclude about those patterns in relation to the study's design (e.g., experimental or quasi-experimental and survey) and data collection (e.g., cross-sectional and longitudinal).

What we do about the missing data we have becomes a more pressing concern. There are a number of available options for dealing with missing data. It helps to know what the defaults and options are for handling missing data in the software programs that we are considering to use in each given research situation. Typically used approaches such as listwise or pairwise deletion, mean substitution, or simple imputation using regression-based techniques (e.g., estimating outcomes with dummy-coded missing data flags to determine whether there were differences in outcomes associated with individuals with missing versus complete data) lead to biased results in most situations (Acock, 2005; Allison, 2002; Larsen, 2011; Peugh & Enders, 2004). Currently, for single-level models, acceptable approaches include full information maximum likelihood (FIML) estimation with the partial data included and multiple imputation (MI) of plausible

values (Asparouhov, 2006; Enders, 2011; Enders & Bandalos, 2001; Peugh & Enders, 2004). Additionally, as Enders notes, inverse probability weighting methods are also gaining attention in the statistics literature (e.g., Little & Rubin, 2002; Robins & Rotnitzky, 1995). It is important to note that few studies have examined the use of these commonly accepted, single-level approaches with multilevel data structures (e.g., see Larsen, 2011; van Buuren, 2011).

Handling missing data in an appropriate manner depends on one's knowledge of the data set and why particular values may be missing. In general, there are three main types of missing data (see Hox, 2010; or Little & Rubin, 2002, for further discussion). Rubin (1976) introduced the notion of the distribution of "missingness" as a way to classify the probability conditions under which missing data can be ignored. These include data that are missing completely at random (MCAR), missing at random (MAR), and nonignorable missing (NIM), which is also referred to as missing not at random (MNAR). For data to be MCAR, strong assumptions must hold. The missing data on a given outcome should be unrelated to either a subject's standing on the outcome or to other observed data or unobserved (missing) data in the analysis. This is the assumption underlying listwise deletion. Typically, this assumption is only met when the data are missing by design, as in the situation where we draw a random sample of the studied population.

In contrast, if the probability of data being missing on the outcome is related to missing data on a covariate, but not to subjects' standing on the outcome, then the data are MAR (Little & Rubin, 2002). It is reasonable to assume there will be some relationships between data that are missing on two or more variables in a study. For example, if students have missing data on attendance for a number of different reasons such as changing schools, having a health condition, or living in a particular region, they may also have missing data on math outcomes. These other variables provide a mechanism for explaining the missing values present. The MAR assumption underlies the MI and FIML approaches to dealing with missing data. Although it is often reasonable to assume that data are MAR, under some circumstances, this assumption may not hold (Enders, 2011).

More problematic is the situation where the probability of missing data on the outcome is related to standing on the outcome for individuals with the *same* value on a covariate. For example, if there is more missing low-math data than missing average- and high-math data among students with the same attendance level, then the data are NIM. Suppose we have 500 students who take a test, but 150 have missing data, and these missing individuals also tend to have relatively high absenteeism (e.g., 20 or more days). Then we might have to acknowledge there is some bias present. Perhaps two thirds of the missing data are students who are in the high-absenteeism group and low-math achievement group. It will now be hard to argue that the missing data on the student absenteeism predictor will not affect the estimation of students' math test scores for the population. This latter type of missing data can produce more bias for model estimation than either of the other situations because the missing data on math achievement are related to actual values of individual achievement for those subjects who do not take the test. We would prefer to be able to say the pattern of missing data on student outcomes is relatively similar for students with high, average, and low absenteeism. This would then indicate data that are MAR.

Other techniques have been developed for data that are NIM. Enders (2011) demonstrates the usefulness of two of the NIM approaches for longitudinal data (i.e., selection models and pattern mixture models) and demonstrates their use on a real data set. More specifically, selection models for longitudinal data combine a substantive model (i.e., a growth curve model) with a set of regression equations that predict missingness, while a pattern mixture analysis stratifies the sample into subgroups that share the same missing data pattern and estimates a growth model separately within each pattern. Interested readers can consult Hedeker and Gibbons (1997, 2006) for one example of a pattern mixture-modeling approach that uses the missing data pattern (represented by one or more dummy variables) as a predictor in the growth model. Their approach can be estimated with standard mixed-modeling procedures (e.g., the MIXED procedures in SPSS and SAS). Choosing an approach for handling missing data (whether assuming MAR or NIM) is a

matter of choosing among competing assumptions. As Enders (2011) concludes, “Researchers should choose a model with the most defensible set of assumptions, and they should provide a logical argument that supports this choice” (p. 15).

As we have cautioned, IBM SPSS is limited in its ability to deal with various patterns of missing data. As a default, the program uses listwise deletion of cases when there is any missing data. This means any individual with data missing on any variable will be dropped from the analysis. As a first step, we suggest examining the amount of missing data on each variable. Even with 5% or less per variable, in some situations, listwise deletion can result in a tremendous loss of data and biased parameter estimation. IBM SPSS does provide a number of options for examining missing data. The standard solutions provided in most routines are listwise, pairwise, or mean substitution. In most situations, however, none of these would be considered as optimal (or acceptable) approaches (Enders & Bandalos, 2001; Peugh & Enders, 2004). For example, listwise deletion leads to inflated standard errors when the data are MCAR and biased parameter estimates when the data are MAR (Allison, 2002; Larsen, 2011). Mean substitution treats individuals with missing data as if they were on the “grand mean” (MCAR), which is also likely to introduce bias in most situations (e.g., by reducing variance). The IBM SPSS Base Statistics program also provides a basic program that provides several replacement methods (e.g., series mean, mean of nearby points, median of nearby points, and linear interpolation). We also note that although “user-missing” values can be specified in IBM SPSS, this approach is typically used for categorical responses, where some possible responses are coded as missing (e.g., “not applicable” in survey questions). If these user-defined missing values are included in the analysis, however, they will also bias parameter estimates.

For users who have access to the SPSS multiple imputation (MI) data module, patterns of missing data can first be identified, and then plausible values can be imputed using the expectation maximization (EM) algorithm. EM is a common method for obtaining ML estimates with incomplete data that has been shown to reduce bias due to missing data (Peugh & Enders, 2004). Obtaining estimates involves an iterative, two-step process where missing values are first imputed and then a covariance matrix and mean vector are estimated. This repeats until the difference between covariance matrices from adjacent iterations differs by a trivial amount (see Peugh and Enders, 2004, for further discussion). The imputed data sets can be saved as separate data sets and then analyzed. It is often the case, for example, that even with 25–35% missing, one can impute plausible values into a number of data sets and do a reasonable job of creating a complete data set with the missing values given random values.

One of the advantages of this approach is that other variables can also be used to supply information about missing data, but they need not be included in the actual model estimation. This approach to missing data is recommended when the assumption that the data are MAR is plausible. The analyst can generate a relatively large number of imputed data sets (Bodner, 2008) and then analyze the complete data sets and report the mean estimates and standard errors across several imputations. The values imputed through MI represent draws from a distribution; in other words, they inherently contain some variation. This parameter variation across multiple imputations is important for creating reasonable distributions of plausible values for variables with missing values. If we assume some degree of normality, we can average the parameter estimates over the imputed data sets. Our practical experience with MI approaches suggests they do pretty well at estimating the total data set where missing values are randomly dispersed across a sizable number of individuals (100–200 or more) found in most published studies. It is important to keep in mind, however, that the MI approach as implemented in SPSS does not assume missing values on *group-level* (Level 2) variables.

For multilevel data, there is less guidance available from previous research (e.g., Daniels & Hogan, 2008; Larsen, 2011; van Buuren, 2011). Larsen recently conducted a study comparing MI and FIML approaches in situations where there were individuals nested within groups. Both approaches were relatively similar in handling Level 1 estimates under the different conditions

examined. More importantly, however, as missing data increased at Level 2, the estimates of the Level 2 predictor from imputed data sets displayed increased parameter bias and decreased standard errors compared to the estimates predicted from the full data set. For Level 2 estimation, Larsen found that FIML estimation with the missing data performed much better than the MI approach. This is because the MI procedure used in his study did not account for random effects. More specifically, the student-level data (Level 1) were assumed to be “randomly sampled” from the same population (i.e., in this case, a classroom), rather than to come from different classrooms (Larsen, 2011).

Van Buuren (2011) provides an introduction to a number of different missing data situations encountered with multilevel data. Where data are missing for Level 2 predictors (e.g., as for a school covariate), for programs defaulting to listwise deletion such as IBM SPSS, this will result in losing all the individuals within those units. These individuals may not have any missing data on the outcome or the Level 1 predictors. This may also complicate the estimate of group-level effects (van Buuren, 2011). We reiterate that the sampling frame through which the data were generated may have an impact on assumptions we make about the distribution of the data at each level. This implies that the nature of missing data at Level 2 in relation to the manner in which the units were selected—whether units themselves were randomly sampled from a population of units or were just an unspecified “collection” of available units—can further complicate the interpretation of the Level 1 results. As van Buuren (2011) notes, although ML methods are quite good at estimating repeated measures values of  $Y$  in longitudinal studies, no generally acceptable approach has been developed for handling missing values on Level 1 and Level 2 predictors (i.e., since the data are assumed to be MAR). If MAR is correct, van Buuren cautions that the variables governing the probability of the missing data should be included in the analysis, for example, in order not to bias the estimate of a treatment effect.

Much of our discussion about missing data suggests that dealing with missing data is not so much about how much missing data is allowable, but rather how to develop a process to deal with the missing data. We favor a strategy of triangulating our results with different approaches that are currently recommended for examining missing data. One possible approach is to do something like the following. First, one can try running the model using listwise deletion (which assumes MCAR). This data set will likely be considerably smaller than the “partially complete” data, but it gives the analyst a baseline view (albeit likely a biased one) for comparing subsequent results. With MIXED, the results with listwise deletion should match the results of the existing (partially complete) data set since the variables with missing values are listwise deleted, unless the data are vertically arranged as in a growth model.

Second, if there is not too much missing data per variable, the listwise results can be compared against a number of complete data sets generated using an MI program, which can be applied to hierarchical data structures (e.g., Mplus or HLM). Some have discussed the use of single-level MI, which ignores the grouping structure in multilevel data; however, this approach may underestimate standard errors (Cheung, 2007; Gibson & Olejnik, 2003). In contrast, Zhang (2005) found it worked reasonably well with missing data up to about 30%. As van Buuren (2011) cautions, there is no consensus yet, and further work is needed in this area. Others have suggested using an individual and group data set to perform separate imputations, while using information from one level to inform the other (Gelman & Hill, 2007; Petrin, 2006).

Analysts may wish to keep in mind the cautions we have mentioned about estimating plausible values when individuals are nested within groups. There are a number of sources that can be consulted for dealing with missing data under MAR and NIM in growth-modeling studies (e.g., see Enders, 2011; Muthén & Muthén, 1998–2006). We also note that van Buuren (2011) examined several types of missing multilevel data situations and found the multilevel multiple imputation used (i.e., which generated multiple imputations from prior distributions of the parameters using the Gibbs sampler) worked generally the best of several approaches (e.g., listwise, single-level MI, and MI with separate groups), but it was not optimal in all situations with

respect to recovering true values by 95% confidence intervals across different cluster sizes and numbers of individuals within clusters.

Third, if the analyst has access to an SEM program (or special multilevel software such as HLM) that will perform multilevel analyses, she or he can also try estimating the model with FIML with the cases with partial data included in the analysis and then comparing these results with the other approaches. If sample weights are available in existing data sets, users should check whether they include adjustments for nonresponse. If so, making use of sample weights at two levels will facilitate accurate estimation, but this option is currently only available in a limited number of multilevel software with which we are familiar. We contrast some of these problems briefly in the next section with two short examples where missing data are encountered.

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## Missing Data at Level 2

It is clear that more work needs to be done on the use of multiple imputation procedures for hierarchical data structures (Larsen, 2011; van Buuren, 2011). We caution in this situation that users should be careful about imputing values for Level 2 variables within the larger data set that contains missing values. This is because the program will create *different* “group” values on Level 2 variables for individuals in the same group. This happens because the imputation routine will borrow information from similar individuals who are *not* in the same group. In this case, if data are missing at the group level (Level 2) for individuals in the data set, we must ensure that the same value is imputed for the group variable for all individuals within the group. We recommend that analysts first check the data for “stray” missing values for Level 2 variables for individuals at Level 1. Values for Level 2 covariates for individuals within each unit should be inserted where they are missing, so that no individuals will be dropped from the unit where that information is available for other subjects within the unit.

The bigger problem is where data on a Level 2 covariate is missing for all members of a particular unit. In two-level designs, the selection of students within a school cannot be considered as independent observations, since the students selected within the school will likely have some common characteristics (Organization for Economic Cooperation and Development, 2009). Assuming independence of observations (i.e., simple random sampling) simplifies analyses but generally tends to lead to mild to severe underestimation of standard errors (Kish, 1987). In Table 1.7 we provide a simple multilevel analysis to illustrate the potential effects of missing data at Level 2 on the analysis. In this case, we start with a complete sample of 1,000 students randomly selected from within 139 schools, which were randomly selected from some 180 schools in the database. We specified a two-level simple model with one predictor at each level and a random intercept.

The coefficients in column 1 of Table 1.7 suggest that community socioeconomic status (*CSES*), which is reverse coded, and *lowSES* are statistically significant ( $p < .05$ ) in explaining students' reading scores. In columns 2–5, we illustrate several possible approaches for dealing with missing data. In column 2, we eliminated data regarding school SES composition in 24 of the 139 schools (17.3%). We assume the school-level data are MAR. To simplify matters, at the individual level, there was no missing data. Because of the missing data on the Level 2 covariate, however, we lose nearly 30% of the Level 1 student data (i.e.,  $289/1,000 = 28.9\%$ ). In columns 3–5, we used FIML estimation or MI to deal with the missing Level 2 data.

In column 2, we can see the listwise results indicate that both *CSES* and *lowSES* affect students' reading scores ( $p > .05$ ); however, with nearly 20% missing data at Level 2 and a resulting 30% missing data at Level 1, the listwise analysis appears to be a bit further away from the original estimates than the other approaches we demonstrated in estimating both the *CSES* and *lowSES* parameters. We suspect this is due to the effect of the missing data on the school SES variable, which we can see may influence both Level 2 and Level 1 estimates. We note that listwise results will typically produce the largest errors in estimating the parameters (since the

**TABLE 1.7 Comparison of SPSS and Mplus Results**

Variables	SPSS Complete	SPSS Listwise	Mplus FIML	Mplus Impute (N = 5)	SPSS Impute (N = 5)
<i>Between Schools</i>					
Intercept	642.58*	641.86*	642.05*	641.88*	642.26*
CSES	-6.09*	-7.73*	-7.39*	-6.05*	-6.09*
<i>Within Schools</i>					
Low SES	-12.22*	-10.41*	-12.16*	-12.49*	-12.37*
<i>Variance Components</i>					
Level 2 Intercept	45.83	60.24	52.70	49.82	54.14
Residual	1,654.55*	1,627.85*	1,643.28*	1,645.73*	1,645.59*
<i>Sample Size</i>					
Level 1 Students	1,000.00	711.00	1,000.00	1,000.00	1,000.00
Level 2 Schools	139.00	115.00	139.00	139.00	139.00

\* $p < .01$ .

sample size may be severely reduced), as well as reduced power, which can both lead to errors in interpreting the results.

In column 3, we present the results of FIML estimation with missing data (estimated with Mplus), which appears to do a reasonable job of recapturing the original estimates in the complete data set presented in column 1. In addition to the intercept, the FIML approach accurately estimates *lowSES*. In contrast, however, the estimate for the *CSES* covariate (-7.39) is not as close to the estimate in column 1, since it was generated from nearly 20% missing data on the covariate at Level 2. In this specific example, we suspect this estimate is a bit stronger because the reverse-coded mean *CSES* of the missing schools was about 0.37 of a standard deviation higher than the set of schools with complete data on this covariate. What this means is that there were higher percentages of students participating in the federal free/reduced lunch program in those schools. This would overattend to the schools where there are lower percentages of low-income students in generating the school-level estimate.

In column 4, we report results using five imputed data sets at Level 2 with specialized multilevel software to estimate the missing data at that level (i.e., Mplus). We obtained these estimates by specifying our multilevel model in Mplus and then using the MI routine to impute missing values (assuming the data at Level 2 were MAR). The estimates in column 4 represent the averaged output from the five data imputations we conducted (Muthén & Muthén, 1998–2006). Mplus has this MI capability beginning in Version 6. We can see that the averaged results across five data imputations produce an estimate of the Level 2 fixed effect for *CSES* that is quite close to the actual estimate with the complete data.

Finally, in column 5, we present results from SPSS using the MI routine for our Level 2 school sample (estimated separately). We remind readers that if they attempt to impute plausible values using the MI routine in IBM SPSS without first separating the Level 2 variables from the larger database, the program will impute different “random” values for the group covariate for individuals within the same unit since it does not recognize “random effects” due to clustering. This, of course, would be impossible in terms of producing an accurate analysis of the effects of Level 2 predictors on the outcomes in a study since the definition of clustering within multilevel analysis is that all individuals within a given unit are assigned the same values on Level 2 predictors. Imputing values into the Level 2 data set might be viable if we know the process through which units were selected at Level 2 (e.g., simple random sample or stratified random sample).

A common problem, however, is that in many multilevel studies the number of Level 2 units available tends to be considerably smaller and, therefore, less likely to represent a “randomly sampled” population of units. We therefore need to keep the likely *distribution* of Level 2 units in mind since our assumption about the nature of the Level 2 units will certainly affect estimates that might be generated. If we can assume the data at Level 2 are MCAR (or more likely MAR), we may be able to provide reasonable estimates for missing data on one or more covariates.

In order to impute plausible values for the Level 2 covariate, we used existing information about *CSES* drawn from the 115 schools with existing data on this variable, as well as information from aggregated unit means for each unit that we created from the complete Level 1 data set. More specifically, we aggregated student data on reading scores and individual SES to generate means on these variables for each unit. Using Level 1 information about each unit helped in generating better “plausible” values for community SES at Level 2 in those units that were missing data. It is generally important to include information about the dependent variable in the imputation model; otherwise, the imputed values will not have the same strength of relationship to the dependent variable that the observed values do.

We reiterate the point that the MI approach has the advantage of incorporating some variability in the Level 2 estimates that are saved into separate data sets. Readers should keep in mind that imputing values at Level 2 may not adequately deal with the nature of the random effects present. Again, this relates back to assumptions about the sampling process through which units and individuals were selected. Even in this simple case, we noted considerable variation in the size of the estimates, their standard errors, and their statistical significance. We suspect that in many situations, it is probably reasonable to consider the Level 2 units as comprising a random sample of a population (i.e., each unit has an equal probability of being selected), even if in practice this assumption may be violated, which can lead to some underestimation of sampling variance (Kish, 1987). Sample weights at Level 2 are created to deal with these types of selection probability issues, but they are not available in many multilevel data sets. Our practical experience with imputing plausible values suggests that this problem can become more important where there are small numbers of Level 2 units available for analysis (or where there are only a few individuals within each unit). For example, smaller samples will lead to greater imputation errors, which necessitate increasing the number of imputations conducted (OECD, 2009). Accurately estimating variance parameters typically takes much more data than estimating fixed effects. Using MI procedures in some fashion would certainly require imputing a sizable number of Level 2 sets of estimates (perhaps 30 or more) where there is 20% or so missing data (e.g., see Bodner, 2008; Larsen, 2011). This would provide a broader distribution of values. This might be a reasonable approach for Level 2 data, where there are a relatively large number of units in the study (as in our example) and the assumption of MAR can be made.

No definitive conclusion should be drawn from this simple illustration in Table 1.7. Our point is simply to suggest that missing data can have a considerable influence on the credibility of our modeling results. It is a problem that should be addressed in preparing the data for analysis. On a positive note, in this example, the estimates for the three fixed parameters and variance components in columns 2–5 all covered the 95% confidence intervals in the original set of estimates in column 1. From this preliminary analysis, if we were preparing the data for further analysis, we would likely conclude that the data are MAR instead of MCAR since the listwise results were different substantively. We might choose either the MI or FIML estimation with missing data as viable approaches to use in examining these data.

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### Missing Data in Vertical Format in IBM SPSS MIXED

As we noted in the previous section, at present IBM SPSS does not generally support FIML estimation in situations where there may be observations missing, as is found in typical SEM software programs. What this means is that the cases with missing values are simply dropped from

**TABLE 1.8 Vertical Data Format**

Subject	Time	Score
01	0	4
01	1	
01	2	6
02	0	5
02	1	8
02	2	
03	0	6
03	1	8
03	2	9
04	0	
04	1	
04	2	7

**TABLE 1.9 Model Dimension<sup>a</sup> Table**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	time	1		1		
Random Effects	Intercept	1	Variance Components	1	Subject	
Repeated Effects	time	3	Identity	1	Subject	4
Total		6		4		

<sup>a</sup> Dependent variable: score.

the analysis. Where the data set is vertically arranged (e.g., where a single individual may have repeated observations that comprise several rows in the data set), however, only that particular occasion will be dropped if there is a missing value present on the outcome. We also illustrate this situation in the next section. If covariates are missing, however, the subject will also be listwise deleted, which will lead to bias if the data are in fact MAR (Larsen, 2011).

Besides repeated measures data, arranging the outcome data vertically can also be useful in situations where an analyst may wish to examine several univariate outcomes (e.g., results on a reading, math, and language test). If there are considerable missing data on each outcome, treating the outcome as multivariate (i.e., with vertical arrangement of the data at Level 1) can result in keeping most of the missing data since only cases where data are missing on all outcomes will be dropped. We note that keeping participants with partial data is important for justifying the MAR assumption (Hox, 2010). We provide a simple illustration of retaining missing cases due to vertical arrangement in Table 1.8, where there are three observations per individual (e.g., three successive math scores) and different patterns of missing data for each particular individual. SPSS can handle different missing data patterns (i.e., missing on the first occasion, the second or third occasion, or various multiple occasions) and amounts of missing data. Some individuals

have no missing observations, some have missing data on one occasion, and some have missing data on two occasions. As long as  $Y$  is not missing on all occasions, the program will come up with an “estimated” growth over each time interval, as well as an initial status (intercept) estimate, even though the initial data point is missing for subject 4.

We provide the model dimension table from this model in Table 1.9. It shows that all four subjects are retained in the analysis. This can be important information for analysts concerning how many individuals in the data set are actually being included in the analysis.

---

## Design Effects, Sample Weights, and the Complex Samples Routine in IBM SPSS

When working with secondary data sets, applying sample weights is important to correct the analyses for features of the sampling design (e.g., probability of selection at multiple levels of the data hierarchy) and data collection problems (Thomas & Heck, 2001). Procedures for the selection of Level 2 units and individuals within those units can vary from being simple (e.g., simple random sampling at each level) to relatively complex. Weights may be available only at the individual level (Level 1) or the group level (Level 2), or they may be available at both levels. Currently, there are no commonly established procedures for applying weights in multilevel analyses, although a considerable number of different approaches have been proposed (e.g., Asparouhov, 2005, 2006; Grilli & Pratesi, 2004; Jia, Stokes, Harris, & Wang, 2011; Pfeffermann, Skinner, Holmes, Goldstein, & Rasbash, 1998; Stapleton, 2002).

The consideration of sample weights and design effects are vitally important in analyses using disproportionate sampling and multistage cluster samples. Disproportionate sampling will lead to samples that overrepresent certain segments of the populations of interest. Typically, this results from the researcher’s interest in including a sufficient number of subjects (or objects) from smaller but important subpopulations. Sampling members of such groups proportionally often results in too few sample members to allow meaningful analyses. We therefore oversample many groups to ensure sufficient numbers in the final sample. The result is that the analytic sample is not representative of the true populations since it has too many sample members from the oversampled groups. Sample weights, typically probability or frequency weights, are used to readjust the sample to be representative of the population from which it was drawn. Failure to use a sample weight in these instances can result in incorrect parameter estimates, biased in the direction of the oversampled members of the population (Thomas & Heck, 2001).

Disproportionate sampling is often found in multistage cluster samples. Cluster sampling is simply where the researcher first draws a sample at a higher level—organizations, for example—and then draws a sample of lower level units within each organization—employees, for example. The units at each level may or may not be drawn proportionate to their presence in the larger population. To illustrate this point, if one were to sample organizations, for some substantive reason related to the research purposes, it might be desirable to oversample rural organizations. If this were to occur, one would want to be sure to adjust for this at the organizational level by using a Level 2 (organizational) weight in the same fashion that the Level 1 individual weight discussed previously was used. Hence, there can be sampling weights for each level of the data, although we note that many currently available data sets do not include weights at multiple levels of the data hierarchy.

To the degree that the observations within each of the higher order clusters are more similar to each other, there will be a *design effect* present that biases the estimated standard errors downward. Because hypothesis tests are based on the ratio of the estimate to its standard error, having standard errors that are too small will lead to a greater propensity to commit a Type I error (i.e., falsely concluding that an effect is statistically significant when it is not) than if the sample were drawn through a simple random-sampling procedure. The design effect is defined as the ratio of the biased standard error to the standard error that would be estimated under a true random sample design. So, for example, if we know that the true standard error was 1.5, but the biased

standard error estimated from the data collected through the multistage cluster sample was 1.2, the calculated design effect would be  $1.5/1.2 = 1.25$ .

One standard measure of this within-unit (dis)similarity is the intraclass correlation. As we have noted earlier in the chapter, the ICC is the proportion of total variance in the outcome due to within-unit differences at higher levels. The higher the ICC, the larger will be the design effect. Hox (2010) notes that the ICC can be viewed as another way to think about the degree of correlation within clusters. An ICC of 0.165, which suggests that 16.5% of the variance in the individual-level outcome exists *between* clusters, could also be viewed as an indication that one might expect a within-cluster correlation of 0.165 between individuals. This conceptual connection between the ICC and within-cluster correlation is important in understanding design effects. In short, the greater the between-cluster variance in the individual-level outcome, the more homogenous will be the individual observations within each of the clusters. To the extent that there exist within-cluster similarities, estimates of the Level 1 variances will be smaller than they would be if the sample were collected through a simple random sample design (where such clustering would be irrelevant to the variance structure). The implication central to our interests is that ignoring the clustering that is part of the sample design will yield downwardly biased estimates of the standard error.

The last several versions of SPSS have made available a COMPLEX SAMPLES module that allows the user to incorporate design information into the model to adjust for the design effects described previously. This module produces results for single-level models incorporating design information and sample weights. As such, the parameter estimates are adjusted for both disproportionate sampling and cluster sampling. In the single-level context, this is the appropriate way to analyze data collected through complex sample designs. In this type of approach, similarities among individuals due to clustering are treated as “noise,” which is adjusted out of the analysis, rather than considered as the focus of the analysis.

Multilevel models, by design, capitalize on the clustered nature of data, and it is quite common to see these models used with large-scale survey data that have been collected through complex sample designs. The same cautions outlined previously apply to estimates produced using various forms of multilevel models. Although multilevel models capitalize on the clustered nature of the data, they do nothing to address disproportionate sampling, and, without proper weighting, they will produce incorrect parameter estimates. Sample weights are often essential to generate accurate estimates.

Weighting for unequal selection is relatively well established for single-level analyses. The COMPLEX SAMPLES module allows adjustments to be made for sample design effects (which can include clustering)—but maintains a single-level analysis after adjustment for features of the sampling scheme. In this type of approach, similarities among individuals due to clustering are treated as unwanted variance that is adjusted out of the analysis. In contrast, standard two-level models can result from two-stage cluster sampling designs (rather than the basic simple random or stratified samples that comprise the majority of single-level analyses). One document that does discuss multilevel models as complex sampling models within IBM SPSS is the *PISA Data Analysis Manual* (OECD, 2009). Because SPSS cannot at present deal with sample weights at two-level results from these types of sampling designs, it can give a preliminary indication of relationships in models where sample weights exist but should not be relied on to provide final, unbiased estimates, even when using plausible values as the dependent variable (OECD, 2009).

In contrast to weighting in single-level analyses, developing weighted analyses in the multilevel context presents a number of more complicated challenges and limitations. Research in this area is ongoing, and important advances have been made over the past 10 years. Most multilevel software programs now include one or more weighting options. Several programs with which we are familiar (HLM 7, LISREL 8.8, and Mplus 7) incorporate design weights that combine information on clustering, the degree of intraclass correlation, and disproportionate sampling

to create a set of scaled weights that will produce accurate estimates at each level of analysis. Although IBM SPSS allows for the incorporation of simple sample weights (an approximate frequency weight) in the MIXED and GENLINUX routines, the current version does not enable a scaling adjustment that accommodates the effects of clustering in the sample design. As we noted earlier in this chapter, this is an important limitation of the IBM SPSS program for conducting multilevel analyses.

A number of factors can influence the estimation of model parameters. These factors include the method of scaling the weights at Level 1 (i.e., how weights are scaled within the clusters), the size of the clusters, the relative invariance of the selection method applied to clusters (referred to as informativeness), the presence of missing data, and the intraclass correlation (Asparouhov, 2006). The scaling of Level 1 sample weights is very important in the multilevel context, helping to improve efficiency and decrease bias in estimation (Pfefferman et al., 1998; Skinner, 2005). Asparouhov explains that the scaling of the weights at Level 1 involves multiplying the weights by a scaling constant, so that the sum of the weights is equal to some kind of characteristic of the cluster (e.g., cluster size).

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## An Example Using Multilevel Weights

We provide one simple example of a comparison between results we obtained with Mplus, which has the capability of incorporating Level 1 and Level 2 weights into multilevel analyses, and the unweighted estimates obtained through using MIXED. In the example, there are 5,952 students nested in 185 schools with a continuous outcome. In Table 1.10, we first provide unweighted estimates for the within- and between-group variables using IBM SPSS MIXED. We next provide unweighted estimates and weighted estimates using Mplus. For Mplus, Level 1 weights were scaled within Level 2 units such that the sum of these weights equals the size of the respective Level 2 unit (Asparouhov, 2006).

The results suggest that providing proper weights can affect both the size of the estimated coefficients as well as the calculation of the standard errors at both Level 1 and Level 2. First, the estimates of average school achievement are quite different in the unweighted versus weighted estimates. For example, the unweighted SPSS and Mplus estimates of the adjusted level of achievement are very similar (543.02 and 542.94, respectively). The weighted estimates, which take into consideration the proper representation of the schools in the stratified sample drawn from the population, are considerably lower at 515.03 than the unweighted estimates.

Second, at the school level, we notice one difference in the pattern of hypothesis testing. Because hypothesis tests are based on the ratio of the estimate to its standard error, on many occasions we can note differences in significance testing of estimates in unweighted versus weighted solutions. In the unweighted solutions, school SES and school type (i.e., private or public) are the only significant predictors of math scores. In the weighted solution, however, the hypothesis test for curriculum orientation (*academic*) is significant (15.60,  $p < .05$ ), but it is not in the two unweighted solutions provided. Regarding the slope model, we can see that students who receive outside tutoring have significantly lower math scores in both the unweighted and weighted solutions. There is a considerable difference in the size of the randomly varying slope (regarding students who receive outside tutoring in math). More specifically, the unweighted estimates are approximately  $-17.4$  versus the weighted estimate of  $-24.0$ . Similarly, the cross\*level interaction (*tutoring\*School SES*) is also different—that is, approximately 31.0 in the unweighted sample and 42.6 in the weighted sample. These results suggest that students who are likely to get tutoring in high SES school settings are more highly achieving than their peers in schools at the grand mean of school achievement. For example, at 1 *SD* above the grand mean, the advantage would be about 18.7 points in the standardized math test ( $-23.98 + 42.63 = 18.65$ ).

Finally, we note considerable differences in calculating the variance components in the weighted and unweighted models in Table 1.10. Despite these differences, however, in all three

**TABLE 1.10 Unweighted and Weighted Level 1 and Level 2 Estimates Explaining Math**

	IBM SPSS		Mplus			
	Unweighted Estimates	SE	Unweighted Estimates	SE	Weighted Estimates	SE
<i>School Model</i>						
Intercept	543.02**	7.97	542.94**	7.56	515.03**	7.75
SchSES	141.80**	9.95	141.82**	9.88	131.36**	11.87
Public	-57.08**	7.18	-57.11**	6.75	-55.57**	6.91
Academic	10.25	7.81	10.41	7.25	15.60*	7.76
City	-2.37	6.94	-2.30	7.06	5.23	8.82
Large City	6.93	8.25	6.82	8.03	8.40	9.23
<i>Individual</i>						
SES	7.75**	1.33	7.76**	1.54	6.87**	2.08
Female	-16.40**	1.81	-16.42**	1.96	-14.04**	2.66
<i>Slope Model</i>						
Tutoring	-17.42**	2.87	-17.41**	2.96	-23.98**	4.58
Tutoring*SchSES	31.02**	7.72	31.10**	7.56	42.63**	8.40
<i>Random Effects</i>						
Residual	3,693.79**	69.59	3,692.37**	89.82	3,849.48**	128.24
Level 2(I)	1,466.58**	168.27	1,413.14**	166.00	1,360.03**	195.40
Level 2(S)	173.09	120.46	158.05	109.05	284.20	188.17

\* $p < .05$ ; \*\* $p < .01$ .

models, after accounting for school SES, we note there is not significant random variance in slopes left to explain across schools ( $p > .05$ ). We emphasize that no definitive conclusions should be drawn from this one simple comparison of unweighted and weighted multilevel results. We provide these results only to make the point that using sample weights, and using them correctly, does make a difference in the accuracy of the estimates obtained and certainly can affect the associated hypothesis tests in multilevel analyses.

Although many of the software programs used for estimating multilevel models enable the appropriate scaling, IBM SPSS does not yet include this feature. If sample weighting is essential to the analysis, it will likely be better currently to use another of the available programs or to revert to a single-level formulation within SPSS through its COMPLEX SAMPLES module. More specifically, as Asparouhov (2006) suggests, if sampling weights present in a secondary data set are designed for a single-level analysis, it may be best to stick with that type of design and conduct a single-level analysis designed for stratified- and cluster-sampling designs. Asparouhov provides two contrasting situations illustrating this point. First, he suggests that when weights are only present at Level 2 (i.e., where clusters have been sampled with unequal probability), we can identify this situation as being within the framework of single-level weighted modeling, and methods available for single-level weighted analysis can be applied with consistent estimations regardless of the size of the clusters. Although the model is multilevel, the nature of the weighting is not. Of course, if sample weights are also provided at Level 1, this will change. Second, he cautions that the situation is different when weights are only provided at Level 1, as the unequal probability of selection is applied to dependent units, and, therefore, the assumptions of the single-level method of analysis will be violated.

The bottom line is that if the single-level sample weights cannot be properly scaled to the multilevel context, it may be better to use the single-level approach. This threat may be more

severe when estimating models with categorical outcomes (Rabe-Hesketh & Skrondal, 2006). We are hopeful that the application of multilevel sampling weights will be included in future versions of the software. In the interim, we call attention to recent work by Chantala, Blanchette, and Suchinindran (2011) providing SAS and Stata routines to generate scaled weights that could be imported into other multilevel software programs (such as IBM SPSS).

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## Summary

In this chapter, we have developed a context and rationale for the use of multilevel models in the social and behavioral sciences. The use of multilevel analysis can add substantive information about how processes unfold at various levels of a data hierarchy. We suggested that multilevel techniques support the specification of more complex theoretical relationships than is possible using traditional single-level regression analyses. Analytic approaches that can be used to model complex relationships have greatly expanded over the past couple of decades, and these analytic alternatives allow us to investigate social processes in more theoretically and methodologically appropriate ways. Substantive progress in a field is often achieved when headway occurs simultaneously on conceptual and methodological fronts.

One attractive feature of SPSS MIXED is that it is not limited in terms of the number of levels in a nested or cross-classified data structure that can be analyzed simultaneously. The ability to specify multiple RANDOM (i.e., randomly varying intercept and slopes) commands at successive levels of a data hierarchy facilitates investigating a variety of multilevel models that are difficult, or not currently possible, to estimate optimally in other software packages. As with most current software programs, however, adding levels to examine in a data hierarchy, randomly varying parameters, and cross-level interactions can greatly increase the amount of time it takes to produce a solution (i.e., from seconds to several hours) and will require increasingly large amounts of memory and disk space as models become more complex.

We see another advantage in MIXED concerning the easy manner in which various types of covariance structures can be specified at different levels of the data hierarchy. Our comparison of IBM SPSS MIXED with other multilevel software suggests that the program produces results substantively consistent with other programs, given similar model specification. Given our assessment of these limitations and several advantages, we feel that using IBM SPSS to investigate multilevel and longitudinal models is a useful way in which to introduce researchers to the uses and benefits of multilevel modeling since it takes immediate advantage of a software program they are likely to have encountered already in their quantitative preparation.

In the next chapter, we take care of a bit of housekeeping by providing an overview of some important data management techniques. Arranging the data for analysis in IBM SPSS is fairly straightforward. We provide readers with a few essential steps necessary to put their data sets in proper order for analysis using MIXED.

## CHAPTER 2

# Preparing and Examining the Data for Multilevel Analyses

Essential to any type of analysis is the organization and vetting of the data that will be analyzed. In this chapter, we identify a number of practical and substantive issues associated with preparing data for analysis in IBM SPSS MIXED and assessing the adequacy of those data for a variety of multilevel analyses.

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### Data Requirements

In this workbook, we deal exclusively with multilevel models using continuous-level outcomes. While other multilevel statistical programs including IBM SPSS can accommodate binary or ordinal outcomes, MIXED is restricted to outcomes measured on a continuous scale. Although we deal only with continuously measured outcomes, predictors can be continuous, ordinal, or dichotomous.

The sample sizes we employ throughout our examples are large at each level of the analysis. The variables used may come from a variety of different sources, many of which are specific to a particular level of analysis. One might, for example, draw on student-level attitudinal, behavioral, or performance data from national surveys such as the *National Educational Longitudinal Study of 1988* (Curtin, Ingels, Wu, & Heuer, 2002). If an objective were to understand the effects of school settings on these individual characteristics, we might assemble school-level data drawing on information from the Common Core of Data (Sable & Noel, 2008). School-level data might include school size, demographic composition, financial characteristics such as state dollars per enrolled student, on-time progression or graduation rates, teacher and administrative numbers, and the like. To carry this to a third level, we could draw on U.S. Census data to define characteristics of the school districts in which the schools at Level 2 were located (e.g., household income, number of people in the household, their levels of education, etc.).

We will show that there are many variants on this modeling framework. We might, for example, want to understand change in some outcome over time. In such an instance, we might conceptualize time points across which we presume change to occur within students who could, in turn, be nested within schools, and so on. However the nesting is conceptualized, each level of analysis will have its own set of variables defining features of the units being measured at that level. While conceptualizing data at discrete levels of the hierarchy may be relatively straightforward, organizing the data set requires an understanding of how the data need to be arranged to represent that hierarchical conceptualization correctly. In the next section, we outline the main organizational features of data sets that can be used in a multilevel analysis. We return to data requirements in more detail in a subsequent section.

## File Layout

We described in the previous chapter an important difference between the single-equation and multiple-equation approaches to estimating multilevel models. The multiple-equation approach (e.g., used in HLM) requires a separate data set for each level of data being analyzed. This can make conceptualization of the different levels clear. If Level 1 consisted of 6,871 students, for example, the Level 1 file would contain student data, including information about higher order group membership (e.g., the classroom or school membership for each of the 6,871 students in the file).

Figure 2.1 shows what such a Level 1 file might look like based on the data set used in Chapter 3. In this particular example (*ch2level-1data.sav*), we have included a student identity (ID) variable (*id*), a school ID variable (*schcode*), and three variables describing student characteristics, gender (*female*), socioeconomic status on a normalized scale (*ses*), and a raw math score (*math*).

If Level 2 in the analysis consisted of schools of which students were members, the Level 2 data set would contain all information about those 419 schools, including a unique school identifier, and perhaps aggregated data from the Level 1 student file given previously.

The Level 2 file (*ch2level-2data.sav*) shown in Figure 2.2 contains a unique school identifier (*schcode*) and two variables describing the characteristics of the school, average socioeconomic status (*ses\_mean*) and the proportion of students planning to attend a 4-year college (*per4yrC*). The data sets are linked through a group-level identifier (in this case, *schcode*) during the multilevel analysis.

In contrast, the single-level approach makes use of one file that combines data from each level (Figure 2.3). In the univariate multilevel model, the file will consist of one record for each Level 1 unit. Values for variables from higher levels will be constant within groups. For example, in a data set (*ch2level-1&2data.sav*) with 6,871 students from 419 schools, there would be a single file of 6,871 records. The values for the student-level variables would vary across all 6,871 students. However, values on the school-level variables would be constant within each of the 419 schools—that is, students within each school would all have the same value on each of the school-level variables.

	schcode	id	female	ses	math	per4yrC	var	val
1	1	6701	1	.59	47.14	.08		
2	1	6702	1	.30	63.61	.08		
3	1	6703	1	-.54	57.71	.08		
4	1	6704	0	-.85	53.90	.08		
5	1	6705	0	.00	58.01	.08		
6	1	6706	0	-.11	59.87	.08		
7	1	6707	0	-.33	62.56	.08		
8	1	6708	1	-.89	47.01	.08		
9	1	6709	0	.21	72.42	.08		
10	1	6710	1	-.34	65.84	.08		
11	1	6711	0	-.17	57.34	.08		
12	1	6712	1	-1.07	62.56	.08		
13	2	3703	0	-.11	61.95	1.00		
14	2	3704	0	1.28	70.22	1.00		
15	2	3705	0	1.06	58.78	1.00		
16	2	3706	0	.80	65.54	1.00		
17	2	3707	0	.73	59.77	1.00		
18	2	3708	0	.13	64.07	1.00		
19	2	3709	0	.68	61.95	1.00		
20	2	3710	0	.92	66.83	1.00		

**FIGURE 2.1** A Level 1 data file (multiple-equation approach,  $N = 6,871$ ).

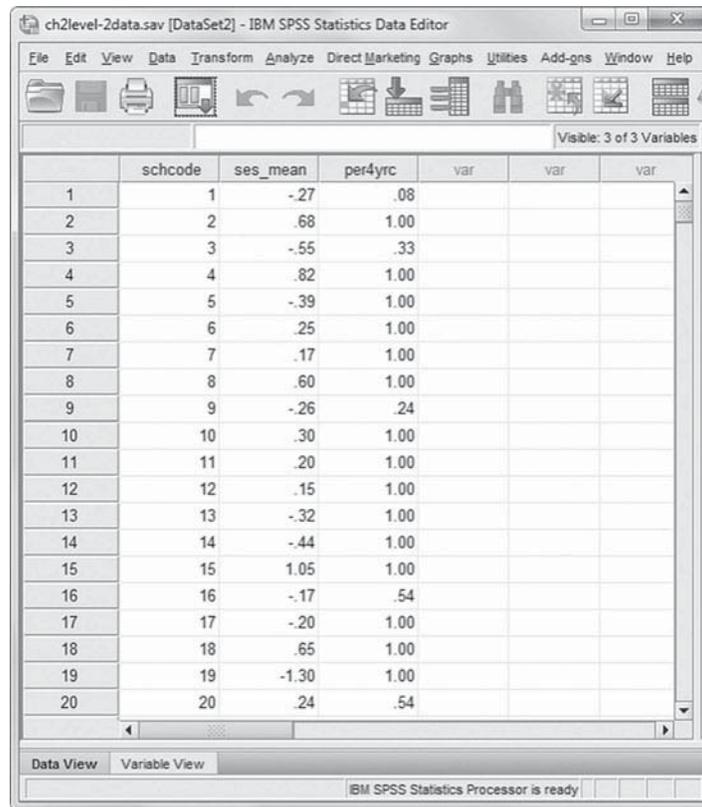


FIGURE 2.2 A Level 2 data file (multiple-equation approach,  $N = 419$ ).

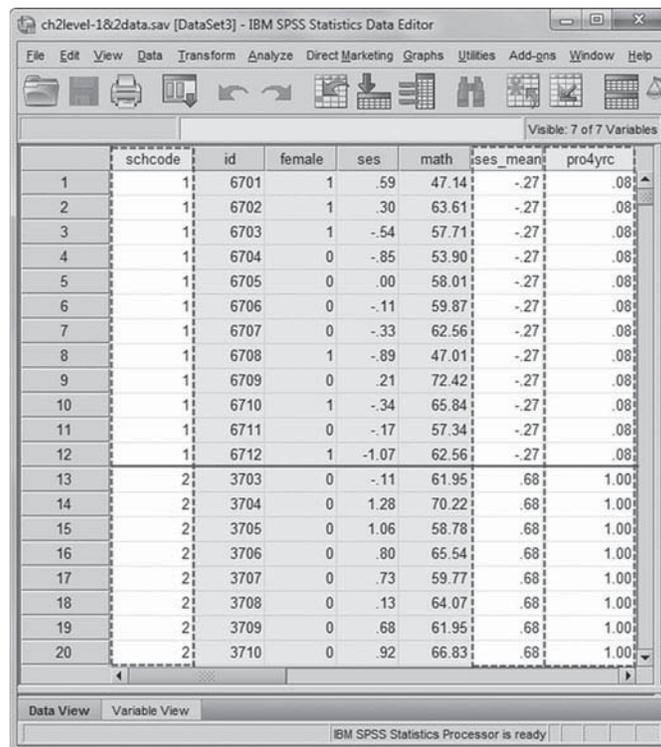


FIGURE 2.3 Combined multilevel data file (single-equation approach,  $N = 6,871$ ).

IBM SPSS MIXED uses the single-equation approach and one omnibus file containing data on each level of the analysis. In the sections that follow, we provide an overview of some of the data management steps within IBM SPSS that will help you organize and prepare your data and files for use within the MIXED routine.

## Getting Familiar with Basic IBM SPSS Data Commands

Organizing and managing the data at various levels is accomplished through five basic IBM SPSS procedures. There are, of course, many other procedures that can be used to modify a data set, but we feel that these are primary to the data management tasks associated with organizing files for multi-level analyses within IBM SPSS. We will have much more to say about the IBM SPSS commands and the menu system itself in the chapters that follow. Here, however, we wish only to introduce these commands and a few principles of data management that we think will prove helpful for getting the most out of the workbook. The five primary procedures are (in order of importance) the following:

1. **RECODE:** Changes, rearranges, or consolidates the values of an existing variable.
2. **COMPUTE:** Creates new numeric variables or modifies the values of existing string or numeric variables.
3. **MATCH FILES:** Combines variables from IBM SPSS-format data files.
4. **AGGREGATE:** Aggregates groups of cases in the active data set into single cases and creates a new aggregated file or creates new variables in the active data set that contain aggregated data. The values of one or more variables in the active data set define the case groups.
5. **VARSTOCASES:** Restructures complex data structures (i.e., in which information about a variable is stored in more than one column) into a data file in which those measurements are organized into separate rows of a single column.

In this section, we will build a multilevel data set using each of the primary commands identified previously.

The data set we use here (*ch2multivarML1.sav*) is based on the example used in Chapter 5 but is modified to exclude missing data. The data set contains three raw test scores taken over time (*test1*, *test2*, and *test3*), dichotomous indicators of teacher effectiveness (*effective*) and gender (*female*), a continuous variable capturing the number of Advanced Placement courses a student has taken (*courses*), and a continuous measure of family socioeconomic status (*ses*). There is also an identifier for students (*id*) and for the schools in which they are enrolled (*nschcode*). The descriptive statistics in Table 2.1 show that there are 8,335 records in the data set. Each record represents a single student. Figure 2.4 displays a partial view of the data structure.

**TABLE 2.1** Descriptive Statistics

	<b>N</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Deviation</b>
id	8,335	1.000	8,670.000	4,308.352	2,510.578
nschcode	8,335	1.000	525.000	261.976	152.817
test1	8,335	24.350	69.250	47.644	6.325
test2	8,335	27.480	74.780	52.379	7.781
test3	8,335	26.960	79.720	57.109	9.450
effective	8,335	0.000	1.000	0.562	0.496
courses	8,335	0.000	4.000	0.748	0.790
female	8,335	0.000	1.000	0.505	0.500
ses	8,335	-2.410	1.870	0.034	0.784
Valid N (Listwise)	8,335				

ch2multivarML1.sav [DataSet4] - IBM SPSS Statistics Data Editor

Visible: 9 of 9 Variables

	id	nschcode	test1	test2	test3	effective	courses	female	ses
1	1	1	49.66	50.09	54.72	.00	.00	1	.59
2	2	1	47.92	58.26	64.33	.00	.00	1	.30
3	3	1	49.12	50.63	55.74	.00	.00	1	-.54
4	4	1	38.77	50.93	46.12	.00	.00	0	-.85
5	5	1	47.54	51.53	60.90	1.00	1.00	0	.00
6	6	1	41.74	48.82	54.55	1.00	2.00	0	-.11
7	7	1	32.83	42.43	48.73	.00	.00	0	-.33
8	8	1	55.91	56.36	60.91	.00	.00	1	-.89
9	9	1	52.93	63.75	68.37	.00	.00	0	.21
10	10	1	39.47	42.30	44.15	.00	.00	1	-.34
11	11	1	42.69	52.43	56.36	.00	.00	0	-.17
12	12	1	48.64	58.98	62.17	.00	.00	1	-1.07
13	13	2	47.54	51.53	59.26	1.00	1.00	0	-.10
14	14	2	46.83	53.70	52.09	.00	.00	0	1.28
15	15	2	45.89	52.95	56.16	.00	.00	0	1.06
16	16	2	47.54	51.53	58.06	1.00	1.00	0	.80
17	17	2	34.11	51.72	43.65	.00	.00	0	.73
18	18	2	47.54	51.53	60.87	1.00	1.00	0	.13
19	19	2	40.04	35.03	34.99	.00	.00	0	.68
20	20	2	36.94	43.72	48.66	.00	.00	0	.92

Data View Variable View

IBM SPSS Statistics Processor is ready

FIGURE 2.4 Horizontal data matrix.

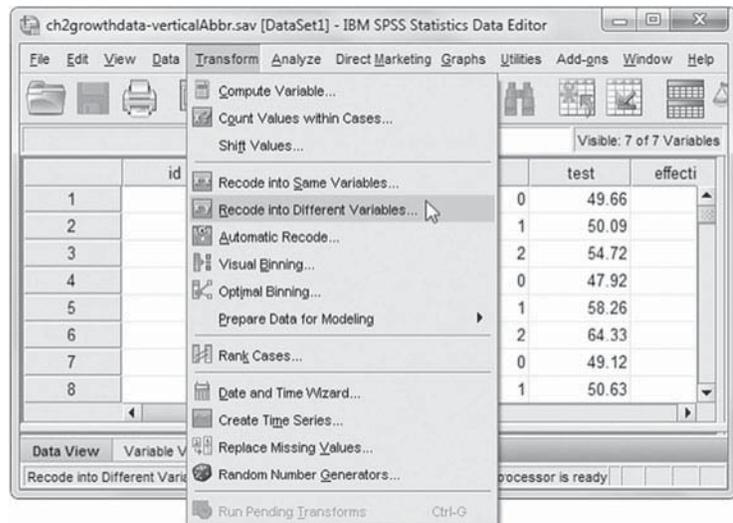
### Recode: Creating a New Variable Through Recoding

We begin by demonstrating recoding old values to new values. For the first example, we recode the variable *time*, which is measured on three linear occasions (0,1,2), to *quadtime*, which is a “squared” quadratic sequence (0,1,4), to capture any changes (acceleration or deceleration) in the rate of change that might occur over the three measurement occasions.

#### Recoding Old Values to New Values

This example uses a truncated version of a data set from Chapter 5 (*ch5growth-data-vertical.sav*). (Refer to Chapter 5 for further discussion and examples on coding time-related variables.)

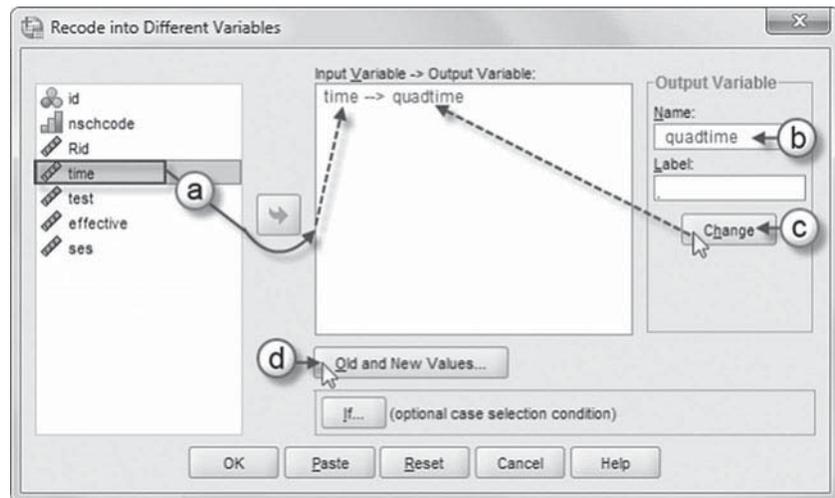
Launch the IBM SPSS application program, and select the data file *ch2growth-data-verticalAbbr.sav*.



1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

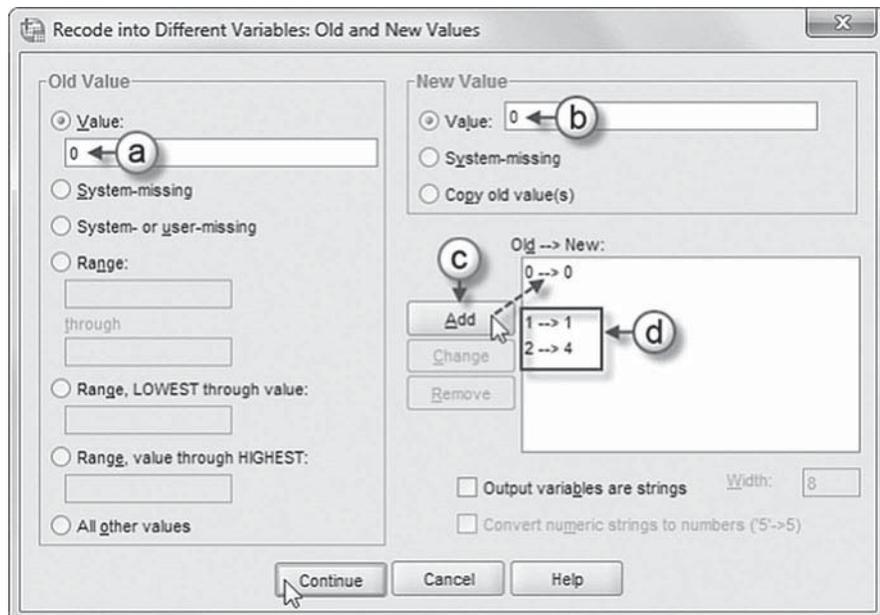
This command will open the *Recode into Different Variables* dialog box.

- 2a. *Recode into Different Variables* enables creating a new variable using a variable from the current data set. Click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.



- b. Now enter the new variable name by typing *quadtime* into the *Output Variable Name* box.
- c. Then click the CHANGE button, which will add *quadtime* and complete the RECODE command for *time → quadtime*.
- d. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

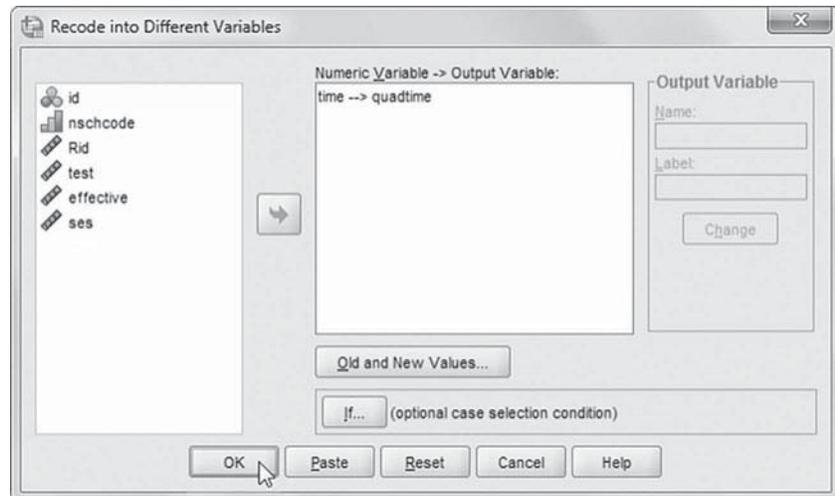
3. Within the *Recode into Different Variables: Old and New Values*, we will begin changing the *time* values (0,1,2) to reflect *quadtime* (0,1,4).



- a. Begin by entering the first value for *time* (0) in the *Value (old)* box.
- b. Next, enter the new value (0) for *quadtime* in the *Value (new)* box.
- c. Click the button to place the first command  $0 \rightarrow 0$  into the *Old → New* box.
- d. Repeat steps 3a to 3c to complete the remaining coding changes for *quadtime* values:
  - $1 \rightarrow 1$
  - $2 \rightarrow 4$

Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

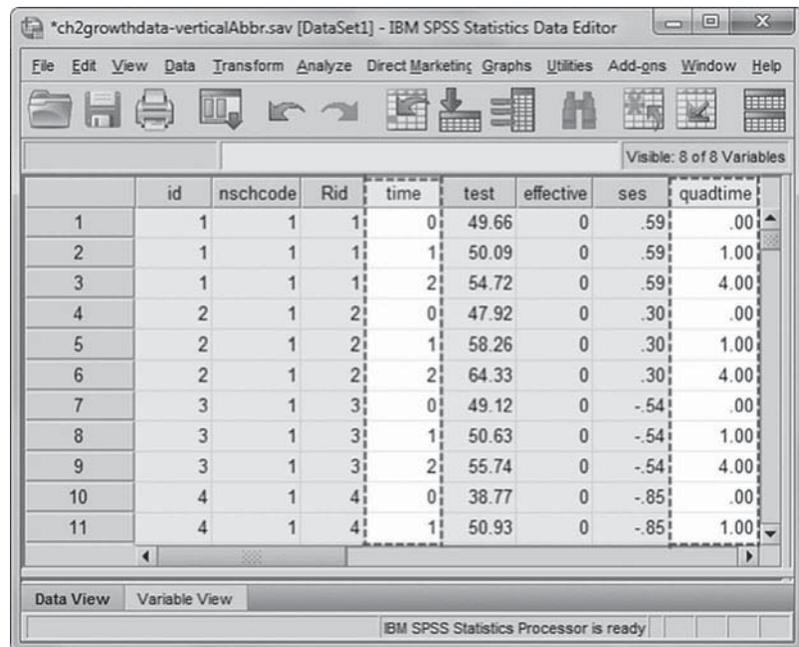
4. Click the OK button to generate the recoded variable *quadtime* and corresponding time values (0,1,4).



5. The new variable *quadtime* with its recoded values will be found in the last column of the data window.

(The two-decimal placement for *quadtime* may be changed in the *Variable View* window.)

**Note:** The asterisk displayed next to the filename at the top of the display window is a reminder that the original file has been changed. If you wish to save the modifications made here, go to the toolbar and select FILE, SAVE.



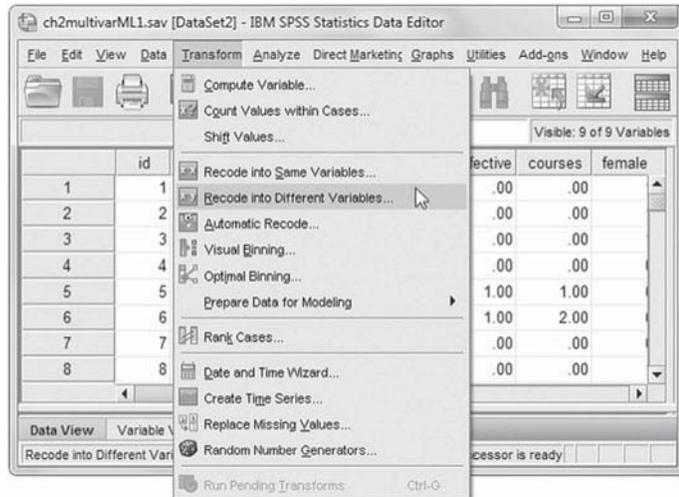
### Recoding Old Values to New Values Using “Range”

Our second task will be to create a new categorical SES variable by recoding *ses* into a variable called *ses4cat* (suggesting that we are going to recode this into a four-category variable). We will use the *Range* feature to recode three somewhat arbitrary cut points and create four categories for our recoded variable: -0.5180, 0.0250, and 0.6130 (these actually represent the 25th, 50th, and 75th percentiles, respectively).

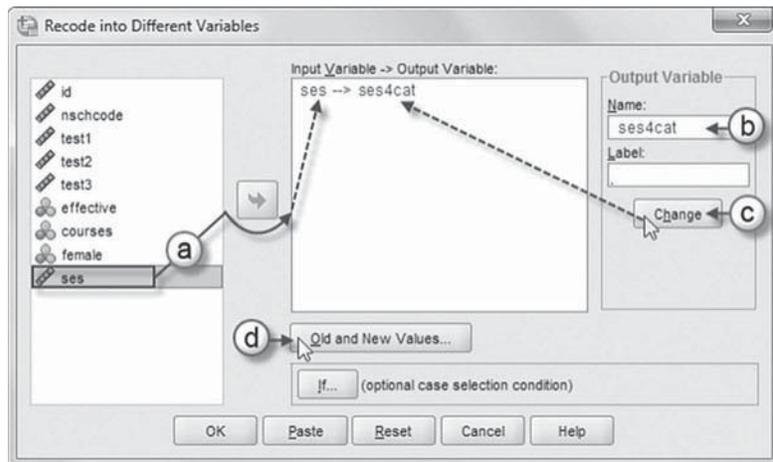
Launch the IBM SPSS application program and select the data file *ch2multi-varML1.sav*

1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.

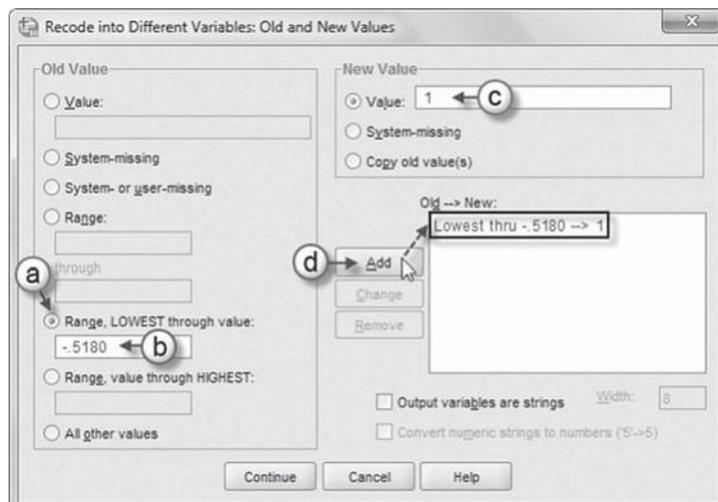


- 2a. *Recode into Different Variables* enables creating a new variable using a variable from the current data set. Click to select *ses* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.

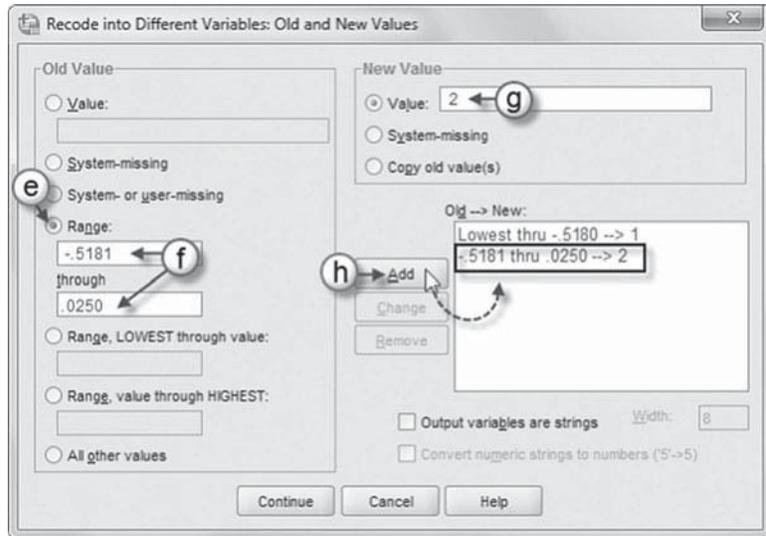


- b. Now enter the new variable name by typing *ses4cat* into the *Output Variable Name* box.
- c. Then click the CHANGE button, which will add *ses4cat* and complete the RECODE command for *ses → ses4cat*.
- d. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

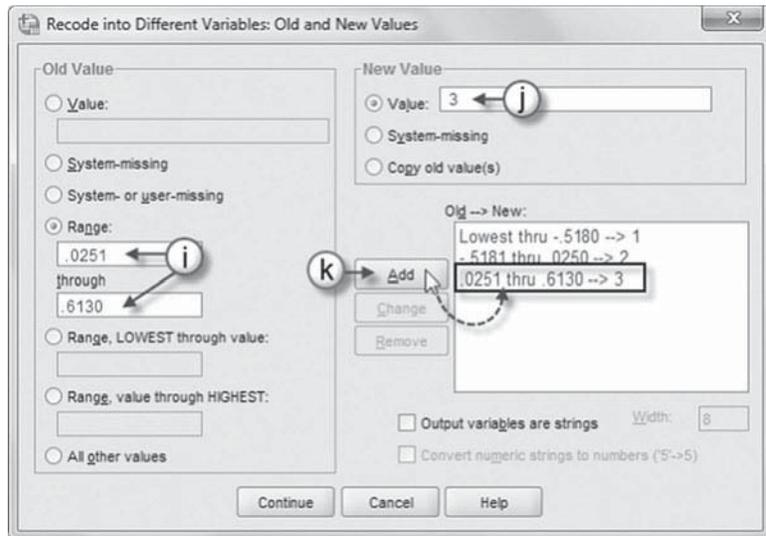
- 3a. The *Recode into Different Variables: Old and New Values* screen displays multiple options. To define the first cut point (-.5180), click to select the option *Range, LOWEST through value*.
- b. Now enter the value: -.5180.
- c. Next, enter "1" as the *New Value*.
- d. Then click the ADD button, which will place the first range command *Lowest thru -.5180 → 1* into the *Old → New* box.



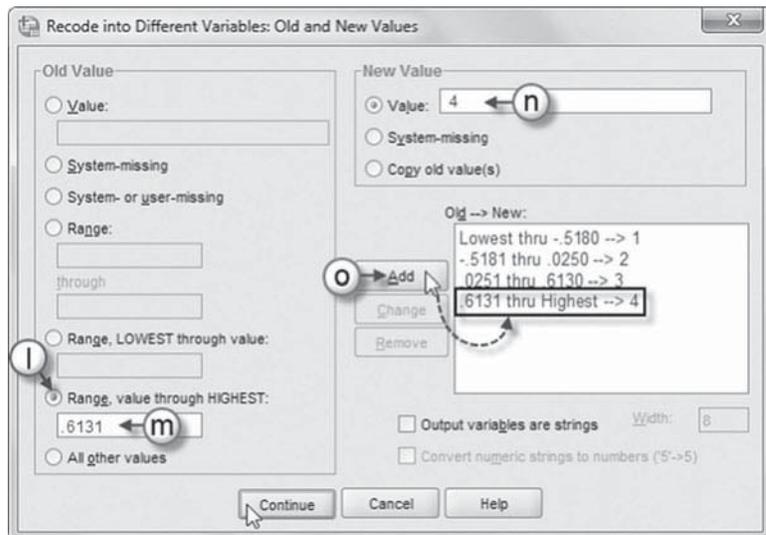
- e. Enter the second range of values by first clicking the *Range* option.
- f. Now enter the low value of  $-.5181$  and then the upper value limit of  $.0250$ .
- g. Next, enter “2” as the *New Value*.
- h. Then click the ADD button, which will place the first range command  $-.5181 \text{ thru } .0250 \rightarrow 2$  into the *Old  $\rightarrow$  New* box.



- i. The third value also uses a range of values, but since the *Range* option was selected previously, only the values need to be entered. First, enter  $.0251$  and then  $.6130$ .
- j. Next, enter “3” as the *New Value*.
- k. Then click the ADD button, which will place the range command  $.0251 \text{ thru } .6130 \rightarrow 3$  in the *Old  $\rightarrow$  New* box.



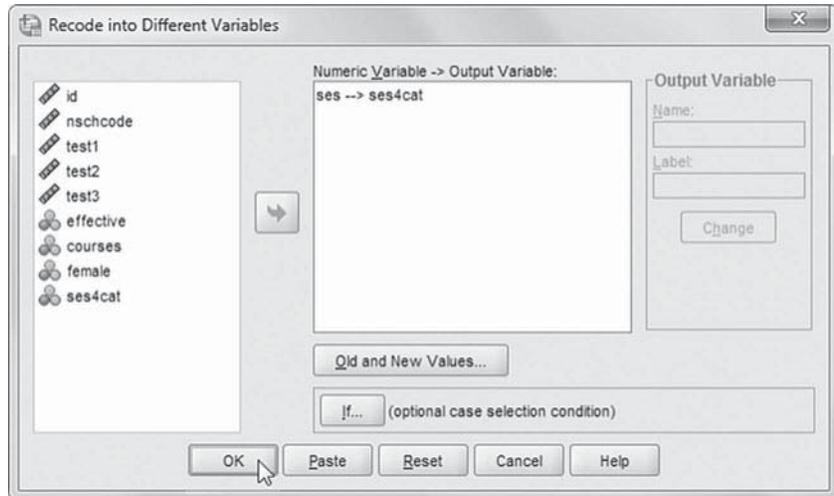
- l. To define the final cut-point value, click to select the *Range, value through HIGHEST* option.
- m. Now enter the value:  $.6131$ .
- n. Next, enter “4” as the *New Value*.
- o. Then click the ADD button, which will place the range command  $.6131 \text{ thru Highest} \rightarrow 4$  into the *Old  $\rightarrow$  New* box.



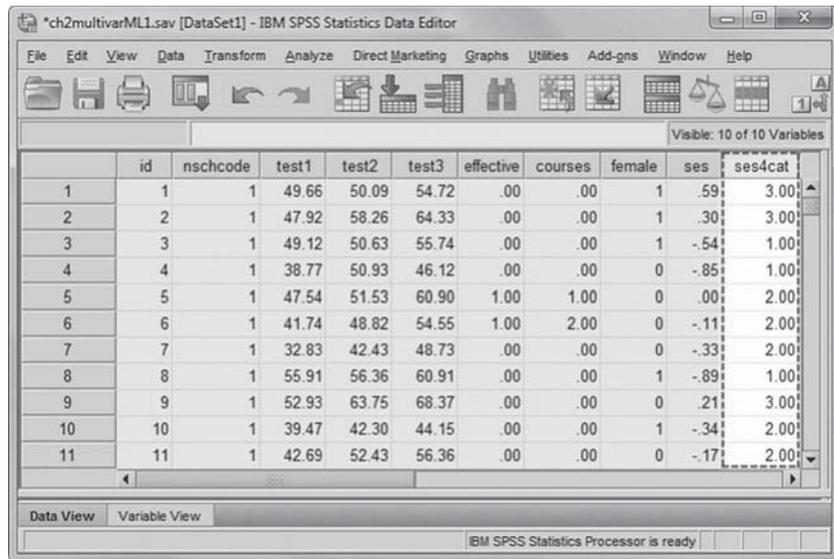
Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

4. Click the OK button to generate the recoded variable *ses4cat* and corresponding values.
5. The new variable *ses4cat* with its recoded categorical scores will be found in the last column of the data window.

You may also verify that the raw *ses* scores conform to the categories defined through the RECODE process.

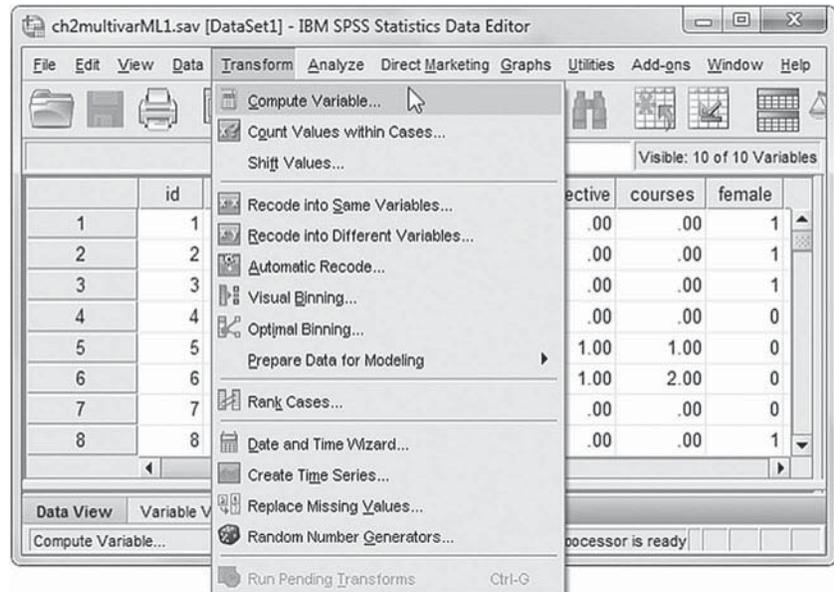


**Note:** The asterisk located next to the data file name at the top of the screen indicates changes were made to the data. At the conclusion of each of the following nine tutorials, we will save the changes by overwriting the *ch2multivarML1.sav* data file (although you may prefer renaming the saved file to preserve the original data file). To save these changes, go to the toolbar, select FILE, SAVE.



### Compute: Creating a New Variable That Is a Function of Some Other Variable

Suppose that we wanted to create a variable that was a summary of the three existing test scores. Using the TRANSFORM, COMPUTE VARIABLE menu command, we call up the COMPUTE VARIABLE dialog box.

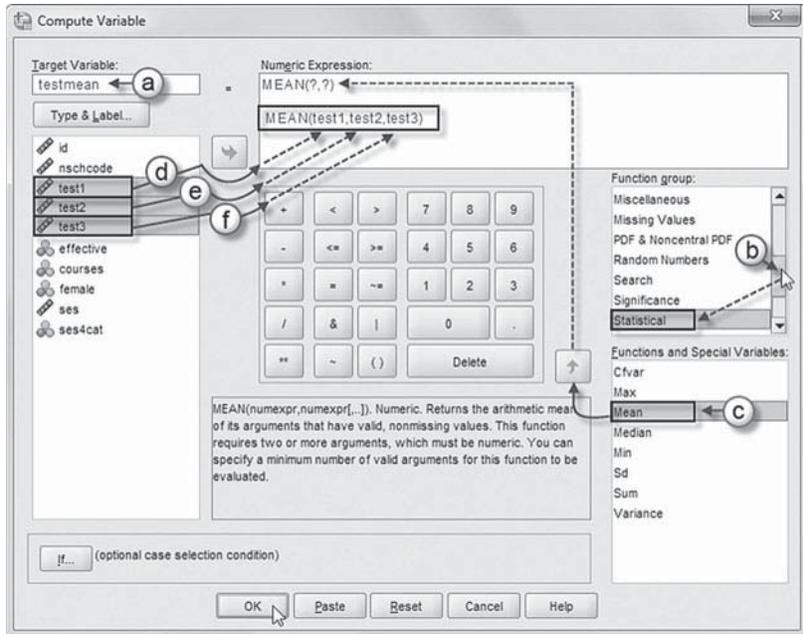


Continue using the *ch2multivarML1.sav* data file.

1. Go to the toolbar and select TRANSFORM, COMPUTE VARIABLE.

This command will open the *Compute Variable* dialog box.

- a. Enter *testmean* as the *Target Variable*.
- b. Scroll down the *Function group* list to locate, and then click to select the *Statistical* group, which displays assorted statistical *Functions and Special Variables*.



- c. Click to select the *Mean* function, and then click the up-arrow button, which will place it in the *Numeric Expression* box formatted as: MEAN(?,?). The question marks indicate the placement for a variable.
- d. The mean will be computed from three variables: *test1*, *test2*, and *test3*. To build the corresponding numeric expression, click *test1*, and then click the right-arrow button to place the variable in the box. Note that a comma must appear following the variable: MEAN(test1,?).
- e. Continue to build the numeric expression by next clicking *test2* and then the right-arrow button to move the variable into the box: MEAN(test1,test2,?).
- f. Complete the numeric expression by clicking *test3* and then the right-arrow button to move the variable into the box: MEAN(test1,test2, test3).

Click the OK button to perform the function.

2. Scroll across the columns, and the new variable *testmean* with its computed mean from the three test scores is found in the last column of the data window.

You may also verify that the raw *testmean* values represent the average of the three test scores for each individual in the data set.

	test2	test3	effective	courses	female	ses	ses4cat	testmean
1	50.09	54.72	.00	.00	1	.59	3.00	51.49
2	58.26	64.33	.00	.00	1	.30	3.00	56.84
3	50.63	55.74	.00	.00	1	-.54	1.00	51.83
4	50.93	46.12	.00	.00	0	-.85	1.00	45.27
5	51.53	60.90	1.00	1.00	0	.00	2.00	53.32
6	48.82	54.55	1.00	2.00	0	-.11	2.00	48.37
7	42.43	48.73	.00	.00	0	-.33	2.00	41.33
8	56.36	60.91	.00	.00	1	-.89	1.00	57.73
9	63.75	68.37	.00	.00	0	.21	3.00	61.68
10	42.30	44.15	.00	.00	1	-.34	2.00	41.97
11	52.43	56.36	.00	.00	0	-.17	2.00	50.49

Although we have created two new variables using the RECODE and COMPUTE commands, our data file is still strictly an individual-level (Level 1) file. More specifically, aside from the school ID, *nschcode*, we do not have any variables that are school specific and therefore do not have anything to analyze at Level 2. Typically, data from other sources would be brought in and merged with our Level 1 file. If we had information about the schools these students attend (e.g., public or private control, etc.), we could very easily merge those data with the individual-level file with which we have been working.

### Match Files: Combining Data From Separate IBM SPSS Files

The MATCH FILES command allows us to combine data from different sources. To demonstrate one use of this command, we will use the existing Level 1 file with which we have been working. MATCH FILES can combine data in two general ways: by adding variables to the existing data set or by adding cases to the data set. We will limit our interest here to the addition of variables that are found in a separate data set. For our example, we are going to show how to merge a file containing school-level information with our existing file containing student-level data. We want to add information about the frequency of Advanced Placement testing at each school. The file containing this information (*apexams.sav*) has two variables in it. The first is a school identifier that is the same as the school identifier used in our Level 1 data file (*nschcode*). The second variable is named *apexams*. It is a ratio of the number of Advanced Placement exams taken by students at the school to the total number of students in the 12th grade at the school. While our student-level (Level 1) file contains 8,335 observations (i.e., the number of students), as shown in Table 2.2, the school-level (Level 2) file shown in Table 2.3 contains 525 observations representing the schools that the 8,335 students attend. Consider the contents of each file given subsequently. The Level 1 file will be the target for the data contained in the Level 2 (*apexams.sav*) file; that is, we are going to merge the data from the Level 2 file onto the records in the Level 1 file.

**TABLE 2.2** Descriptive Statistics

	<b>N</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Deviation</b>
id	8,335	1.000	8,670.000	4,308.352	2,510.578
nschcode	8,335	1.000	525.000	261.976	152.817
test1	8,335	24.350	69.250	47.644	6.325
test2	8,335	27.480	74.780	52.379	7.781
test3	8,335	26.960	79.720	57.109	9.450
effective	8,335	0.000	1.000	0.562	0.496
courses	8,335	0.000	4.000	0.748	0.790
female	8,335	0.000	1.000	0.505	0.500
ses	8,335	-2.410	1.870	0.034	0.784
Valid N (Listwise)	8,335				

**TABLE 2.3** Descriptive Statistics

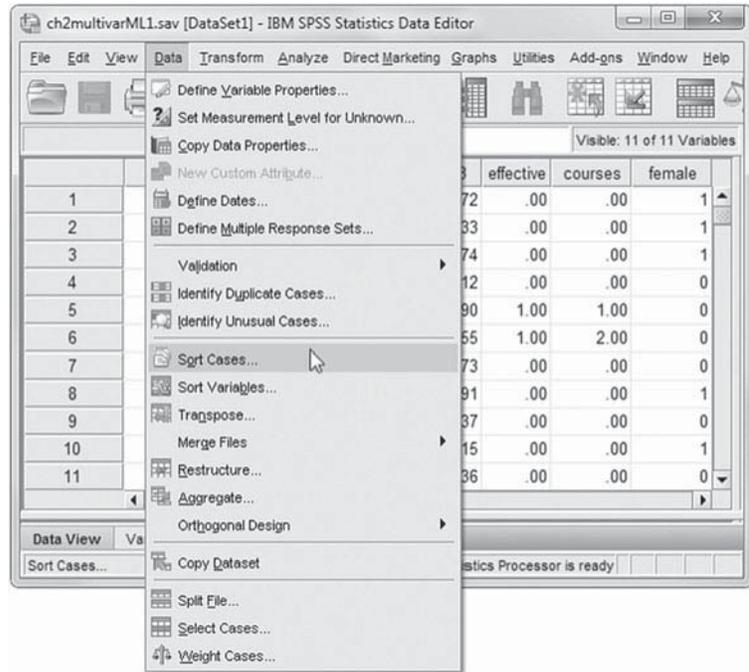
	<b>N</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Deviation</b>
nschcode	525	1.000	525.000	263.000	151.699
apexams	525	0.000	0.800	0.164	0.133
Valid N (Listwise)	525				

Because this is not a one-to-one match (i.e., there are not the same number of records in each file), we will have to identify a “key” to be used to match the data from the schools to the data from the students. Notice that matching student data to the schools would require aggregating student-level variables because there are fewer schools than students in this case. We will use the single common identifier variable, *nschcode*, which represents the schools in both files. Both data sets will need to be sorted on the key variable. This can be accomplished by opening each data set and choosing the DATA, SORT CASES menu and dialog box. For each file, the sort should be on the variable *nschcode*, and each file needs to be saved after sorting.

This example will combine data from two files: *ch2multivarML1.sav* (primary file) and *apexams.sav* (secondary file). When combining data from separate files, we recommend that the cases are sorted before you begin this procedure to prevent interruption of the workflow.

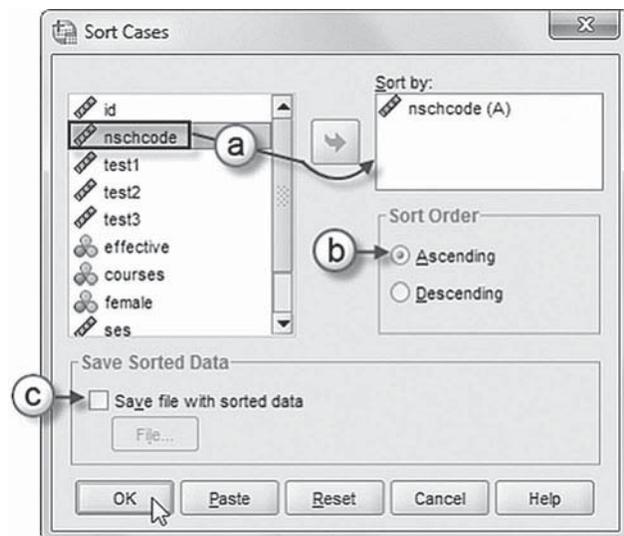
If you are continuing from the prior section concerning the COMPUTE function, the *ch2multivarML1.sav* file is already open. If not, locate and open the data file.

1. Go to the toolbar and select DATA, SORT CASES.



This command will open the *Sort Cases* dialog box.

- 2a. Within the *Sort Cases* dialog box, select *nschcode* from the left column, and then click the right-arrow button to transfer the variable into the *Sort by* box.
- b. The default *Sort Order* setting is *Ascending* (low to high), which we will retain.
- c. Later versions of IBM SPSS (version 20.0 forward) provide an option for saving the sorted data directly to a file. We will forgo using this option for the current example.

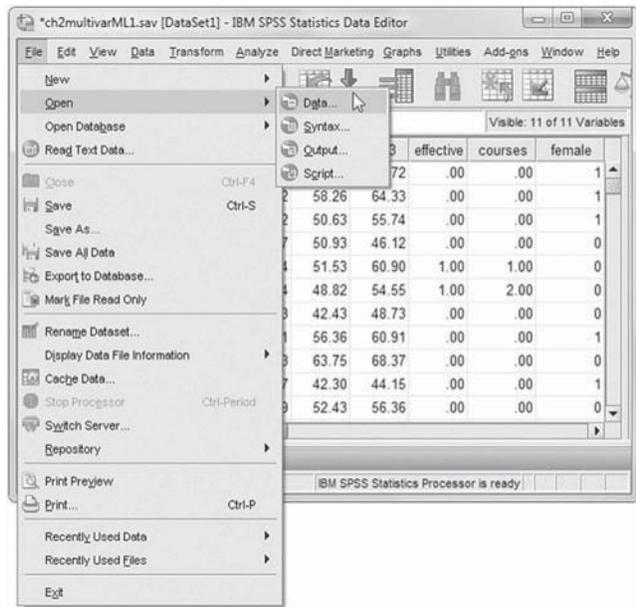


Click the OK button to begin sorting the cases. The next step is to sort the cases for the secondary file, *apexams.sav*.

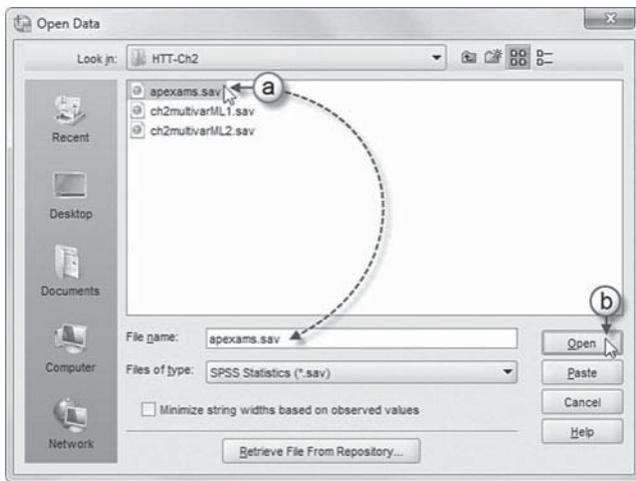
3. Begin by opening the secondary file (*apexams.sav*) while keeping the *ch2multivarML1.sav* file open.

Go to the toolbar and select FILE, OPEN, DATA.

This command will then display the *Open Data* screen.

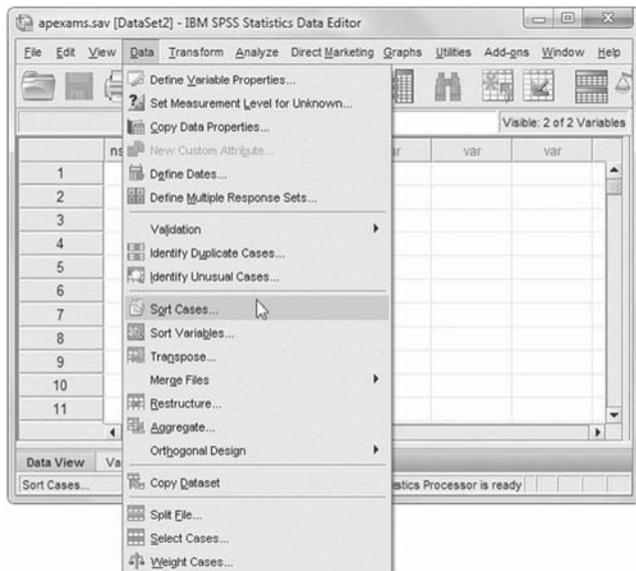


- 4a. Locate and click to select the data file (*apexams.sav*).
- b. Then click the OPEN button.



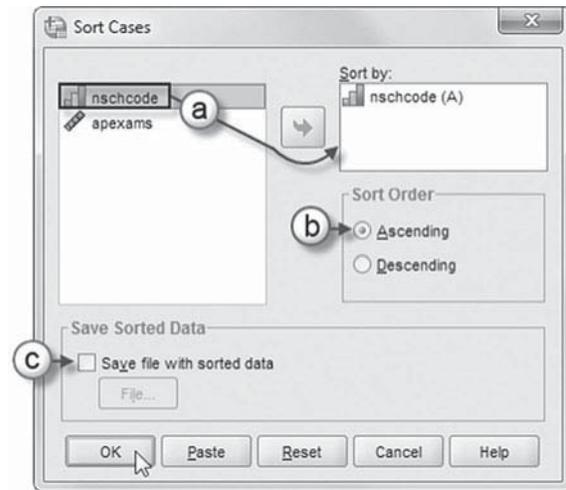
5. With the data file *apexams.sav* opened, go to the toolbar and select DATA, SORT CASES.

This command will open the *Sort Cases* dialog box.



- 6a. Within the *Sort Cases* dialog box, select *nschcode* from the left column, and then click the right-arrow button to transfer the variable into the *Sort by* box.
- b. The default setting for sorting is the *Ascending* order, which we will retain.
- c. Later versions of IBM SPSS (version 20.0 forward) provide an option for saving the sorted data directly to a file. We will forgo using this option for the current example.

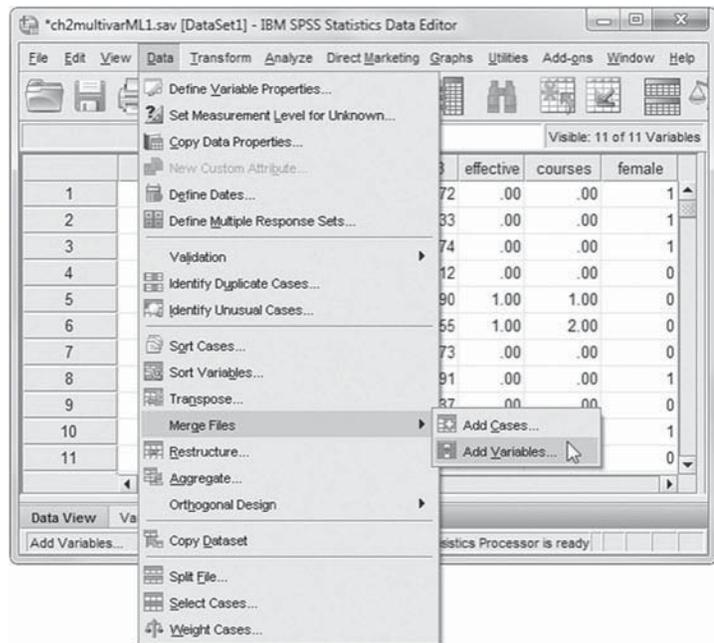
Click the OK button to begin sorting the cases.



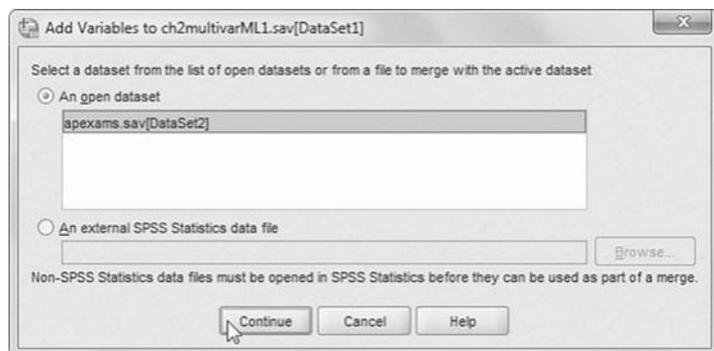
7. Once the files are sorted, then merging may begin.

Return to the primary data file (*ch2multivarML1.sav*). Then go to the toolbar and select DATA, MERGE FILES, ADD VARIABLES.

This command will open the *Add Variables* dialog box.



8. Since the data file *apexams.sav* is an opened file, the option *An open dataset* is preselected and displays the file name in the box below. If *apexams.sav* had not been opened, the option *An external SPSS Statistics data file* would have been preselected instead, requiring you to locate and identify the file using the BROWSE button.

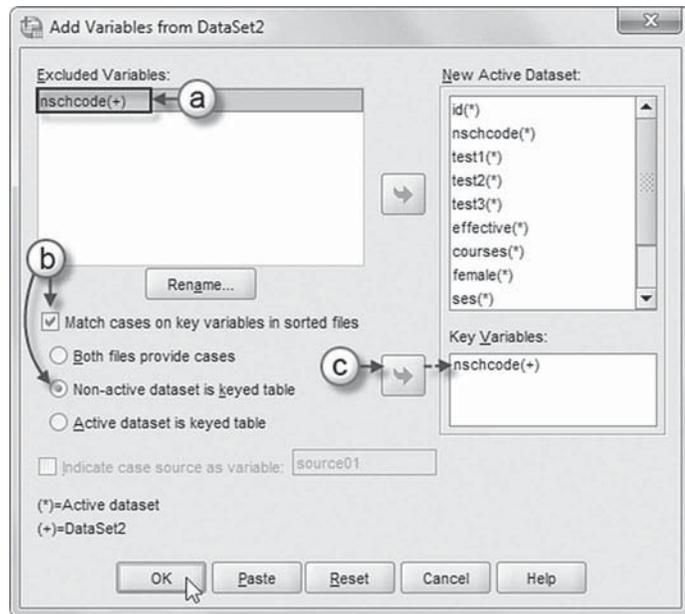


Now click to select *apexams.sav(DataSet2)*, and then click the CONTINUE button to access the *Add Variables from DataSet2* screen.

- 9a. Within the *Add Variables from DataSet2* display screen, click to select *nschcode(+)* from the *Excluded Variables* box. This action will activate the *Match cases on key variables in sorted files* option below, enabling the box to be checked.
- b. Click to select the *Non-active dataset is keyed table* option.

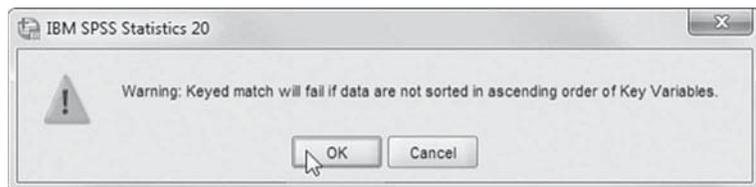
**Note:** A “keyed” table or table lookup file is a file that contains data for each case that can be applied to numerous cases in another data file.

- c. Now click the right-arrow button to move *nschcode(+)* into the *Key Variables* box.



Click the OK button to process this function.

10. A warning to presort the data appears but may be disregarded since the data from both files had been sorted using the *nschcode* variable at the outset.



Click the OK button.

11. Scroll across the columns, and the merged variable *apexams* taken from the *apexams.sav* data file is found in the last column of the *ch2multivarML1.sav* data window.

	test3	effective	courses	female	ses	ses4cat	testmean	apexams
1	54.72	.00	.00	1	.59	3.00	51.49	.08
2	64.33	.00	.00	1	.30	3.00	56.84	.08
3	55.74	.00	.00	1	-.54	1.00	51.83	.08
4	46.12	.00	.00	0	-.85	1.00	45.27	.08
5	60.90	1.00	1.00	0	.00	2.00	53.32	.08
6	54.55	1.00	2.00	0	-.11	2.00	48.37	.08
7	48.73	.00	.00	0	-.33	2.00	41.33	.08
8	60.91	.00	.00	1	-.89	1.00	57.73	.08
9	68.37	.00	.00	0	.21	3.00	61.68	.08
10	44.15	.00	.00	1	-.34	2.00	41.97	.08
11	56.36	.00	.00	0	-.17	2.00	50.49	.08

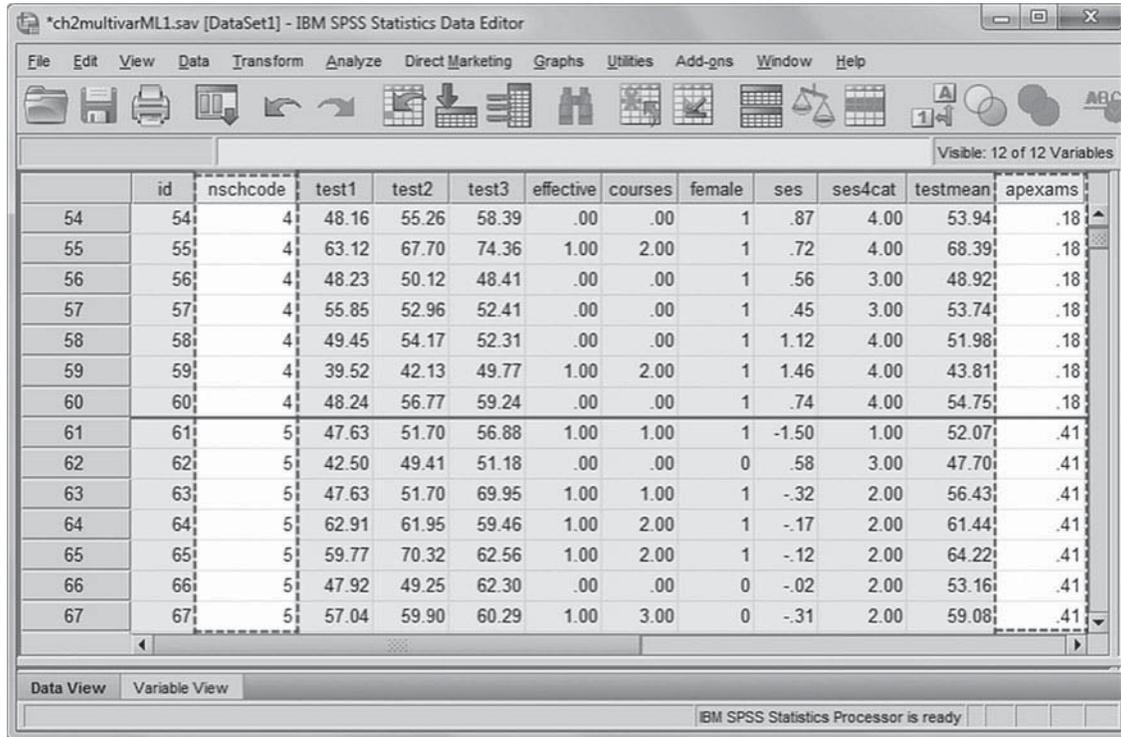


FIGURE 2.5 Data matrix after performing the MATCH FILES function.

Once the merge is complete, the new variable, *apexams*, will appear in the active data set window as shown in Figure 2.5. Notice that while the individual-level variables vary across all cases within the window, the new school-level variable, *apexams* is constant within each school (compare *nschcode* 4 with *nschcode* 5 in the subsequent screen shot).

Descriptive statistics on the data set, summarized in Table 2.4, will now show a value for *apexams* for each of the 8,335 Level 1 observations. Notice that the descriptive statistics provide no hint of the lack of variance within each *nschcode* for the *apexams* variable. It now looks like an individual-level variable.

TABLE 2.4 Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
id	8,335	1.000	8,670.000	4,308.352	2,510.578
nschcode	8,335	1.000	525.000	261.976	152.817
test1	8,335	24.350	69.250	47.644	6.325
test2	8,335	27.480	74.780	52.379	7.781
test3	8,335	26.960	79.720	57.109	9.450
effective	8,335	0.000	1.000	0.562	0.496
courses	8,335	0.000	4.000	0.748	0.790
female	8,335	0.000	1.000	0.505	0.500
ses	8,335	-2.410	1.870	0.034	0.784
apexams	8,335	0.000	0.800	0.166	0.130
Valid N (Listwise)	8,335				

## Aggregate: Collapsing Data Within Level 2 Units

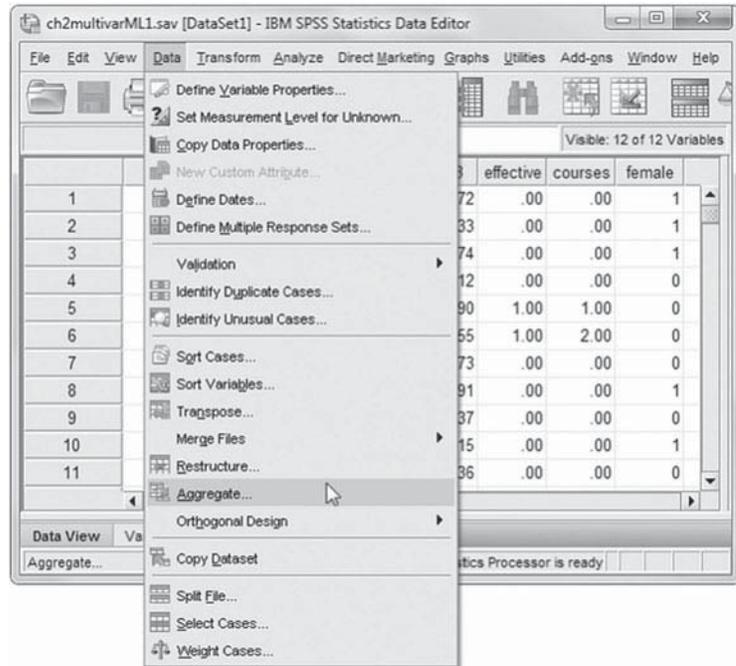
Many times the analyst will be interested in creating group-level measures by aggregating the characteristics of individuals within each group. One candidate variable might be student socioeconomic status (*ses*). Another might be gender (*female*) (aggregating this to the mean will yield the proportion of students at Level 1 who are female). In this section, we will use the AGGREGATE command to create both of these variables.

The objective in this instance is to take the within-school means of the variables *female* and *ses*. This will yield the proportion of females in the sample for each school and the average socioeconomic status of students in the sample for each school.

If you are continuing from the prior section concerning the MERGE FILES function, the *ch2multivarML1.sav* file is already open. If not, locate and open the data file.

1. Go to the toolbar and select DATA, AGGREGATE.

This command will open the *Aggregate Data* dialog box.

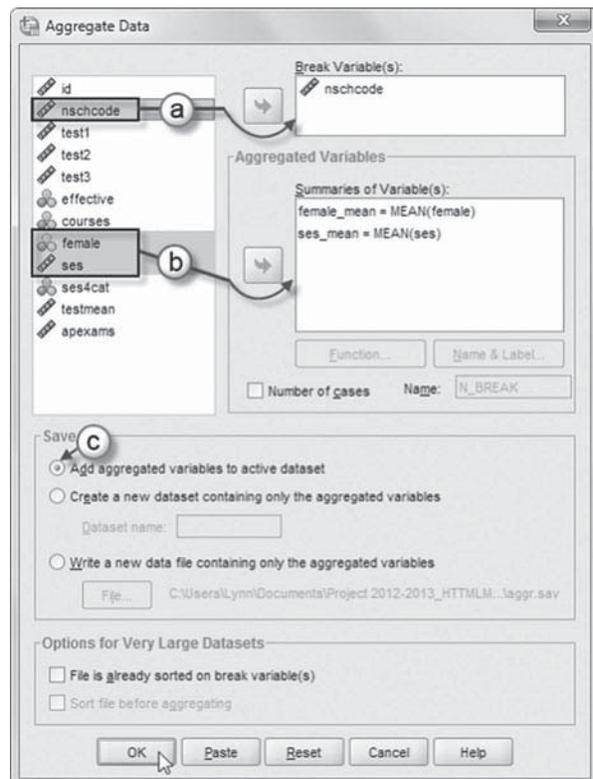


- 2a. Click to select *nschcode* from the left column of the *Aggregate Data* dialog box, and then click the right-arrow button to move the variable into the *Break Variable(s)* box.
- b. Now click to select *female* and *ses* from the left column, and then click the right-arrow button to move the variables into the *Summaries of Variable(s)* box.

**Note:** IBM SPSS uses MEAN as the default function, which will be used for this example. Other functions besides MEAN are also available to generate a variety of aggregated data.

- c. Click the option *Add aggregated variables to active dataset*, which will add the two variables *female\_mean* and *ses\_mean* directly into the active Level 1 data set.

**Note:** Besides the *Add aggregated variables to the active dataset* option, we could also choose two other options: *Create a new dataset containing only the aggregated variables* or *Write a new data file*



containing only the aggregated variables. The latter choices would require us to use the MATCH FILES routine to merge the new aggregated variables back onto the individual-level data set. Choosing to have the variables written straight to the active data set is much more efficient and reduces the risk of errors.

Click the OK button to run the aggregation and merge the new variables to the student-level data set (shown in the active data window).

3. Scroll across the columns, and the new aggregated variables *female\_mean* and *ses\_mean* appear in the last two columns of the *ch2multivarML1.sav* data window.

Notice that school 4 (*nschcode* = 4) has an all-female sample (*female\_mean* = 1.00) while less than half of the school 5 sample was female (*female\_mean* = .47). The SES average for school 4 was also higher than the average for school 5 (.82 vs. -.39, respectively).

	id	nschcode				female_mean	ses_mean
	54	4	48.16	55.26	58.39	1.00	.82
	55	4	63.12	67.70	74.36	1.00	.82
	56	4	48.23	50.12	48.41	1.00	.82
	57	4	55.85	52.96	52.41	1.00	.82
	58	4	49.45	54.17	52.31	1.00	.82
	59	4	39.52	42.13	49.77	1.00	.82
	60	4	48.24	56.77	59.24	1.00	.82
	61	5	47.63	51.70	56.88	.47	-.39
	62	5	42.50	49.41	51.18	.47	-.39
	63	5	47.63	51.70	69.95	.47	-.39
	64	5	62.91	61.95	59.46	.47	-.39
	65	5	59.77	70.32	62.56	.47	-.39
	66	5	47.92	49.25	62.30	.47	-.39

The RECODE, COMPUTE, MATCH FILES, and AGGREGATE commands provide all the tools necessary for structuring our IBM SPSS files for a multilevel analysis within IBM SPSS MIXED. We will expand on some of the commands in subsequent sections to create variables that can be very useful in multilevel analyses. Before turning our attention to those additional examples, however, we introduce a different data structure that enables multivariate analyses and analyses of change over time using the multilevel model through IBM SPSS MIXED.

### VARSTOCASES: Vertical Versus Horizontal Data Structures

The data sets we created in the previous sections are arranged horizontally; that is, each observation is contained on a single row with variables arrayed across the columns. In the horizontal data sets, we have the variables arranged in such a way that the lower level units can be seen as nested within high-level units (e.g., students within schools where student values vary within and between schools while school-level values vary only between schools).

In multivariate and time-varying models, we reconceptualize the nesting and deal with vertical rather than horizontal data structures. Instead of students being our Level 1 unit of analysis, we might nest time periods or indicator variables within students who could, in turn, be nested within schools. In this case, our time periods or indicator variables become Level 1, students become Level 2, and schools become Level 3. In terms of the data structure, what this means is that each individual (i.e., students, to continue our example) will have multiple records. If there are three occasions of interest, each student will have three records, one for each occasion. Similarly,

Visible: 14 of 14 Variables

	id	nschcode	test1	test2	test3	effective	courses	female	ses
1	1	1	49.66	50.09	54.72	.00	.00	1	.59
2	2	1	47.92	58.26	64.33	.00	.00	1	.30
3	3	1	49.12	50.63	55.74	.00	.00	1	-.54
4	4	1	38.77	50.93	46.12	.00	.00	0	-.85
5	5	1	47.54	51.53	60.90	1.00	1.00	0	.00
6	6	1	41.74	48.82	54.55	1.00	2.00	0	-.11
7	7	1	32.83	42.43	48.73	.00	.00	0	-.33
8	8	1	55.91	56.36	60.91	.00	.00	1	-.89
9	9	1	52.93	63.75	68.37	.00	.00	0	.21
10	10	1	39.47	42.30	44.15	.00	.00	1	-.34
11	11	1	42.69	52.43	56.36	.00	.00	0	-.17
12	12	1	48.64	58.98	62.17	.00	.00	1	-1.07
13	13	2	47.54	51.53	59.26	1.00	1.00	0	-.10
14	14	2	46.83	53.70	52.09	.00	.00	0	1.28
15	15	2	45.89	52.95	56.16	.00	.00	0	1.06
16	16	2	47.54	51.53	58.06	1.00	1.00	0	.80
17	17	2	34.11	51.72	43.65	.00	.00	0	.73
18	18	2	47.54	51.53	60.87	1.00	1.00	0	.13
19	19	2	40.04	35.03	34.99	.00	.00	0	.68
20	20	2	36.94	43.72	48.66	.00	.00	0	.92

Data View Variable View

IBM SPSS Statistics Processor is ready

FIGURE 2.6 Horizontal data matrix.

if we are interested in defining a latent variable with five indicators, each student would have five records. So if we have a data set with 1,000 students and we are interested in looking at change, say in test scores, over three occasions, our data set will have three occasions  $\times$  1,000 students = 3,000 records. Using the latent variable example, if we had 1,000 students and five indicators, we would have 5,000 records in the data set. This is quite in contrast to the univariate outcome models in which individuals are nested in successively higher organizational levels. In those models, the individual defines Level 1 of the analysis, and there are as many records as there are individuals.

Rearranging the data to accommodate the change and multivariate models is quite straightforward. Let's refresh our memory of the data set we have been using. Note that in the subsequent data view illustrated in Figure 2.6 there is a single record for each student, and we have three test scores across successive time periods.

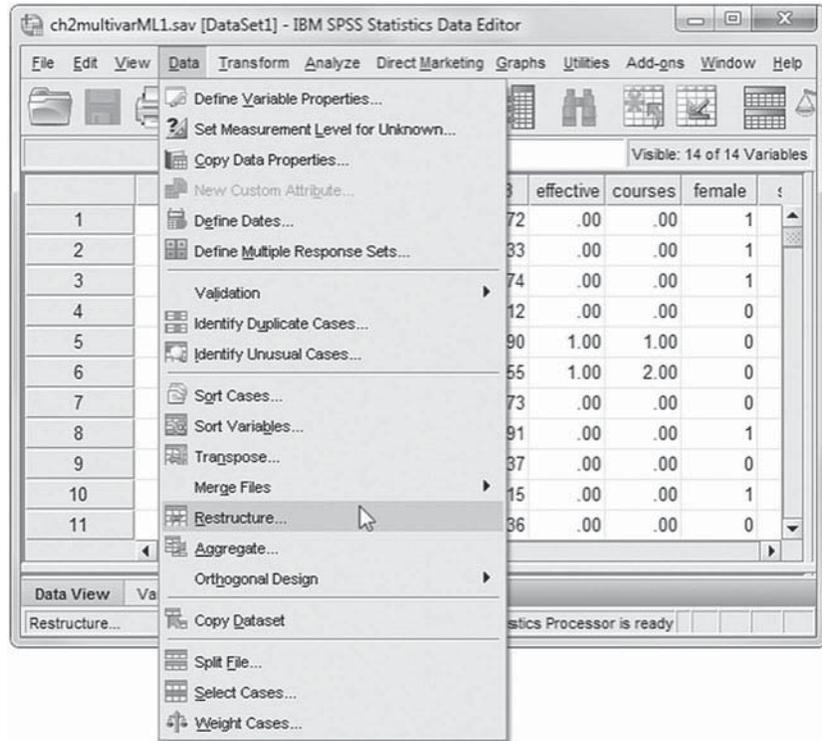
Our objective is to create three records for each student, each representing a distinct time point for each individual in the sample. In other words, we are going to nest these three testing occasions within each student (students will still be nested within their schools). To accomplish this, we will use the VARSTOCASES routine that is contained in the *Restructure Data Wizard*.

If you are continuing from the prior section concerning the MERGE FILES function, the *ch2multivarML1.sav* file is already open. If not, locate and open the data file.

1. Go to the toolbar and select DATA, RESTRUCTURE.

**Note:** If you're prompted to save the file before proceeding, you may save it or elect not to do so.

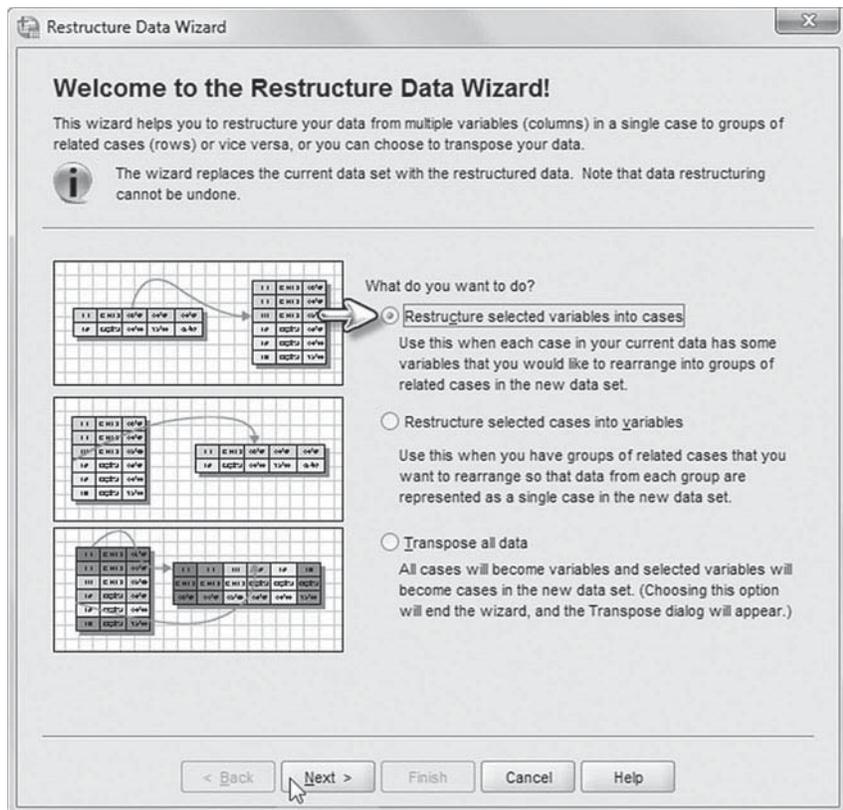
This command will open the *Welcome to the Restructure Data Wizard* dialog box.



2. The *Restructure Data Wizard* presents three options: *Restructure selected variables into cases*, *Restructure selected cases into variables*, and *Transpose all data*.

For this situation, we want to treat the three test variables (*test1*, *test2*, and *test3*) as a single grouped variable, so will use the default setting: *Restructure selected variables into cases*.

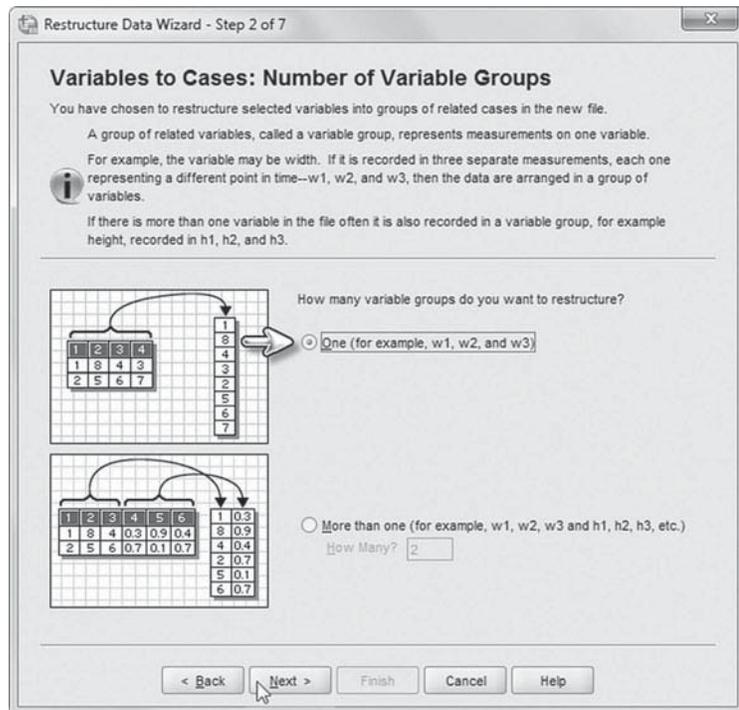
Click the NEXT button to go to the *Variables to Cases* screen.



3. The *Variables to Cases: Number of Variable Groups* screen allows defining the number of variable groups to create in the new data file.

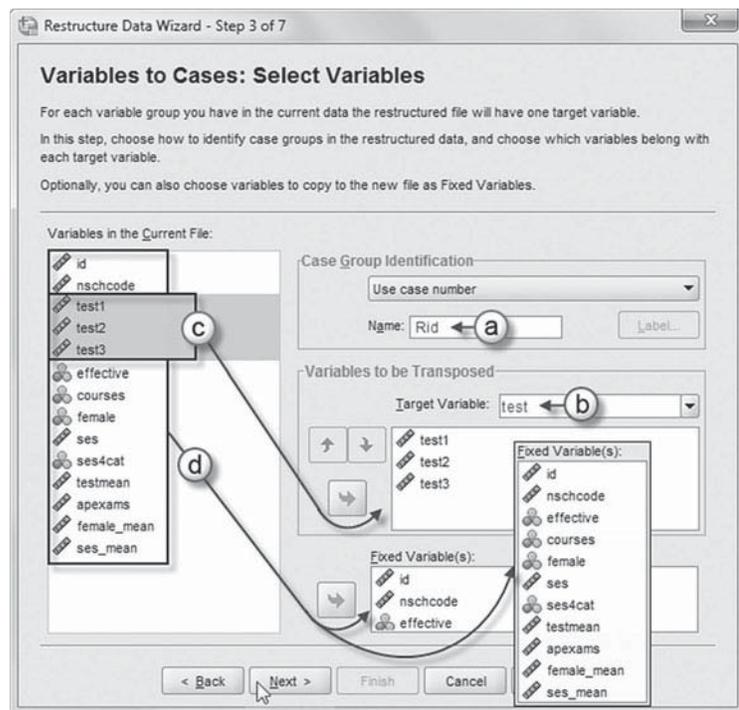
In this case, the three tests (*test1*, *test2*, and *test3*) are to be treated as a single group, so the default setting of *One* will be used.

Click the NEXT button to continue to the next screen.



4. The *Variables to Cases: Select Variables* screen allows defining the variables for the new data. This includes specifying a new group ID (*Case Group Identification*), the variables to define the transposition (*Variables to be Transposed*), and the variables to include with each new record [*Fixed Variable(s)*].

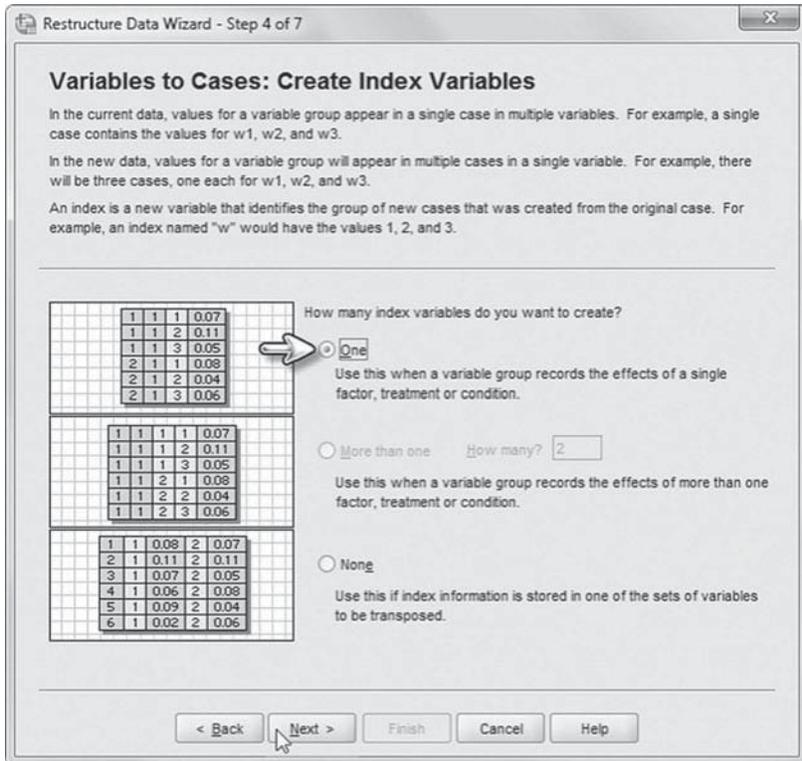
- Begin by entering *Rid* (recoded ID) as the *Case Group Identifier Name*.
- Enter *test* as the *Target Variable*.
- Click to select *test1*, *test2*, and *test3* from the left column, and then click the right-arrow button to move the variables into the *Variables to be Transposed* box. The three variables will be combined to form a single variable (*test*) that is related to each record (row).
- Click to select all the remaining variables (excluding *test1*, *test2*, and *test3*) from the left column, and then click the right-arrow button to move them into the *Fixed Variable(s)* box. [Refer to the *Fixed Variable(s)* boxed insert.] These variables will then be appended to each of the new rows in the data set.



Click the NEXT button to continue to the *Variables to Cases: Create Index Variables* screen.

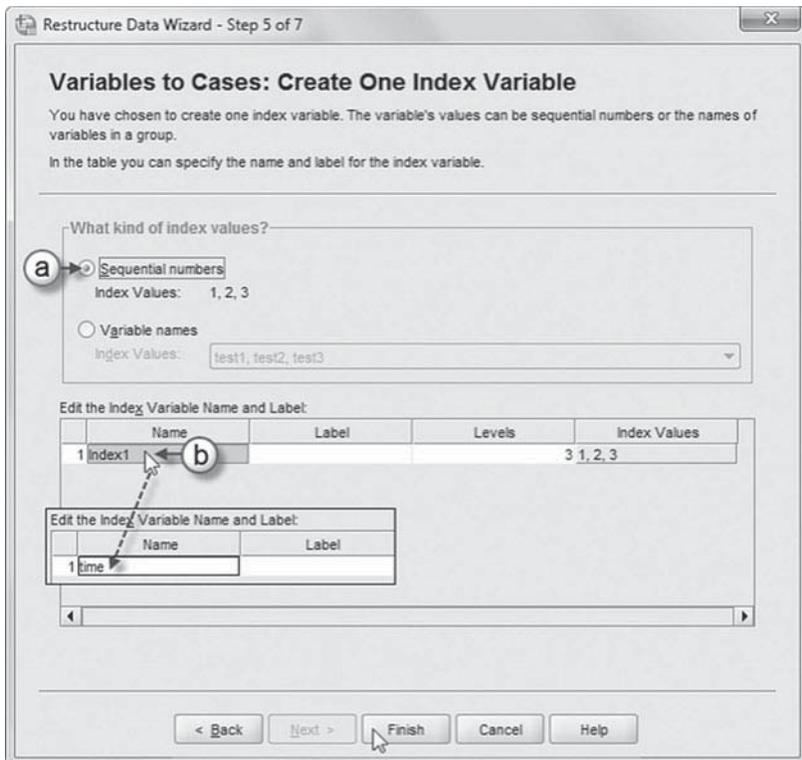
5. The *Variables to Cases: Create Index Variables* screen allows for creating one or more index variables.

In this case, we want to create one indexed variable encompassing the “timing” of each test. Since *One* is the default option, click the NEXT button to continue to the *Variables to Cases: Create One Index Variable* screen.



6. Further information may be specified for the indexed variable.
  - a. *Sequential numbers* is the default setting, which will be retained for this example.
  - b. The variable name may be changed by clicking on *Index1* and then changing it to *time*.

Click the FINISH button to indicate that no further changes will be made.



7. After clicking the FINISH button, a message immediately appears warning that some of the original data will remain in use and that care should be administered to ensure use of the appropriate data set for subsequent analyses.



Click the OK button to proceed with generating the new data set.

8. The new data set appears, and scrolling across the columns, the *time* and *test* variables are found in the last two columns. A truncated view of the data set shows that each student (as identified by *Rid* and *id*) now has three lines of data, one line for each of the three tests and time periods. A more complete picture is presented in Figure 2.7.

	Rid	id	nschcode	test
1	1	1	1	49.66
2	1	1	1	50.09
3	1	1	1	54.72
4	2	2	1	47.92
5	2	2	1	58.26
6	2	2	1	64.33
7	3	3	1	49.12
8	3	3	1	50.63
9	3	3	1	55.74
10	4	4	1	38.77
11	4	4	1	50.93

Notice that each student now has three lines of data—one for each test. The test scores (Level 1) vary across all respondents, student characteristics vary across all students (Level 2) but are constant within the group of three test scores, and school characteristics vary across all schools (Level 3) but are constant within the group of students attending each school.

	Rid	id	nschcode	effective	courses	female	ses	ses4cat	testmean	apexams	female_mean	ses_mean	time	test
1	1	1	1	.00	.00	1	.59	3.00	51.49	.08	.50	-.27	1	49.66
2	1	1	1	.00	.00	1	.59	3.00	51.49	.08	.50	-.27	2	50.09
3	1	1	1	.00	.00	1	.59	3.00	51.49	.08	.50	-.27	3	54.72
4	2	2	1	.00	.00	1	.30	3.00	56.84	.08	.50	-.27	1	47.92
5	2	2	1	.00	.00	1	.30	3.00	56.84	.08	.50	-.27	2	58.26
6	2	2	1	.00	.00	1	.30	3.00	56.84	.08	.50	-.27	3	64.33
7	3	3	1	.00	.00	1	-.54	1.00	51.83	.08	.50	-.27	1	49.12
8	3	3	1	.00	.00	1	-.54	1.00	51.83	.08	.50	-.27	2	50.63
9	3	3	1	.00	.00	1	-.54	1.00	51.83	.08	.50	-.27	3	55.74
10	4	4	1	.00	.00	0	-.85	1.00	45.27	.08	.50	-.27	1	38.77
11	4	4	1	.00	.00	0	-.85	1.00	45.27	.08	.50	-.27	2	50.93
12	4	4	1	.00	.00	0	-.85	1.00	45.27	.08	.50	-.27	3	46.12
13	5	5	1	1.00	1.00	0	.00	2.00	53.32	.08	.50	-.27	1	47.54
14	5	5	1	1.00	1.00	0	.00	2.00	53.32	.08	.50	-.27	2	51.53
15	5	5	1	1.00	1.00	0	.00	2.00	53.32	.08	.50	-.27	3	60.90

FIGURE 2.7 Data matrix after performing the VARSTOCASES function.

**TABLE 2.5** Descriptive Statistics

	<i>N</i>	Minimum	Maximum	Mean	Std. Deviation
Rid	25,005	1.000	8,335.000	4,168.000	2,406.155
id	25,005	1.000	8,670.000	4,308.352	2,510.477
nschcode	25,005	1.000	525.000	261.976	152.811
effective	25,005	0.000	1.000	0.562	0.496
courses	25,005	0.000	4.000	0.748	0.790
female	25,005	0.000	1.000	0.505	0.500
ses	25,005	-2.410	1.870	0.034	0.784
ses4cat	25,005	1.000	4.000	2.496	1.118
testmean	25,005	28.027	74.583	52.377	6.964
apexams	25,005	0.000	0.800	0.166	0.130
female_mean	25,005	0.000	1.000	0.505	0.153
ses_mean	25,005	-1.297	1.416	0.034	0.500
time	25,005	1.000	3.000	2.000	0.817
test	25,005	24.350	79.720	52.377	8.843
Valid <i>N</i> (Listwise)	25,005				

Notice also that when looking at the descriptive statistics for the new data set in Table 2.5, there are 25,005 observations. The original data set had 8,335 observations. From our earlier description, we can see that 8,335 (students) multiplied by 3 (test scores) equals 25,005 records in the new data set.

### Using “Compute” and “Rank” to Recode the Level 1 or Level 2 Data for Nested Models

From this it becomes apparent how quickly data sets can expand as models become more complicated. As the models become more complicated, the computing time necessary for the solution to converge within each level can become substantial. This is especially true in the multivariate and change models where variables or time periods are nested within individuals. Leyland (2004) notes that one can save a great deal of the time it takes to estimate a proposed model by reindexing the Level 1 (and, in some cases, the Level 2) identifiers within each school. Reindexing the individual-level data in these models can yield significant savings in run time and, in some cases, may make a difference in whether the model can even be estimated within the confines of the computer’s available memory. The objective of reindexing is to create a new set of individual identifiers that are numbered only with reference to each group. So, for example, the 10 individuals in group 1 would be numbered from 1 to 10 (assuming there were only 10 in the group), and the 14 individuals in group 2 would be numbered from 1 through 14, and so forth (1,2,...,  $n$ ).

Situations in which this may prove beneficial will be identified in later chapters. For now, however, we want to introduce the steps involved. This reindexing can be easily accomplished using the IBM SPSS RANK command. From the TRANSFORM > RANK CASES menu within IBM SPSS, we can call up the *Rank Cases* dialog box.

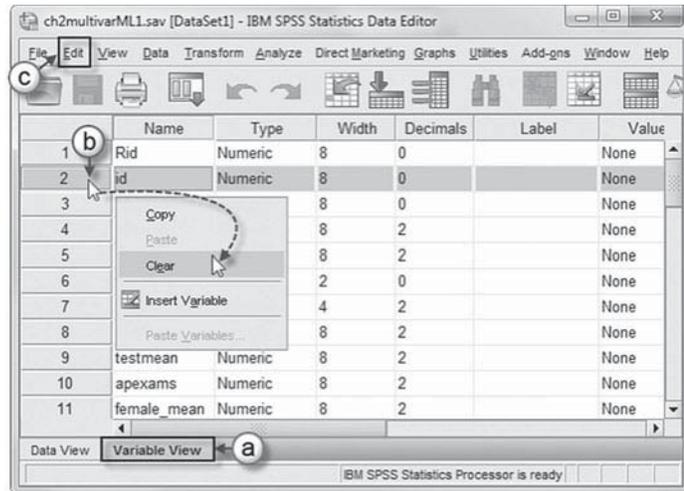
#### Creating an Identifier Variable

Creating identification variables is straightforward in IBM SPSS. We will first show how to generate an ID variable for each case. This can be useful in those instances when an identifier is not found in the file being used. For this first example, we will use the *ch2level-1data.sav* file we introduced at the beginning of this chapter (see Figure 2.1 on page 36). You may recall that this data set is single-level and cross-sectional with 6,871 observations. We have removed the *id* variable that appeared in the version of the data set we used earlier. Our first task will be to recreate an individual-level identifier using the COMPUTE command.

## Creating an Individual-Level Identifier Using “Compute”

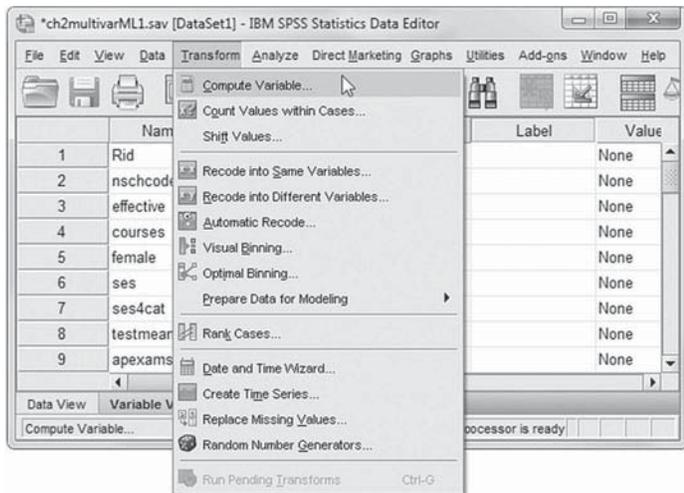
Continue using the *ch2multivarML1.sav* data set.

1. Begin by deleting the current *id* variable from the data set.
  - a. First, click on the *Variable View* tab, which displays the variables in the data set.
  - b. To delete *id*, first locate and click to select the row (2). You may then delete *id* by one of two methods: Right-click your mouse to display the submenu, and select *Clear*, which will delete the variable; or
  - c. An alternative method to deleting the variable is to go to the toolbar and select EDIT, CLEAR.



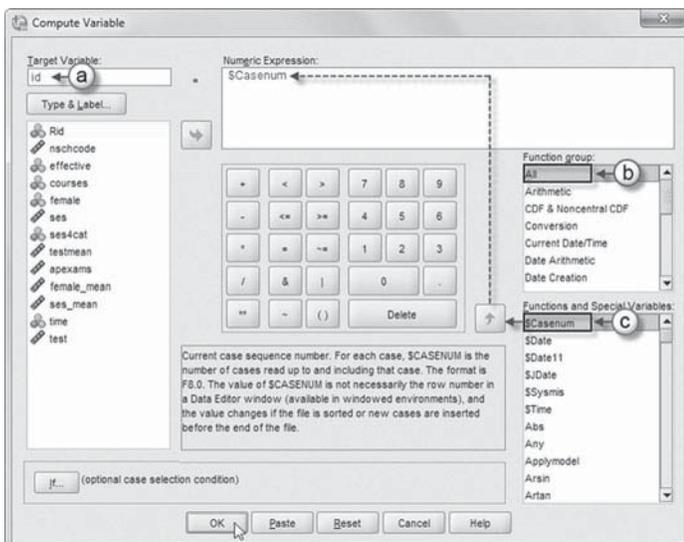
2. After removing *id* from the data set, go to the toolbar and select TRANSFORM, COMPUTE VARIABLE.

This command will open the *Compute Variable* dialog box.



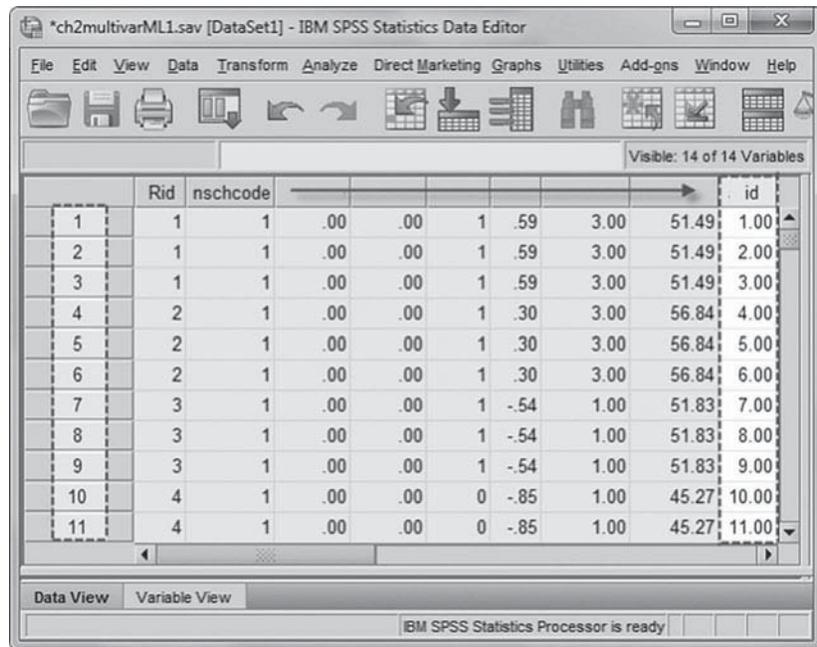
- 3a. Enter *id* as the *Target Variable*.
- b. Within the *Function group* list, click to select *All*, which will display assorted functions in the *Functions and Special Variables* box.
- c. Click to select the *\$Casenum* function, and then click the up-arrow button, which will place the term into the *Numeric Expression* box.

Click the OK button to perform the function.



4. Scroll across the columns, and the new variable *id* is found in the last column of the data window and corresponds to the case number (or row number) for each case.

**Note:** The decimal place setting may be adjusted by clicking on the *Variable View* tab and changing the number to the desired setting (i.e., “0”).

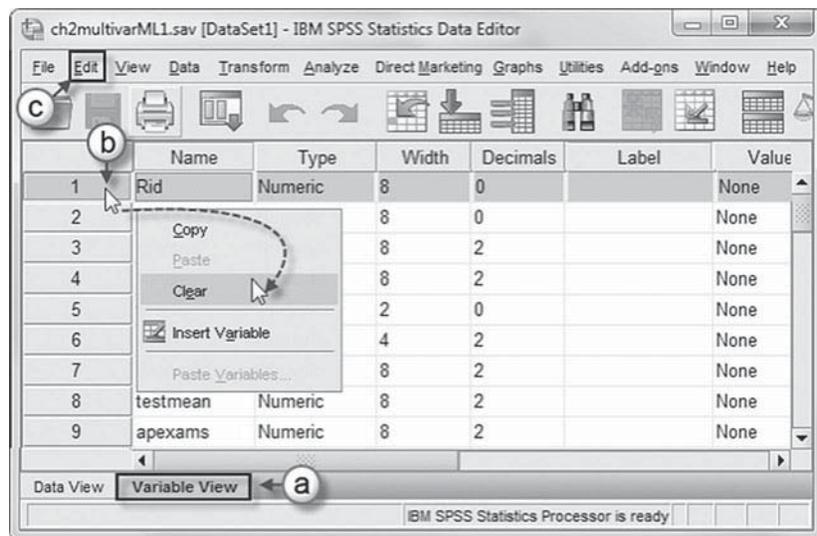


Once a Level 1 identifier exists, it is quite easy to create a variety of different within-group identifiers. These within-group identifiers will become very important in later chapters where we use more complex models. Using the TRANSFORM, RANK CASES commands from the IBM SPSS menu, we will now create another identifier assigning a sequential ID variable *Rid* within each group. The *Rid* variable will range from 1 through *n* within each Level 2 unit.

#### Creating a Group-Level Identifier Using “Rank Cases”

Continue using the *ch2multivarML1.sav* data set.

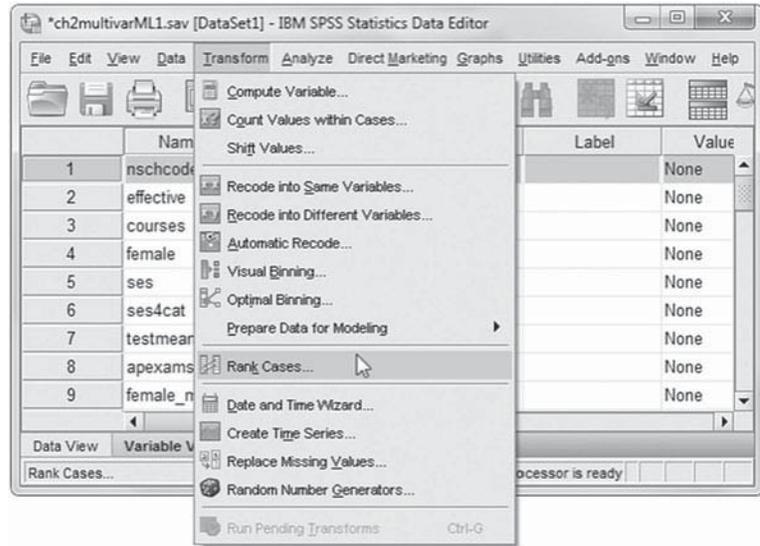
1. Begin by deleting the current *Rid* variable from the data set.
  - a. First, click on the *Variable View* tab, which displays the variables in the data set.
  - b. To delete *Rid*, first locate and click to select the row (1). You may then delete *Rid* by one of two methods: Right-click your mouse to display the submenu, and select *Clear*, which will delete the variable; or



- c. An alternative method to deleting the variable is to go to the toolbar and select EDIT, CLEAR.

2. After removing *Rid* from the data set, go to the IBM SPSS toolbar and select TRANSFORM, RANK CASES.

This command will open the *Rank Cases* dialog box, which enables creating new variables containing ranks, normal and Savage scores, and percentile values for numeric variables (IBM Corporation, 2012).

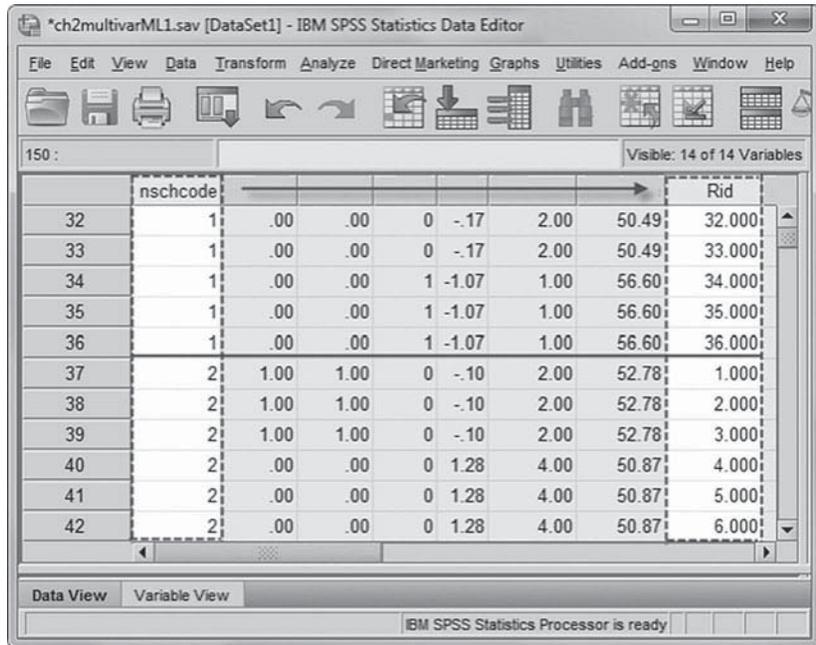


- 3a. Click to select the Level 1 identifier (*id*) from the left column, and then click the right-arrow button to move it into the *Variable(s)* box.
- b. Next, click to select group identifier (*nschcode*) from the left column, and then click the right-arrow button to move the variable into the *By* box.
- c. In the *Assign Rank 1 to* section, confirm that *Smallest value* (default setting) is selected.
- d. Click the RANK TYPES button, which will open the *Rank Cases: Types* dialog box.
- e. There are several ranking methods with a simple *Rank* as the default setting. We will use *Rank*, so click the CONTINUE button to close the dialog box.
- f. Now click the TIES button, which will open the *Rank Cases: Ties* dialog box; this box controls the method for assigning rankings to cases with the same value on the original variable (IBM Corporation, 2012).
- g. Among the options listed for *Rank Assigned to Ties*, click to select *Sequential ranks to unique values*. The option must be selected to ensure that each person is assigned an identifier within each unit. Then click the CONTINUE button to close the box.

Now click the OK button to create the *Rid* Level 1 group identifier.

4. Scroll across the columns, and the *Rid* Level 1 group identifier variable is located in the last column of the data window.

Note that each Level 2 unit (*nschcode*) will have its own sequence of *Rid* beginning with 1.

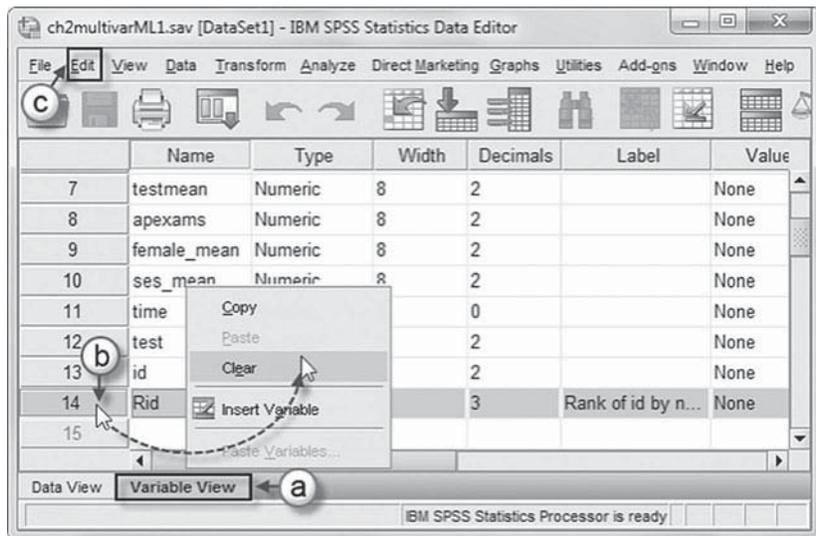


Finally, we will take this one step further by creating a new within-group id (*Rid*) variable for the multivariate data set shown in *ch2multivarML1.sav* (see Figure 2.7 on page 58).

### Creating a Within-Group-Level Identifier Using “Rank Cases”

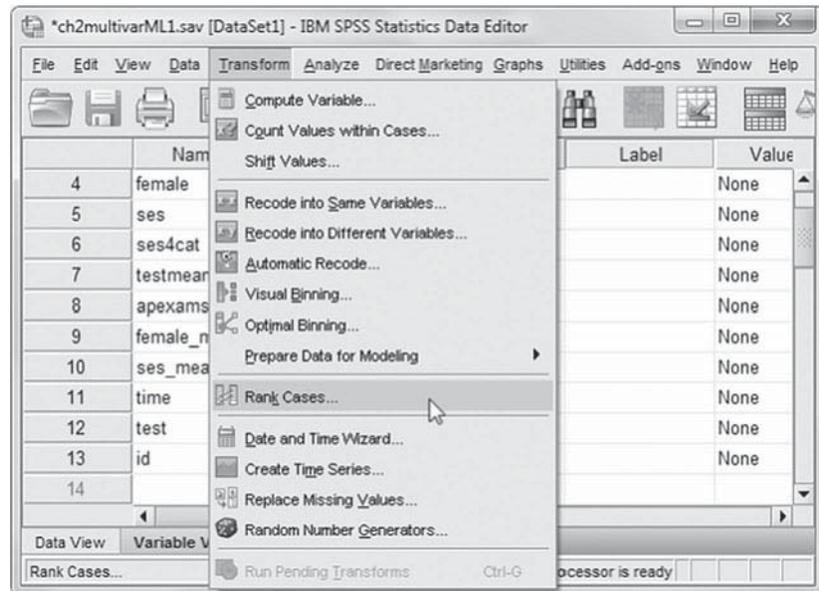
Continue using the *ch2multivarML1.sav* data set.

1. Begin by deleting the current *Rid* variable from the data set.
  - a. First, click on the *Variable View* tab, which displays the variables in the data set.
  - b. To delete *Rid*, first locate and click to select the row (14). You may then delete *Rid* by one of two methods: Right-click your mouse to display the submenu, and select *Clear*, which will delete the variable; or
  - c. An alternative method to deleting the variable is to go to the toolbar and select *EDIT*, *CLEAR*.



2. After removing *Rid* from the data set, go to the IBM SPSS toolbar and select TRANSFORM, RANK CASES.

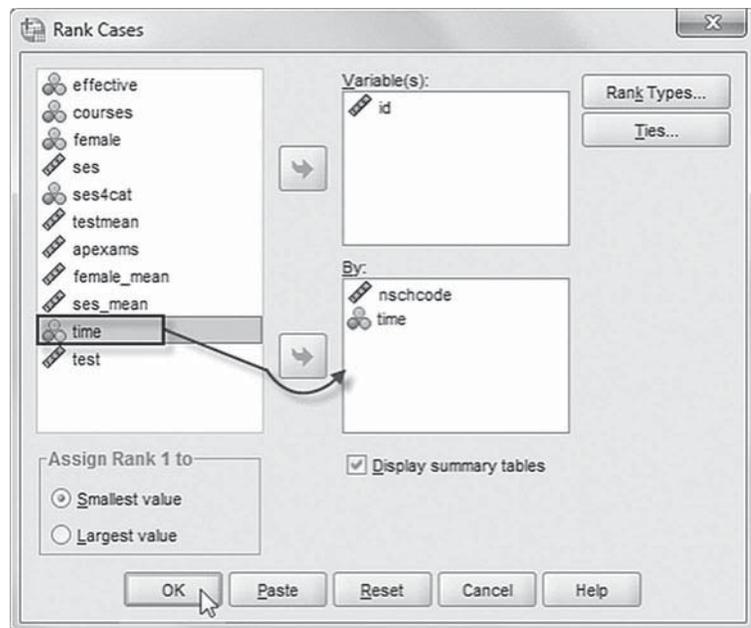
This command will open the *Rank Cases* dialog box.



3. The default settings from the prior example display *id* within the *Variable(s)* box and *nschcode* in the *By* box.

In the previous example, we selected only one group identifier, *nschcode*. Since the data in the multivariate example are vertically arranged (i.e., there are multiple records for each individual), we need to generate sequential, within-group identifiers for each person, constant across the three time points (recall that we had three time points nested within each person).

We will now add the other grouping variable (*time*) to the *By* box. This will generate a sequential identifier for each person across the three time periods within each group.



The settings for *Rank Types* and *Ties* remain the same as the prior example, so skip over these buttons and click OK.

4. Scroll across the columns, and the *Rid* variable is shown in the last column of the data viewer.

Notice that the *Rid* variable reflects the number of observations in the data set but not the number of individuals.

**Note:** If you would like to view the changes made to the *ch2multivarML1.sav* data set after completing the tutorial sections to this point, please view *ch2multivarML1-tutorials.sav*.

	nschcode	effective	courses				Rid
1	1	.00	.00	1	.59	3.00	1.000
2	1	.00	.00	1	.59	3.00	1.000
3	1	.00	.00	1	.59	3.00	1.000
4	1	.00	.00	1	.30	3.00	2.000
5	1	.00	.00	1	.30	3.00	2.000
6	1	.00	.00	1	.30	3.00	2.000
7	1	.00	.00	1	-.54	1.00	3.000
8	1	.00	.00	1	-.54	1.00	3.000
9	1	.00	.00	1	-.54	1.00	3.000
10	1	.00	.00	0	-.85	1.00	4.000
11	1	.00	.00	0	-.85	1.00	4.000

In this section we have shown the primary commands necessary to create new variables, re-code existing variables, merge new data onto the existing data set, create group-level aggregates from individual-level variables, and restructure the data set for multivariate analyses using IBM SPSS MIXED. In the sections that follow, we use these data management procedures to offer instruction on the creation of new variables that will be needed for the analyses presented in subsequent chapters.

## Centering

The basic multilevel model (i.e., a random intercept model) treats the Level 1 intercept as an outcome with variance that can be explained using variables from a higher level. While we develop this model in some detail in the following chapter, we want to convey the importance of the intercept here. Consider as a starting point the traditional fixed-effect ordinary least squares (OLS) regression model using *test1* as a predictor for some outcome.

$$Y_i = \beta_0 + \beta_1 \text{test1}_i = \varepsilon_i \quad (2.1)$$

From the earlier descriptive statistics table (Table 2.2), we know that values on *test1* range from 24.35 to 69.25 with a mean of 47.64. Recall that the intercept in a model such as that in Equation 2.1 is the value of the outcome (*Y*) when the predictor (*test1* in this example) is equal to 0. When additional terms are added to the model, the interpretation generalizes to the value of the outcome (*Y*) when each of the predictors in the model is equal to 0. So in instances where the predictor cannot be 0 (such as this example with the test score), the intercept is of little substantive use. This is fine when the emphasis is on the interpretation of slopes that are constant across groups that may exist in the sample.

But imagine a scenario in which, for some reason, we did have an interest in an interpretable slope. One way to ensure a meaningful and interpretable intercept is to alter the predictor in a way that makes 0 a meaningful value. This is often accomplished by “centering” the predictor on 0 (or some other value). First, consider the results of the OLS model specified previously. We will use scores on *test1* to predict scores on *test3* (which has a mean of 57.11).

TABLE 2.6 Coefficients<sup>a</sup>

Model	Unstandardized Coefficients		Standardized Coefficients		Sig.
	B	Std. Error	Beta	t	
1 (Constant)	13.272	0.620		21.414	.000
test1	0.920	0.013	0.616	71.351	.000

<sup>a</sup> Dependent variable: test3.

From these results in Table 2.6, we can see that a 1-point increase in performance on *test1* is associated with a 0.92-point increase on *test3*. The intercept value is 13.272. With these two values, one could calculate a predicted value for *test3* given knowledge of performance on *test1*. The intercept is used only to generate that predicted value for *test3* and has no real interpretative use because 0 is not a valid option for *test1* performance.

By centering the *test1* variable on 0, we can make the intercept more interpretable. To do this, we simply subtract the mean of *test1* from the *test1* score for each student in the data set; that is,

$$(X_{ij} - \bar{X}_{..}) \quad (2.2)$$

We will show how to do this in a moment. For now, let us just consider the change in the OLS results.

In Table 2.7, notice that the *test1* slope coefficient (and standard error) remains the same as in the previous model but that the intercept is now 57.11 or the raw mean for *test3* (see previous discussion). So when *test1* is equal to 0 (and 0 is now the overall mean for *test1*), *test3* is equal to its overall mean. The intercept now has a useful interpretation.

Because the multilevel model treats the intercept as an outcome, as we will show in the next chapter, it is very important that the Level 1 model yield an interpretable value for  $\beta_0$ . Centering makes this possible and therefore is an important feature of the multilevel model. There are two types of centering that we will be concerned with throughout the workbook: grand-mean centering, such as that used in the previous example; and group-mean centering, which centers the variable on the mean of each higher level group. In the sections that follow, we demonstrate how to create these within IBM SPSS MIXED. We further develop the rationale of each of these methods at length in subsequent chapters. Here, however, we focus on how to compute the variables that will be needed.

TABLE 2.7 Coefficients<sup>a</sup>

Model	Unstandardized Coefficients		Standardized Coefficients		Sig.
	B	Std. Error	Beta	t	
1 (Constant)	57.108	0.082		700.254	.000
g <sub>m</sub> test1	0.920	0.013	0.616	71.351	.000

<sup>a</sup> Dependent variable: test3.

TABLE 2.8 Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
test1	8,335	24.350	69.250	47.644	6.325
gctest1	8,335	-23.294	21.606	0.000	6.325
Valid N (Listwise)	8,335				

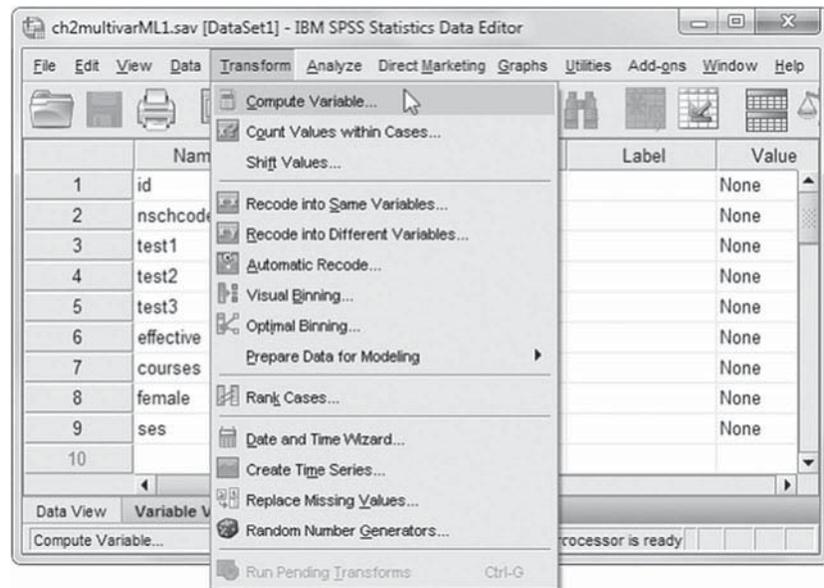
### Grand-Mean Centering

As in our previous example, variables in a multilevel model are most frequently grand-mean centered. For example, in Table 2.8 the grand mean for *test1* is 47.64. Using COMPUTE in IBM SPSS, one can name the new variable *gctest1* and then compute the new variable by subtracting the grand mean of *test1* from students' scores on that variable (i.e.,  $test1 - 47.64$ ). This will transform the scores in terms of the grand mean of the sample. So, if a student has a *test1* score of 50.64, her new score will be 3 ( $50.64 - 47.64 = 3$ ), which carries the meaning that its relative position is three points above the grand mean with respect to other students in the sample. Grand-mean centering results in unit-level means that have been adjusted for differences among individuals within the units. Notice that the distribution remains exactly the same when we center. The only thing that shifts is the scale itself.

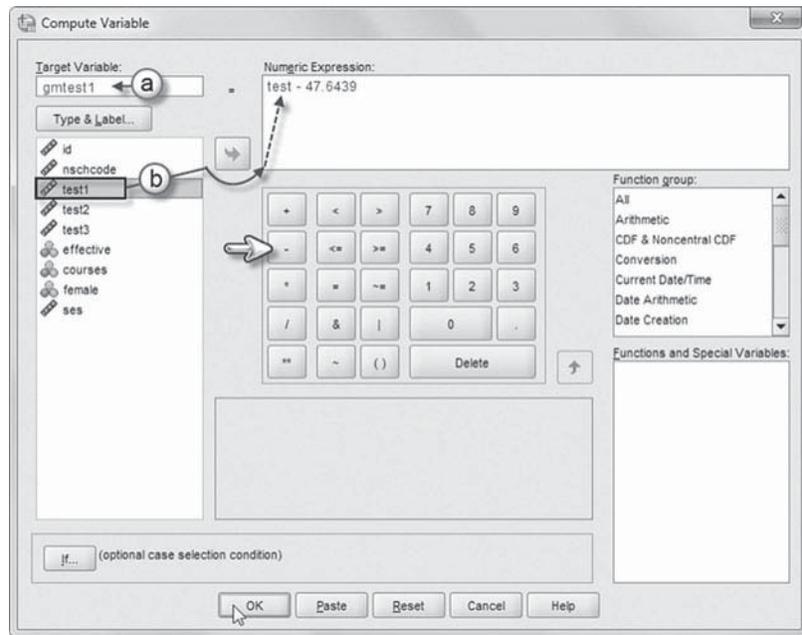
For this tutorial we will use the original nine-variable “horizontal” *ch2multivarML1.sav* data file for this section.

1. Go to the toolbar and select TRANSFORM, COMPUTE VARIABLE.

This command will open the *Compute Variable* dialog box.



- 2a. In the *Compute Variable* display screen, enter *gmtree1* as the *Target Variable*.
- b. Now select *test1*, and click the right-arrow button to move the variable into the *Numeric Expression* box. Use the numeric keypad (or enter from your keyboard) to complete the equation:  $test1 - 47.6439$ . Insert a minus sign (-) by clicking on the key.



**Note:** Use ANALYZE, DESCRIPTIVE STATISTICS, DESCRIPTIVES to obtain the mean of *test1* (47.6439).

Click the OK button to create the computed variable *gmtree1*.

3. Scroll across the columns, and the computed variable *gmtree1* is found in the last column of the *ch2multi-varML1.sav* data window.

	id	nschcode						gmtree1
1	1	1	49.66	50.09	54.72	.00	.00	2.02
2	2	1	47.92	58.26	64.33	.00	.00	.28
3	3	1	49.12	50.63	55.74	.00	.00	1.48
4	4	1	38.77	50.93	46.12	.00	.00	-8.87
5	5	1	47.54	51.53	60.90	1.00	1.00	-.10
6	6	1	41.74	48.82	54.55	1.00	2.00	-5.90
7	7	1	32.83	42.43	48.73	.00	.00	-14.81
8	8	1	55.91	56.36	60.91	.00	.00	8.27
9	9	1	52.93	63.75	68.37	.00	.00	5.29
10	10	1	39.47	42.30	44.15	.00	.00	-8.17
11	11	1	42.69	52.43	56.36	.00	.00	-4.95

A quick look at the descriptive statistics of the original *test1* variable and its grand-mean centered counterpart in Table 2.8 show that although the standard deviation remains the same, the means and ranges stay the same while the minimum and maximum values have changed. Again, this simply demonstrates that only the scale has changed and that the distribution (or variability around the mean) itself remains the same. Note that we have not standardized the variable, but rather we have simply readjusted it so its mean is equal to 0.

## Group-Mean Centering

Group-mean centering of variables yields values that represent the unadjusted mean for group  $j$ . So rather than using the overall mean as the reference, the group mean is used instead.

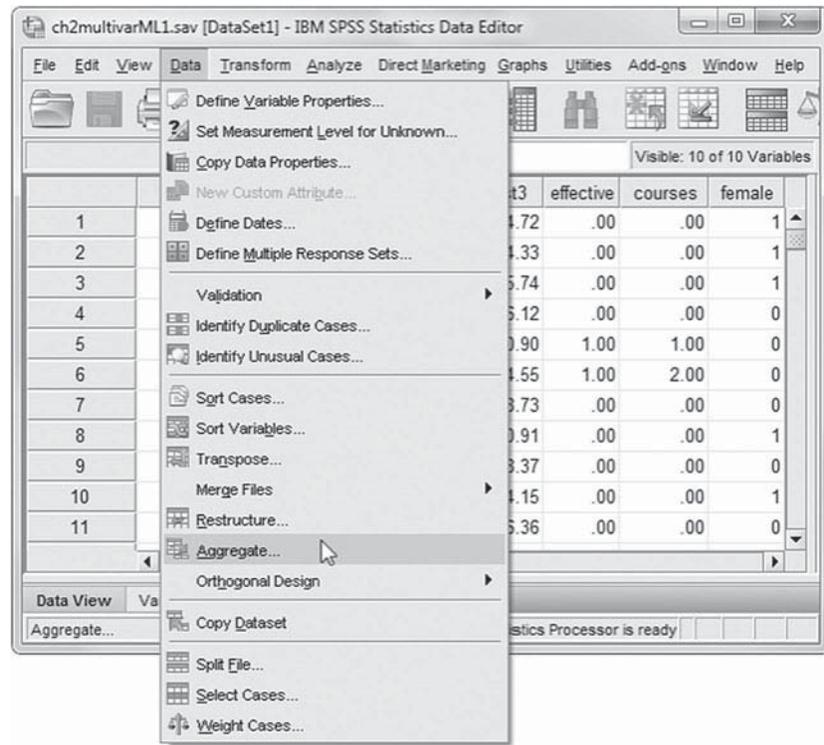
$$X_{ij} - \bar{X}_j \quad (2.3)$$

Group-mean centering of variables is a two-step process that first involves the aggregation of the focal variable to the group level and then follows a logic similar to that used for grand-mean centering described previously. We will use the AGGREGATE and COMPUTE commands to accomplish this.

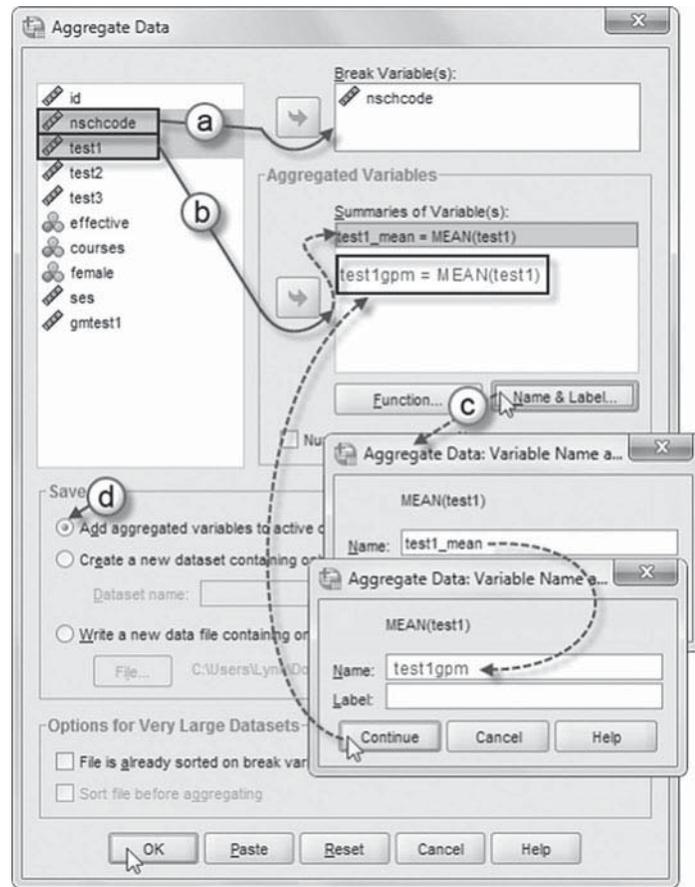
Continue using the *ch2multivarML1.sav* data file, which now contains 10 variables at the conclusion of the prior tutorial.

1. Go to the toolbar and select DATA, AGGREGATE.

This command opens the *Aggregate Data* dialog box.



- 2a. Within the *Aggregate Data* dialog box, click to select *nschcode* from the left column, and then click the right-arrow button to move the variable into the *Break Variable(s)* box.
- b. Next, click to select *test1* from the left column, and then click the right-arrow button to move the variable into the *Summaries of Variable(s)* box. (IBM SPSS uses MEAN as the default function, which will be used for this example.)
- c. Change the output variable name by clicking on the NAME & LABEL button, which will open the *Aggregate Data: Variable Name and Label* box. Then replace the initial variable name *test1\_mean* with the current *test1gpm*. Click the CONTINUE button to close the box when completed to return to the *Aggregate Data* main dialog box.



Click the OK button to perform the aggregation and create the new *test1gpm*.

3. Scroll across the columns, and the new aggregated variable *test1gpm* appears in the last column of the *ch2multivarML1.sav* data window.

Notice that this new variable is constant within each Level 2 unit (schools); that is, the value represents the mean *test1* score within each school. These values provide the reference value for the group centering of *test1* (just as the overall mean provided the reference value for grand-mean centering). With the group-mean value known, it is easy to calculate the group-centered *test1* variable, which we will call *group<sub>test1</sub>*.

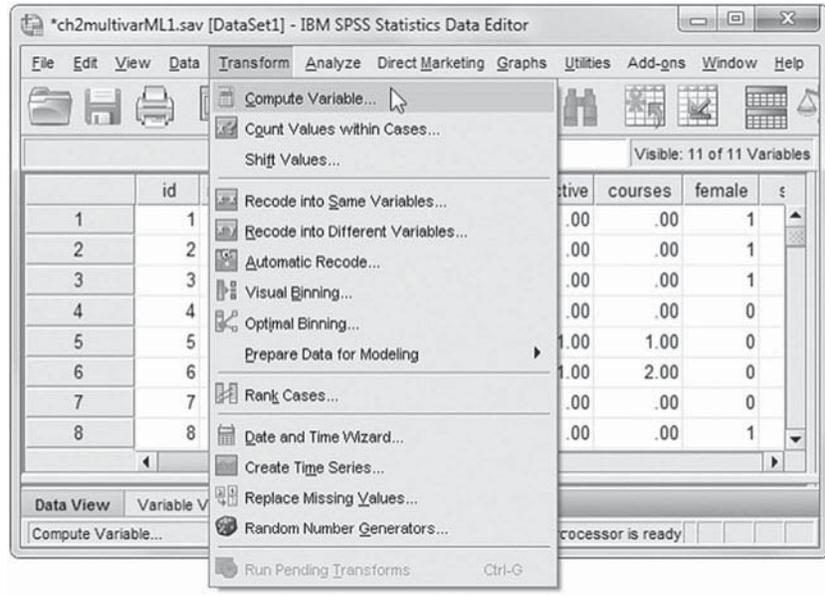
	id	nschcode	test1	test2	test3		test1gpm	
6	6	1	41.74	48.82	54.55	1.00	2.00	45.60
7	7	1	32.83	42.43	48.73	.00	.00	45.60
8	8	1	55.91	56.36	60.91	.00	.00	45.60
9	9	1	52.93	63.75	68.37	.00	.00	45.60
10	10	1	39.47	42.30	44.15	.00	.00	45.60
11	11	1	42.69	52.43	56.36	.00	.00	45.60
12	12	1	48.64	58.98	62.17	.00	.00	45.60
13	13	2	47.54	51.53	59.26	1.00	1.00	44.59
14	14	2	46.83	53.70	52.09	.00	.00	44.59
15	15	2	45.89	52.95	56.16	.00	.00	44.59
16	16	2	47.54	51.53	58.06	1.00	1.00	44.59

We will specify the target variable we wish to create (*group<sub>test1</sub>*) and create the Numeric Expression *test1 - test1gpm*.

Continue using the *ch2multi-varML1.sav* data file.

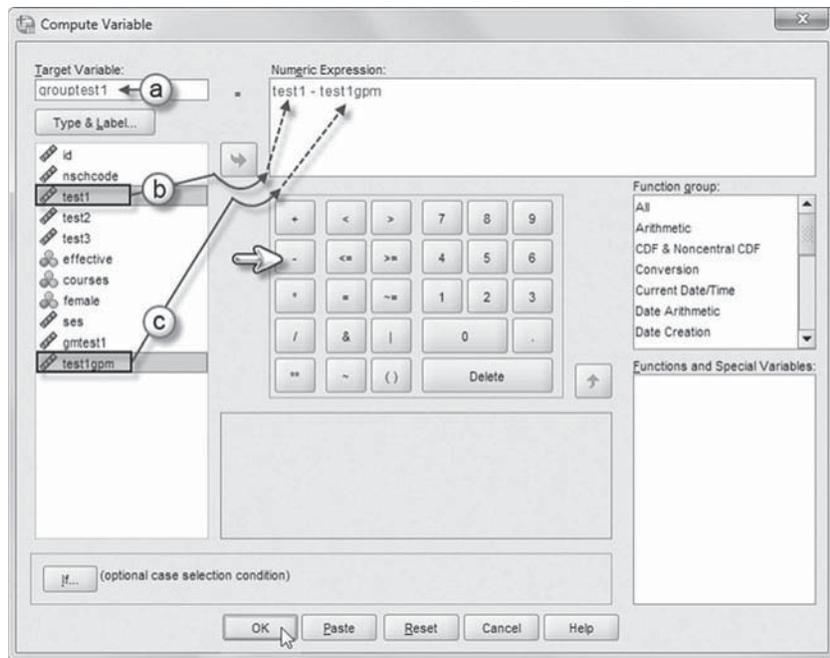
1. Go to the toolbar and select TRANSFORM, COMPUTE VARIABLE.

The command opens the *Compute Variable* dialog box.



**Note:** The formula (*test1 - 47.6439*) used in the preceding exercise may appear in the *Numeric Expression* box. To remove the formula, click the RESET button at the bottom of the screen.

- 2a. Enter *grouptest1* as the *Target Variable*.
- b. Click to select *test1* from the left column, and then click the right-arrow button to move the variable into the *Numeric Expression* box. Insert a minus sign (-) by clicking on the key.
- c. To complete the numeric expression, click *test1gpm* from the left column, and then click the right-arrow button to move the variable into the box. This completes the *Numeric Expression* of subtracting *test1gpm* from *test1*.



Click the OK button to perform the function.

3. Scroll across the columns, and the new *grouptest1* variable appears in the last column of the data window.

We will specify the target variable we wish to create (*grouptest1*) and create the Numeric Expression  $test1 - test1gpm$ .

We can now compare the three variables: (a) *test1*, (b) *gmtest1*, and (c) *grouptest1*.

The screenshot shows the IBM SPSS Statistics Data Editor window for a file named 'ch2multivarML1.sav [DataSet1]'. The window displays a data grid with 12 columns and 11 rows of data. The columns are labeled 'id', 'nschcode', 'test1', 'test2', and 'grouptest1'. The data values are as follows:

id	nschcode	test1	test2	grouptest1
1	1	49.66	50.09	4.06
2	2	47.92	58.26	2.32
3	3	49.12	50.63	3.52
4	4	38.77	50.93	-6.83
5	5	47.54	51.53	1.94
6	6	41.74	48.82	-3.86
7	7	32.83	42.43	-12.77
8	8	55.91	56.36	10.31
9	9	52.93	63.75	7.33
10	10	39.47	42.30	-6.13
11	11	42.69	52.43	-2.91

**TABLE 2.9** Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
test1	8,335	24.350	69.250	47.644	6.325
gmtest1	8,335	-23.294	21.606	0.000	6.325
grouptest1	8,335	-22.897	22.904	0.000	5.879
Valid N (Listwise)	8,335				

Notice in Table 2.9 that while the uncentered *test1* and the grand-mean-centered *test1* variables share the same distribution (but different means), the group-mean-centered *test1* variable has a mean of 0 (like the grand-mean version) but a different standard deviation. This results from the variance in *test1* means across the schools in the sample—an artifact that will become very important in subsequent analyses.

For reasons we will discuss in Chapter 4, researchers often enter dummy variables in the model as uncentered (although there are occasions when analysts may also wish to grand-mean center them), and they grand-mean center the continuous variables at Level 1 and Level 2. We will also show how this logic generalizes to situations in which there are more than two levels of analysis.

## Checking the Data

The diligent analyst always takes great care to examine the data thoroughly. As data sets for multilevel analyses can become quite complex, in terms of structure and content, great attention to detail should be given when reviewing the contents of the data set. IBM SPSS has a rich set of tools for the exploratory analysis of the data. The IBM SPSS EXPLORE routine, for example, can provide rich detail on data coding, distributions, missing data, and the like (this can be accessed through the ANALYZE, DESCRIPTIVE STATISTICS, EXPLORE menu). From

Chapter 1, it should be clear that missing data can prove to be a real headache when using IBM SPSS MIXED or any other multilevel-modeling routine. We highly recommend that missing data be carefully assessed to determine any patterns that may exist and to find remedies where at all possible.

Beyond a careful examination of the data, thought should be given to model specification and the distribution of residuals. Raudenbush and Bryk (2002, p. 253) point out that model specification assumptions apply at each level and that misspecification at one level can impact the results at other levels. Moreover, as with OLS regression at a single level, there are assumptions about the distribution of the residuals that apply at each level in the multilevel model. Although we do not spend a great amount of time addressing these important issues in this workbook, we do recommend that the reader familiarize him- or herself with the possibilities that exist for model checking in the multilevel framework.

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## A Note About Model Building

As will become clear in the chapters that follow, developing and testing multilevel models requires a great deal of thought as well as trial and error. Even with a basic two-level model, there are many intermediate steps over which the model evolves. Keeping track of this evolution is essential for understanding the way the model is behaving and for replicating the models in subsequent steps.

We have over the years developed a fairly simple naming scheme for our models and take care to document each model as fully as possible. Moreover, while we may use the IBM SPSS graphical user interface to develop the model, we always have IBM SPSS export the syntax so we can save it for future reference. Our naming scheme is applied to the syntax files themselves. At a glance, we can determine the type of model specified through the syntax (e.g., from a simple ANOVA model to a fully specified three-level, random-slopes and intercept model). With the syntax for each model saved and annotated, we can always document the evolution of model specification and easily modify models at any point in the future. We will have more to say about this throughout the workbook.

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## Summary

This chapter has provided an overview of the data management tools necessary for understanding and working with the hierarchical data files used in multilevel modeling. We have introduced five primary commands for manipulating data files to suit the needs of univariate and multivariate analyses using IBM SPSS MIXED. There is, of course, a great deal more than could be presented. Our main purpose in this workbook is, however, the modeling techniques themselves rather than treating the more universal data management skills used to structure data within IBM SPSS. The treatment provided in this chapter is designed to highlight the elementary skills associated with data management relating to the specification of the multilevel model within IBM SPSS.

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# CHAPTER 3

## Defining a Basic Two-Level Multilevel Regression Model

This chapter introduces the basic approach to two-level multilevel modeling. The material is challenging because the models are more complex than the general linear model, which most readers will be familiar with from their basic statistics courses. Like everything else, however, one has to start somewhere. The general concepts we present in this chapter become more familiar as one reads more research that makes use of multilevel techniques. After first reviewing some basic concepts of the single-level multiple regression model, we develop the basic steps of conducting a multilevel regression analysis using an extended example. Our intent is to develop the rationale behind the specification of this general class of models in a relatively nontechnical manner and to illustrate its use in an applied research situation. The methods presented in this chapter should provide a basis for the application of these techniques to a wider set of research problems in the chapters that follow.

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### From Single-Level to Multilevel Analysis

Statistical modeling depends on a family of probability distributions for outcome variables (Agresti, 2007). We often use the term *random variable* to describe possible values that an outcome may have. A probability distribution is a mathematical function that links a particular observed outcome obtained in a sample to the probability of its occurrence in a specific population. The most common example involves the sampling distribution of the mean from which the probability of obtaining particular samples with a particular mean can be estimated (Azen & Walker, 2011). Most commonly, the observed mean is assumed to result from a normal distribution of possible values around the population mean, which has some variance. We assume that the general shape of this distribution resembles a bell-shaped curve; that is, the majority of individuals are closer to the mean, and there are fewer individuals as the distance from the mean increases. The normal distribution is commonly associated with continuous outcomes (i.e., variables measured on an interval or ratio scale). Other types of categorical (discrete) variables (e.g., nominal, ordinal, and count) have different probability distributions and require other types of analytic methods for optimal analysis. Examples include being proficient or not proficient in reading, purchasing a sports car (among several possible types of cars), expressing a level of agreement or disagreement on a political issue, or receiving a number of traffic tickets during a specified interval of time. Random values of these different types of categorical outcomes are obtained from probability distributions (i.e., binomial, multinomial, and Poisson) other than the normal distribution (for further discussion, see Agresti, 2007; Azen & Walker, 2011; Heck, Thomas, & Tabata, 2012; Hox, 2010).

Linear models (e.g., analysis of variance [ANOVA], analysis of covariance, multiple regression, and multivariate analysis of variance [MANOVA]) have long been used in the social sciences to analyze data from experimental, quasi-experimental, and nonexperimental designs. Univariate analysis, such as multiple regression, is concerned with examining the variability in a single continuous outcome (or dependent) variable from information provided by one or more predictor (or independent) variables. Multivariate analysis (e.g., MANOVA and factor analysis) is the more general case of univariate analysis; that is, it facilitates the examination of multiple independent and dependent variables in one simultaneous model. A commonality between these univariate and multivariate approaches, however, is that they are confined to single-level analyses; that is, either individuals are the unit of analysis or groups are the unit of analysis.

Multiple linear regression requires a continuous dependent variable (i.e., measured on an interval or ratio scale) and can handle both continuous and dichotomous (e.g., gender) independent variables. It cannot handle categorical variables (i.e., referred to as factors in analysis of variance terminology) without recoding them in some way. There are two broad conceptual approaches to the regression model, predictive and explanatory. Through the predictive approach, the analyst uses the multiple regression model to optimize predictions about an outcome based on values of a set of independent variables. The linear regression model assumes that a unit increase in the independent variable is related to an expected constant change in the dependent variable. For example, we might wish to predict someone's likely starting salary in a new job if she or he has a certain level of education and experience. For the linear model to hold, it is assumed that an increase in education (or experience) will bring an expected similar change in starting salary, regardless of where someone starts in terms of education. In this type of single-level regression model, the coefficients that describe the prediction equation (i.e., the intercept and slope coefficients for each predictor in the model) are generally considered as fixed values in the population estimated from the sample data. For this type of research purpose, the focus of the analysis is primarily on the efficiency of the prediction and the parsimony of variables included in the prediction equation. In other words, the analyst hopes to make the best predictions using the smallest number of variables.

The second broad approach is explanatory rather than predictive. Through the explanatory approach, the analyst sets out to determine how a set of independent variables affects a dependent variable and to estimate the magnitude of the effects for each independent variable. For example, existing research may suggest that a particular model (e.g., consisting of identified market processes, individual background, and perhaps organizational factors) interacts in a way that influences beginning salary. The focus in this type of study rests on the correct specification and testing of a theoretical model that is under consideration. In this case, it is important to include in the model a set of variables identified as important by theory and previous research. More specifically, the researcher formulates a model from theory, tests the model against the data, and determines how well the empirical test of the model conforms to theoretical expectations.

Of course, these goals are not mutually exclusive. We distinguish between these two goals, however, because in predictive studies variables might be retained in a model only because they are statistically significant and dropped simply because they are not (Heck & Thomas, 2009). In other words, theory would not enter into decisions about model efficiency. In contrast, in the explanatory approach, the specification of the theoretical model should be carefully considered, and subsequent changes (i.e., whether to add or remove a variable from a model) should be made sparingly with careful attention to theory. Otherwise, it may be difficult to attach any substantive meaning to the final model. This latter point has particular relevance to the investigation of multilevel data (i.e., data on individuals and the groups they define) that tend to go with more complex theories about how processes operate across multiple social groupings.

Although researchers were aware of problems due to the nesting of individuals within higher level units of the data hierarchy in the past, the presence of similarities among individuals in the

same groups did not enter directly into single-level analyses. For example, in analyses of large-scale survey data, the analyst typically applied sample weights to address the oversampling of some subgroups in the data set (e.g., by socioeconomic status [SES] or by ethnicity). Failure to account for similarities among individuals (due to grouping) within the study, however, can lead to biased estimates of model parameters and therefore erroneous conclusions about the effects of some predictors in the model (Thomas & Heck, 2001).

Multilevel modeling represents a compromise between modeling each unit separately and modeling all unit contexts simultaneously within the same model (Kreft & de Leeuw, 1998). These models obviate the forced choice of conducting either an individual-level analysis or a group-level analysis. We use the term *multilevel model* with respect to two separate statistical objectives described within one model. The first objective concerns inferences made about a model's structural parameters (Morris, 1995), often referred to as the model's fixed effects. The second objective concerns inferences about the unknown variance parameters in the model, referred to as the random parameters (Morris, 1995). Although researchers are generally most interested in the model's structural parameters, the distribution of a model's random parameters (e.g., variances and covariances) is also of interest.

There are several advantages of multilevel analysis over traditional single-level univariate and multivariate approaches (Heck & Thomas, 2009). First, as we have stated, multilevel analysis helps researchers avoid the choice of individuals or groups as the unit of analysis. Second, it allows researchers to deal with more complicated sampling strategies. Single-level analyses are based on the assumption of simple random samples. In many data collection strategies, however, individuals may be sampled within the same neighborhoods or schools, or subgroups of individuals (e.g., by ethnicity or SES) may be oversampled compared with their representation in the population. Such complex sampling strategies create clustering effects that violate the assumptions of simple random sampling (i.e., that every individual has an equal chance of being selected in the sample). Third, where similarities among individuals are present (e.g., clustering effects due to sharing similar circumstances), multilevel models are acknowledged to provide more accurate estimates of model parameters than single-level analyses (Hox, 2002). This is primarily due to their greater accuracy in calculating standard errors associated with parameter estimates. Because hypothesis tests are based on the ratio of the unstandardized estimate to its standard error, ignoring the presence of nested data structures can lead to underestimating standard errors and, therefore, false inferences about the significance of model parameters (Thomas & Heck, 2001). Fourth, multilevel analysis allows the researcher to define variables at their correct theoretical level of the data hierarchy. So, for example, in a two-level hierarchy, a variable such as school size can be determined with respect to the number of schools in the sample, while a variable like gender can be evaluated with respect to the number of individuals in the sample. Finally, multilevel modeling allows researchers to ask more complex questions about the data. One is about the distribution of outcomes (e.g., means or regression slopes) across a sample of groups (such as schools). We may attempt to determine what types of school variables might reduce gaps in student learning due to socioeconomic status or previous skill levels. Examples might include the quality of a school's teachers, its leadership practices, and its classroom learning activities.

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## Building a Two-Level Model

The basics of multilevel modeling involve the investigation of randomly varying outcome parameters. These typically include variation in the levels of the outcome (intercepts) and the strength of within-group relationships indicated by regression coefficients (slopes) across groups. Once we identify that variation exists in the parameters of interest, we can build models to explain this variation. As we suggested in Chapter 1, in some cases, we may have a specific theoretical model in mind that we wish to test; in others, however, we might be trying to explore possible new mechanisms that explain this variation.

Consider an analysis where the researcher wishes to determine whether there is an association between a predictor,  $X$ , such as SES, and an outcome,  $Y$ , such as a math test score. Because the current educational policy context in the United States demands increasing accountability for student outcomes, schools are accountable for reducing gaps in achievement due to students' social backgrounds. Such concerns are related to the social distribution of learning within schools (e.g., see Lee & Bryk, 1989). Ideally, we may wish to identify school settings where achievement is generally high for all the students in the school and where there is little or no relationship between student SES and outcomes. Such schools would be considered both effective (i.e., producing high achievement outcomes) and equitable (i.e., having little or no social distribution of learning due to students' social backgrounds). In contrast, we may also wish to identify schools where achievement is consistently low for students and where student SES background is consequential for their outcome levels. We might be able to intervene effectively (e.g., increase teacher quality and reallocate resources) if we could identify such settings that are ineffective and inequitable for students.

### Research Questions

Our first research question focuses on whether student achievement in math varies across schools. We can ask simply: Does student math achievement vary across schools? We might then investigate the relationship between students' socioeconomic status and their math achievement. Second, we might ask whether the effects of individual SES tend to compound at the school level to influence student math achievement; that is, do both individual-level SES and school-level aggregate (or average) student SES influence math achievement? Third, we investigate whether features of schools' contexts (i.e., student composition and type of school) and their academic environments (i.e., the proportion of students planning to attend 4-year universities after they graduate from high school) affect the relationship between individual student SES and math achievement. More specifically, we ask: Do features of schools' contexts and academic environments moderate the relationship between individual student SES and math achievement? Our research questions, therefore, provide an illustration of building a two-level model to investigate (a) a randomly varying intercept (math achievement level) and, subsequently, (b) a randomly varying slope (i.e., the individual SES–math achievement relationship).

### The Data

The data set used in this example consists of 6,871 secondary students in 419 schools (see Table 3.1).

### Specifying the Model

We will begin with a single-level analysis (i.e., considering only the students and not their nesting within schools) as a starting point. One typical research question for a single-level analysis might be: Is there a relationship between students' SES background and their achievement levels in math? We might hypothesize that socioeconomic status is positively related to the subject's score on the math test. The single-level multiple regression model to explain an individual's ( $i$ ) math achievement outcome would be

$$Y_i = \beta_0 + \beta_1 SES_i + \varepsilon_i, \quad (3.1)$$

where  $\beta_0$  is the intercept,  $\beta_1$  is a slope parameter, and  $\varepsilon_i$  represents error in predicting individual outcomes from the equation. The intercept represents the expected math achievement score for a student whose SES is 0. It is important to consider the scaling of the independent variable or variables in a model. In this simple case, because SES (which is a continuous variable) was

**TABLE 3.1** Data Definition of *ch3multilevel.sav* (*N* = 6,871)

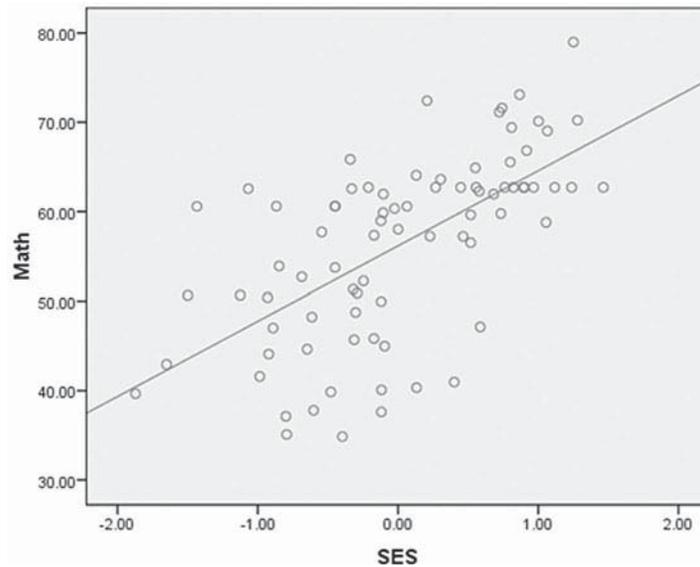
Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>schcode</i>	School	School identifier (419 schools).	Integer	Ordinal
<i>Rid</i>	Individual	A within-group level identifier representing a sequential identifier for each student within 419 schools.	1 to 37	Ordinal
<i>id</i>	Individual	Student identifier (6,871 students).	Integer	Ordinal
<i>female</i>	Individual	Demographic predictor variable representing student's gender.	0 = Male 1 = Female	Scale
<i>ses</i>	Individual	Predictor interval variable (z-score) measuring student socioeconomic status composition within the schools.	-2.41 to 1.87	Scale
<i>femses</i>	Individual	Predictor variable (grand-mean centered) measuring student socioeconomic status by gender (female).	-2.41 to 1.85	Scale
<i>math</i>	Individual	Student math achievement test score.	27.42 to 99.98	Scale
<i>ses_mean</i>	School	Predictor variable (grand-mean centered) measuring student socioeconomic status.	-1.30 to 1.44	Scale
<i>pro4yrc</i>	School	Variable measuring the school's academic program (i.e., aggregate percentage of students who intend to study at 4-year universities).	0.00 to 1.00	Scale
<i>public</i>	School	Dichotomous variable identifying school type.	0 = Other 1 = Public School	Scale

<sup>a</sup> Individual = Level 1; school = Level 2.

standardized (i.e., rescaled into a standardized score, or *z*-score), the mean is 0 and the standard deviation (*SD*) is 1.0. This is often a convenient scale for continuous variables in multilevel modeling, since a score of 0 on SES, therefore, represents the score for a person whose SES background is equal to the SES grand mean for the sample. We discuss these types of centering options in further detail at the end of this chapter and also in Chapter 4. The slope coefficient ( $\beta_1$ ) represents the expected change in math achievement for a 1-unit (in this case, 1 *SD*) change in SES.

The key point about a single-level model is that the estimates of the intercept and slope are each fixed to one value that describes the average for the sample. For example, the slope expressing the relationship between SES and math scores will be the same across all cases. This also means that the errors ( $\epsilon$ ) in estimating the intercept and slope parameters are assumed to be independent, to be normally distributed, to have constant variance, and to have a mean of 0.

To examine her research question preliminarily, the researcher might first develop a scatterplot of the relationship between student SES and math achievement. To illustrate this in relation to single and multilevel designs, we will develop a scatterplot for the first 80 students in the data set. The resulting graph is summarized in Figure 3.1. The figure suggests that as student SES increases, so do math scores. The goal of the overall analysis is to determine the best-fitting line that describes the relationship between student SES and test scores in this sample. This is accomplished by estimating values for the intercept and slope. Of course, once we estimate the predicted values for each subject on the two variables, there will be a discrepancy between the predicted values (which would lie on the line) and subjects' actual values on the SES and math test score measures. The difference between observed and predicted values is represented as error. The intercept coefficient represents the average level of student scores when SES is 0 (which represents a mean adjusted for SES), and the slope represents the average effect of SES on the math score across the sample of students. These values become "fixed" for the entire sample; that is, because individuals are randomly sampled, it is assumed that the value represents population averages.



**FIGURE 3.1** Regression line describing the fixed intercept and slope for student SES and math achievement.

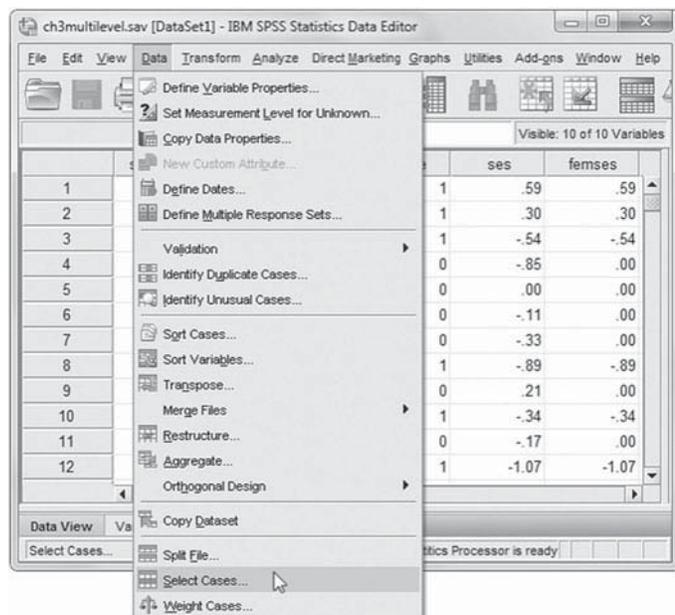
Fixing the values of the intercept and slope results in the regression line in Figure 3.1 that summarizes the relationship between SES and math test scores. The principle of least squares states that the correct regression line is the one that best fits the data points. Model fit is assessed by summing the squared distances of each observed value from the predicted value that rests on the regression line. The line that minimizes the sum of these squared distances (they are squared to cancel out positive and negative errors above or below the line) is said to fit the data best; hence, the term *least squares regression* (Neter, Kutner, Nachtsheim, & Wasserman, 1996). In the linear regression model, the error term is a random source of variation, which we assume is 0 on average and normally distributed, varies independently of  $X$  and has constant variance across all levels of  $X$ . Interested readers can reproduce the scatterplot in Figure 3.1 in IBM SPSS menu commands.

### **Graphing the Relationship Between SES and Math Test Scores with IBM SPSS Menu Commands**

Launch the IBM SPSS application and select the data file: *ch3multilevel.sav*.

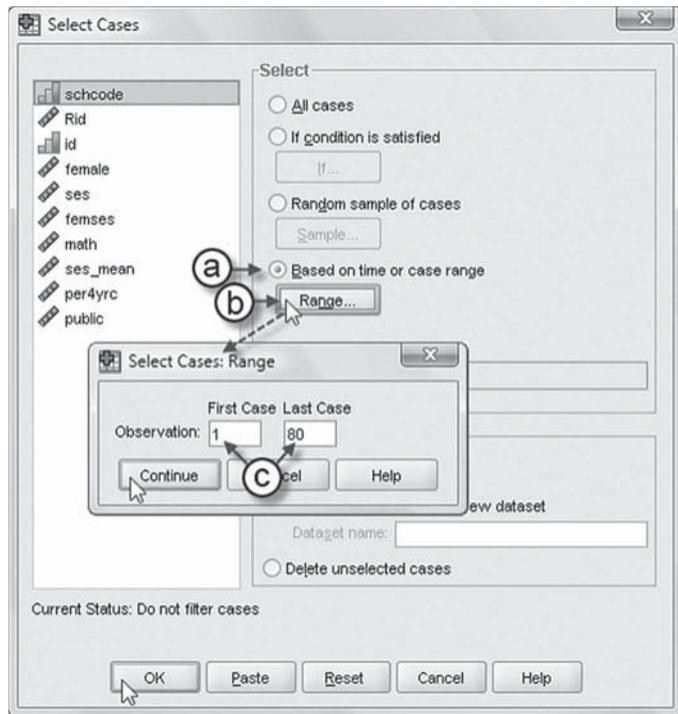
1. Go to the toolbar and select DATA, SELECT CASES.

This command will open the *Select Cases* dialog box.



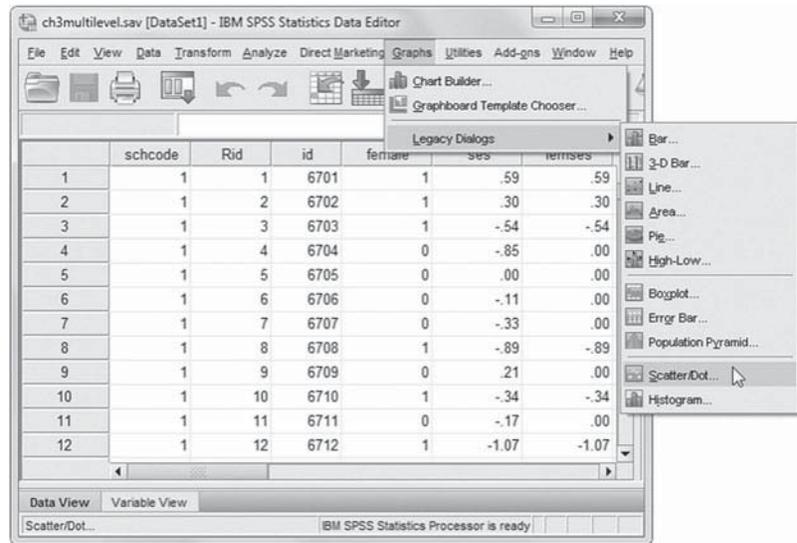
- 2a. Within the *Select Cases* dialog box, select the option *Based on time or case range*, which will activate the RANGE button.
- b. Click on the RANGE button, which will open the *Select Cases: Range* box.
- c. Enter “1” as the *First Case* and “80” as the *Last Case*. This setting will select the first 80 individuals in the study. Then click the CONTINUE button, which will close the *Select Cases: Range* box.

Click the OK button to close the main *Select Cases* dialog box and return to the IBM SPSS main screen.

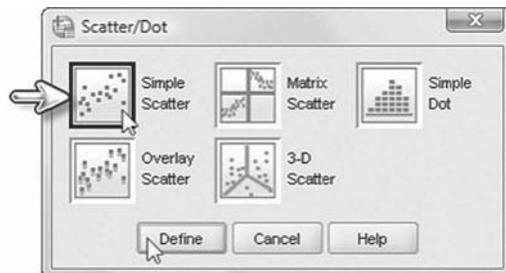


3. In the IBM SPSS toolbar, select GRAPHS, LEGACY DIALOGS, SCATTER/DOT.

This command will open the *Scatter/Dot* box.

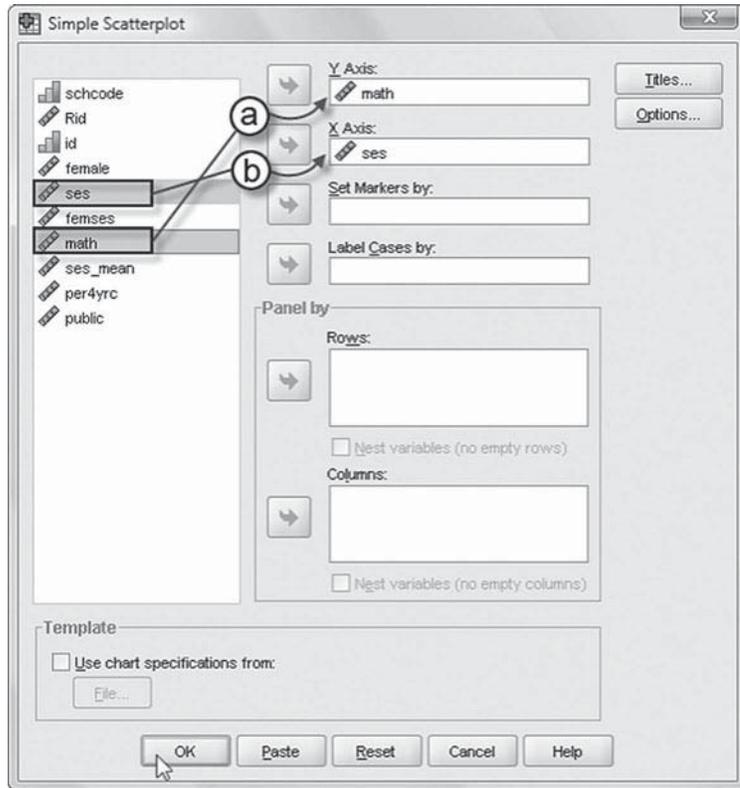


4. Click the SIMPLE SCATTER option in the *Scatter/Dot* box. Then click the DEFINE button to open the *Simple Scatterplot* box.



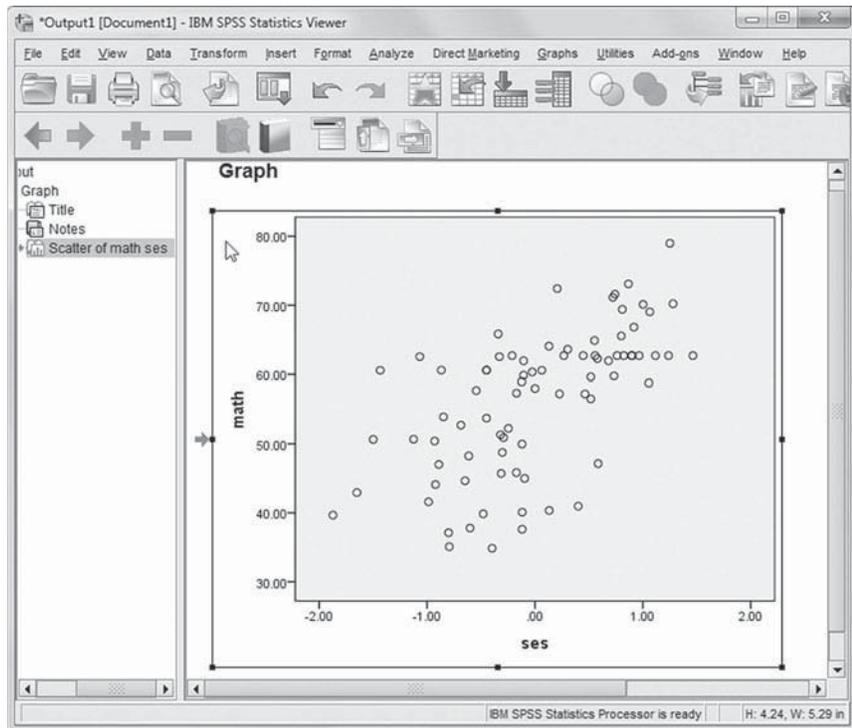
- 5a. Within the *Simple Scatterplot* box, click to select the *math* variable from the left column. Then click the right-arrow button to move *math* into the *Y Axis* box.
- b. Next, click *ses* to select the variable from the left column, and then click the right-arrow button to move the variable into the *X Axis* box.

Click OK to generate the scatterplot.

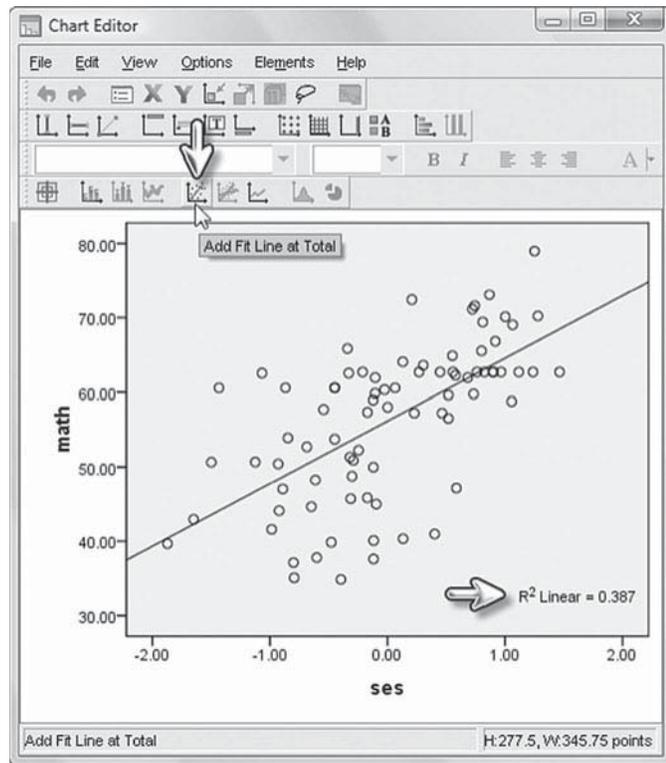


6. The IBM SPSS output will display the scatterplot.

Place your cursor on the image and double-click to open the *Chart Editor*.



7. On the *Chart Editor* short-cut icon bar, click the ADD FIT LINE AT TOTAL icon that has the  $X$  and  $Y$  axes. This will generate the fit line and the linear  $R$ -square of 0.387 for this subset of the sample.



As Figure 3.1 emphasizes, in a single-level linear model, the coefficients describing the intercept and slope are generally considered as fixed values in the population estimated from the sample data. As shown in Table 3.2 on page 84 (based on an analysis of the full sample), the intercept is estimated as 57.598. This can be interpreted as the sample mean adjusted for individual SES (note that, while not shown, the sample mean unadjusted for SES is 57.734). Referring back to Equation 3.1, we obtain the following estimated linear relationships to explain achievement:

$$\hat{Y}_i = 57.798 + 4.255 * SES,$$

where  $\hat{Y}_i$  (or  $Y$  hat) indicates that the *predicted* value of  $Y_i$  is equal to the estimated intercept plus the coefficient for SES (plus some unknown error). The unstandardized regression coefficient describing the effect of individual SES ( $\beta_1$ ) on math achievement is 4.255. The slope coefficient suggests that, on average, as student SES goes up by 1 unit (in this example, 1  $SD$ ), the predicted student test score would be expected to increase by 4.255 points to 62.053 [ $57.798 + 4.255(1)$ ]. This is arrived at by multiplying the regression weight (4.255) by the desired 1-unit increase in SES. If we wanted to know the predicted value, a student's test score for a 2-unit increase in SES (i.e., in this case, a 2- $SD$  increase), we would multiply as follows:

$$\hat{Y}_i = 57.798 + 4.255(2),$$

where the predicted of  $Y_i$  would then be 66.308 [ $57.798 + (4.255)(2) = 57.798 + 8.510$ ]. We note in the output that in the single-level analysis we also have a standardized regression weight (0.378), which provides a common metric for examining the effects of predictors on the outcome.

TABLE 3.2 Coefficients<sup>a</sup>

Model		Unstandardized Coefficients		Standardized Coefficients	<i>t</i>	Sig.
		<i>B</i>	Std. Error	Beta		
1	(Constant)	57.598	0.098		586.608	.000
	ses	4.255	0.126	0.378	33.858	.000

<sup>a</sup> Dependent variable: math.

TABLE 3.3 ANOVA<sup>a</sup>

Model		Sum of Squares	<i>df</i>	Mean Square	<i>F</i>	Sig.
1	Regression	75,813.316	1	75,813.316	1146.382	.000 <sup>b</sup>
	Residual	454,265.500	6,869	66.133		
	Total	530,078.816	6,870			

<sup>a</sup> Dependent variable: math.

<sup>b</sup> Predictors: (Constant), ses.

For a continuous predictor, such as SES in this case, this can be interpreted as a 1-*SD* increase in the predictor producing a 0.378-*SD* increase in the math outcome. As we noted in Chapter 1, most multilevel software programs do not produce standardized estimates as part of the output, given various ways of standardizing estimates and the changes in the model's variance components that can result from standardizing estimates when there are random slopes in the model (Heck & Thomas, 2009; Hox, 2010).

The analysis also provides a table describing the variability in math achievement accounted for by the predictors in the model (Table 3.3). This can be used to estimate the variance accounted for in achievement by mean student composition and individual SES. From the table, we can calculate  $R^2$  as the ratio of the regression variance to the total variance ( $75813.3/530078.8 = 0.143$ ).

As we have noted, however, there would likely be problems in the accuracy of this analysis. The assumptions necessary for multiple regression models to yield the best, unbiased estimates are most realistic when the data have been collected through simple random sampling. Random sampling assumes that subjects in a study are independent of each other. As groups are added as a feature of a study, however, this assumption becomes more tenuous. In large-scale educational research, for example, simple random sampling would rarely be used. Instead, various types of complex sampling strategies are employed to select schools, classrooms, and students. These can include multistage-sampling strategies where individuals may be sampled within various groups and the likely oversampling of some groups. Clustered data, therefore, result from the strategies used in large-scale databases, as well as the natural groupings of students in classrooms and schools.

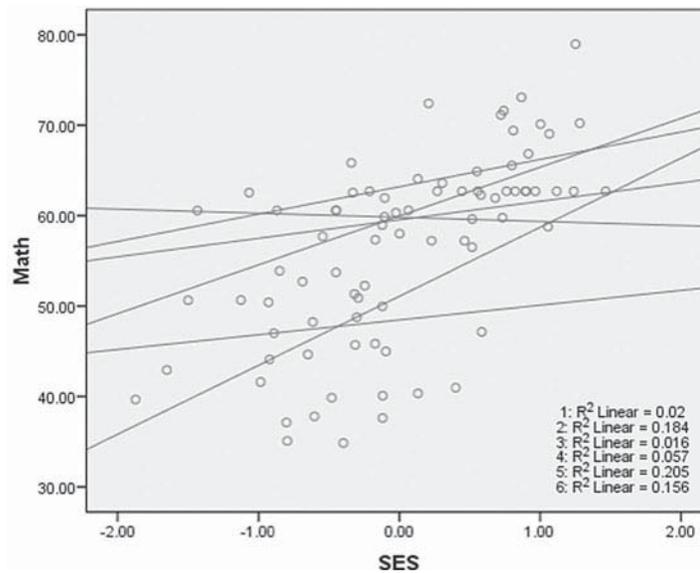
Single-level analyses are appropriate when there is little interest in considering group-level influences on the outcomes. The key point about a single-level model is that the estimates of the intercept and slope are each fixed to one value that describes the average for the sample.

The single-level regression model cannot take into consideration that the students may be clustered within a number of schools with other students having similar backgrounds and math

scores. In our example, the multiple regression analysis does not take into consideration that students are nested within the 419 schools in the sample (or that the estimated slope might be different between these schools). Multilevel models imply that individuals are “clustered” in higher order social groupings (a department, a school, or some other type of organization). In these types of studies, simple random sampling does not hold because people clustered in groups will tend to be “similar” in some ways. In this example, the size of the student samples within their schools ranges from 12 to 37 (mean = 17.4,  $SD = 4.74$ ). If the clustering of students is ignored, it is likely that bias will be introduced in estimating the coefficients and their standard errors.

We can again develop a scatterplot of the relationship between students’ SES and math achievement from Figure 3.1, this time taking into consideration their schools (i.e., in this case the 80 students are in six schools). We can therefore estimate a separate regression equation for each school. Each school would have its own intercept (describing the level of its students’ outcomes adjusted for SES) and a slope (describing the relationship between SES and math achievement within that school). Where data hierarchies exist, the school intercepts (i.e., average math scores adjusted for student SES) would likely vary across the sample of schools in the study. Moreover, there might be some schools where the effect of student SES on math achievement (represented by the slope of regression line) is greater than or less than the average (or fixed) effect. There might also be some schools where there is no relationship at all. In other words, where there are clustered data, it is likely that there is a distribution of both intercepts and slopes around their respective average fixed effects. In this situation, we might wish to investigate the “random” variability in intercepts and slopes across the sample of higher level units in the study (e.g., classrooms, schools).

Figure 3.2 presents the relationship between SES and math achievement for the previous subset of 80 individuals nested in six schools. The figure suggests that the slope relationship accounts for differing amounts of variance within each unit (i.e., with  $R^2$  coefficients ranging from 0.02 to 0.205). This suggests considerable social distribution of math learning within these six schools.



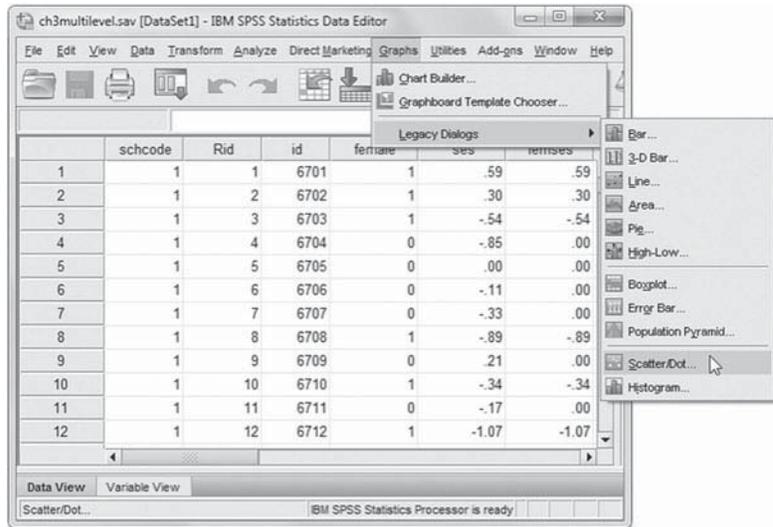
**FIGURE 3.2** Randomly varying SES-achievement slopes in six schools.

### Graphing the Subgroup Relationships Between SES and Math Test Scores with IBM SPSS Menu Commands

(Settings will default to the prior scatterplot.)

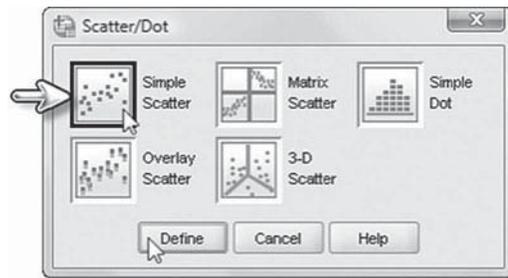
1. In the IBM SPSS toolbar, select GRAPHS, LEGACY DIALOGS, SCATTER/DOT.

This command will open the *Scatter/Dot* box.



2. The default selection from the prior scatterplot is set to *Simple Scatter*.

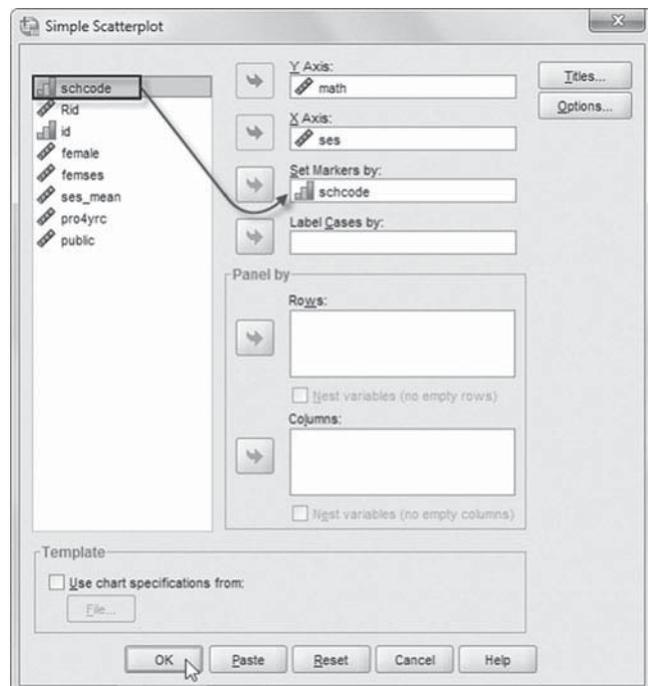
Click the DEFINE button to open the *Simple Scatterplot* box.



3. The *Simple Scatterplot* box displays the variables *math* (Y Axis) and *ses* (X Axis).

Click to select *schcode* from the left column. Then click the right-arrow button to move *schcode* into the *Set Markers by* box.

Click OK to generate the scatterplot.



Multilevel modeling can be used to specify a hierarchical system of regression equations that take advantage of the clustered data structure (Heck & Thomas, 2009). Multilevel regression modeling involves first estimating a Level 1 model within each higher level unit (e.g., schools) and then estimating a series of between-unit models using the within-unit estimates (i.e., intercepts or slopes) as dependent variables. In contrast to a single-level regression model, where the impact of a variable like SES is assumed to be fixed across all individuals in the sample, within multilevel regression modeling one can specify a single-level regression equation that can be estimated within each school (as shown in Figure 3.2). In this way, the researcher can determine whether the effect of SES on math achievement is stronger or weaker in some schools. The formal test of this is whether the distribution of slopes (i.e., the  $R^2$  coefficients in Figure 3.2) varies significantly different across units.

In this case, the researcher might be interested in estimating the average intercept and the SES effect on math achievement across the set of schools, as well as determining how particular schools deviate from the overall average intercept and average SES-achievement slope. Although the researcher's primary concern is usually with a model's structural parameters (e.g., the level of achievement and the average effect of SES on achievement), in some cases examining the appropriateness and distribution of the random variance parameters (e.g., variance associated with intercepts, slopes, and covariances between intercepts and slopes) may be of equal interest to the researcher. In a multilevel model, the math achievement intercept and the SES-achievement slope can be defined to vary as probability distributions across the set of schools. The variability of intercepts (i.e., where each unit's regression line crosses the  $Y$  axis) and slopes (the steepness of each institution's regression line) gives rise to different types of questions that can be asked of the data. In addition to our initial research question (Is there a relationship between student SES and math achievement?), we can now ask other questions of the data.

If we inspect Figure 3.2 visually, it looks like the levels of the intercepts vary considerably across the six schools. For example, in one school, a student somewhat below 2  $SD$  in SES would have a math score of about 34.0. In contrast, in another school, a similar student (somewhat below 2  $SD$  in SES) would score approximately 61.0 in math. This observed variability in intercepts gives rise to a second question: Do the average math scores students receive vary across schools in the sample? Answering this question would provide information about each school's effectiveness, given the SES background of its students. The figure also suggests the possibility of randomly varying slopes in the data set. Five of the six schools in our illustrative subsample have positive slope relationships, and one school has a negative slope relationship. One can also notice that the  $R^2$  coefficients associated with the slope relationship within each unit, which summarize the strength of the relationship between student SES and math, also vary considerably in the first six schools (i.e., from 0.02 to 0.21). This small data set suggests that there may be schools where there is little gap in math achievement due to students' SES background. These schools could be viewed as more equitable in terms of the social distribution of learning due to students' social backgrounds (e.g., SES or perhaps gender and race/ethnicity if we were to include those variables).

In other schools, students' social background might be very consequential in determining their achievement. A second question that could be posed is the following: Do the slopes (i.e., the strength of relationship between student SES and math achievement) vary across schools? Answering this question would provide information about schools' equity in producing outcomes, given the social backgrounds of their students. We might examine whether the relationship between individual SES and math achievement is stronger or weaker in schools of differing average social composition. Moreover, we can investigate whether there is a difference in the strength of association between individual SES and achievement in public and private schools or in schools having a stronger academic focus, after controlling for the mean SES level at the school.

Parameters that are proposed to vary randomly across units are referred to as random effects or random coefficients from various statistical perspectives. In experimental research, for example, a *random effect* describes a situation where the levels of a treatment are assumed to represent a

sample drawn from a universe of treatments or treatment levels. Because the effect is considered as randomly varying across a universe of treatment levels, the intent is to make inferences beyond the specific treatment levels included in the study. The effects, therefore, are not assumed to be constant. In contrast, a *fixed effect* describes the situation where all possible treatments are present in the experiment (Kreft & de Leeuw, 1998). In this latter case, inferences can only be made about the specific treatments used. The effects are considered to be constant and measured without error because all possible cases are included.

Unlike single-level (ordinary least squares [OLS]) regression, where random errors are assumed to be independent, to be normally distributed, and to have constant variance, in multilevel models the error structures are more complex. The individual-level errors are dependent within each unit because they are common to every individual within that unit (Heck & Thomas, 2009). Errors do not have constant variance because the residual components describing intercepts and slopes may also vary across units. The estimation of these unknown random parameters associated with intercepts or slopes may also depend on characteristics of the data (e.g., sample size, degree of imbalance in sample sizes of higher level units, and degree of similarity among individuals within groups), the type of analysis conducted, and the measurement scale of the dependent variables (Muthén & Muthén, 1998–2006; Raudenbush & Bryk, 2002). Because the model's random parameters must be estimated with group samples containing differing numbers of individuals, iterative estimation procedures must be used to obtain efficient estimates (Muthén & Muthén, 1998–2006).

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## **Building a Multilevel Model with IBM SPSS MIXED**

We can use IBM SPSS MIXED to run a variety of different multilevel cross-sectional and longitudinal (e.g., growth) models. The program is flexible and can be used to estimate a number of different types of models with random intercepts (i.e., means that vary across groups) and random slopes (i.e., within-group regression coefficients that vary across groups). It is also useful in looking at individual change over repeated measurements or in studies of changes of individuals within organizations over time. In addition, it can also be applied to situations where individuals may be cross-classified by higher level groupings (e.g., in several different classrooms within schools or within various high schools and subsequent universities). In the chapters that follow, we develop each of these possibilities in more detail. In the remainder of this chapter, we focus on the univariate cross-sectional multilevel model.

There are several ways to develop models using IBM SPSS MIXED. Some users prefer the IBM SPSS graphical user interface (GUI), while others favor using syntax statements to define the model. We will build models in the chapters that follow using the GUI, and we also provide some of the key syntax in Appendix A. As we mentioned in Chapter 2, syntax can be very useful on occasions since one does not have to return to the specific window to make successive changes in a model that is being investigated. Syntax also provides a record of what has been done previously. This record can be saved and used on subsequent occasions without having to set the model up again through the IBM SPSS menu system (i.e., GUI). We have found that small differences may result depending on whether models are developed with syntax commands or the GUI (e.g., owing to some default commands that users may not specify with the syntax commands). Also, we note that if users have different versions of IBM SPSS (e.g., Version 15 forward), estimates may be slightly different (owing to different default rounding procedures). In this chapter, we take the reader through the steps in the IBM SPSS GUI to develop the basic two-level regression model. An alternative to writing your own syntax is to develop models using the IBM SPSS GUI and then tell IBM SPSS to generate the syntax (which is then pasted into a syntax window). Although we use the GUI interface, we will also demonstrate how to generate syntax through the menu system. We follow this convention throughout the remainder of

the book. Note that all screen shots from the IBM SPSS GUI provided here are based on IBM SPSS for Windows Version 21.0.0.1 (64 bit).

### Step 1: Examining Variance Components Using the Null Model

There are three distinct steps in developing the multilevel model. We develop these in this chapter in the following order: (a) specification of the null, or no predictors, model; (b) specification of the Level 1 model; and (c) specification of the Level 2 model. This latter step can include the model to explain intercepts and the model or models to explain randomly varying slopes. The first step in a multilevel analysis usually is to develop a null (or no predictors) model to partition the variance in the outcome into its within- and between-groups components. This will help the researcher determine how much of the variance in math achievement lies between the schools in the sample. Notice that in Equation 3.2, we add a subscript for schools ( $j$ ). The null model for individual  $i$  in school  $j$  can be represented as

$$Y_{ij} = \beta_{0j} + \varepsilon_{ij}, \quad (3.2)$$

where  $\beta_{0j}$  is the intercept and  $\varepsilon_{ij}$  represents variation in estimating individual achievement within groups. Between groups, variation in intercepts can be represented as

$$\beta_{0j} = \gamma_{00} + u_{0j}. \quad (3.3)$$

Through substitution, the null model can be written as

$$Y_{ij} = \gamma_{00} + u_{0j} + \varepsilon_{ij}. \quad (3.4)$$

The null model therefore provides an estimated mean achievement score for all schools. It also provides a partitioning of the variance between Level 1 ( $\varepsilon_{ij}$ ) and Level 2 ( $u_{0j}$ ). Altogether there are three effects to estimate: the intercept, the between-school variation in intercepts ( $u_{0j}$ ), and the variation in individual scores within schools ( $\varepsilon_{ij}$ ).

This model also provides a measure of dependence within each Level 2 unit by way of the intraclass correlation ( $\rho$ ). The intraclass correlation (or ICC) describes the proportion of variance that is common to each unit, as opposed to variation that is associated with individuals within their units. It can be thought of as the population estimate of the amount of variance in the outcome explained by the grouping structure (Hox, 2002). The proportion of variance found between groups can be calculated in IBM SPSS by using either the Variance Components or MIXED procedures. Both will give the same estimation of within-groups and between-groups variance components. As we noted in Chapter 1, the ICC can be represented as

$$\rho = \sigma_B^2 / (\sigma_B^2 + \sigma_W^2), \quad (3.5)$$

where  $\sigma^2$  represents the variance and  $B$  and  $W$  stand for between groups and within groups, respectively. (Refer to Appendix C for a syntax routine that enables calculating rho from a model's variance components.) Stated differently, the ICC is the ratio of between-groups variance to the total variance. The higher the ICC, the more homogeneous are the units (i.e., there exists substantial variability between schools). In contrast, if the ICC is quite small (i.e., researchers often

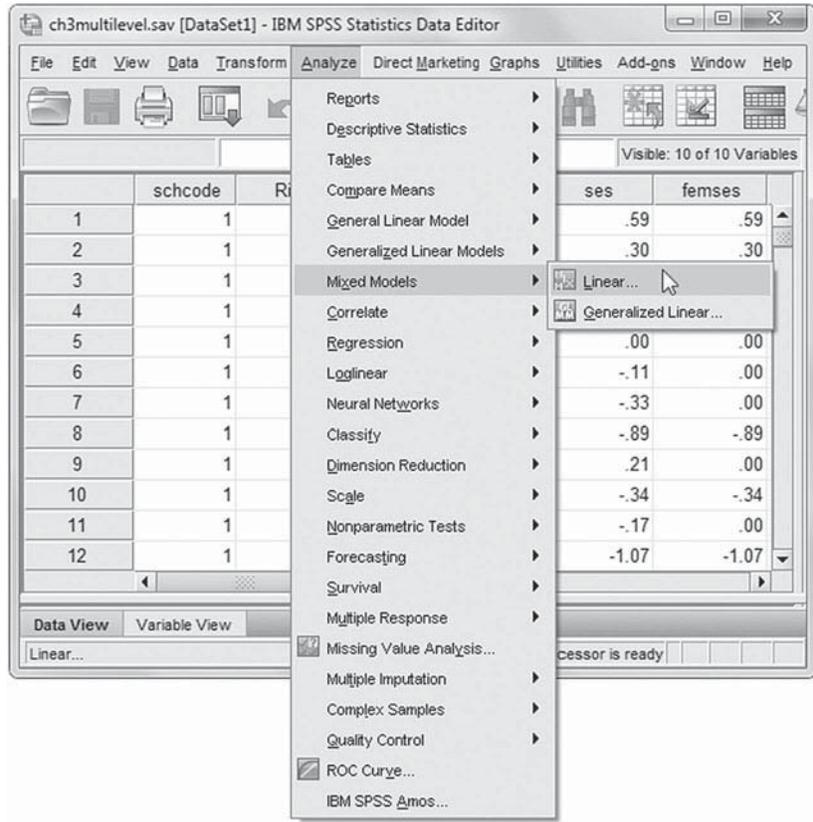
use 0.05 as a rough “cutoff” point), then there would be little advantage to conducting a multi-level analysis. Simply put, the higher level grouping does not affect the estimates in any meaningful way. In these cases, a single-level analysis conducted at the individual level would suffice.

**Defining Model 1 (Null) with IBM SPSS Menu Commands**

Launch the IBM SPSS program application and select the *ch3multilevel.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

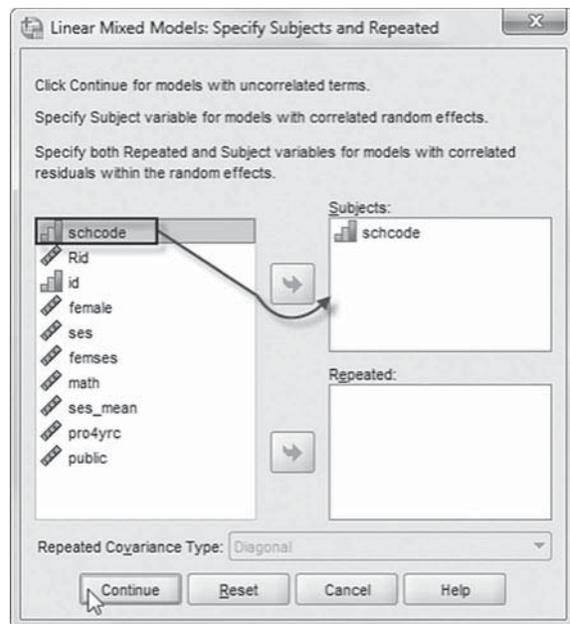


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.

A subject is an observational unit that may be independent of other subjects. For this model, we will designate *schcode* as a subject identifier for the model.

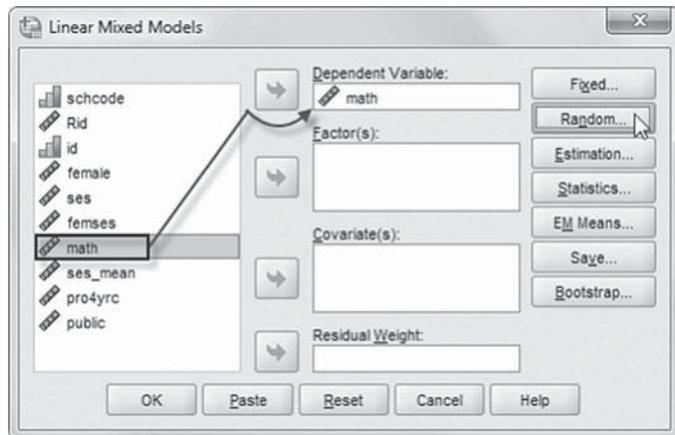
Click to select *schcode* from the *Variables* column and then click the right-arrow button to move the variable into the *Subjects* box.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main screen enables specifying the dependent variable; factors; covariates; access to dialog boxes for defining *Fixed* and *Random* effects; and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

At Level 1, we will use *math* as the dependent variable. Click to select the *math* variable from the left column listing. Then click the right-arrow button to transfer *math* into the *Dependent Variable* box.



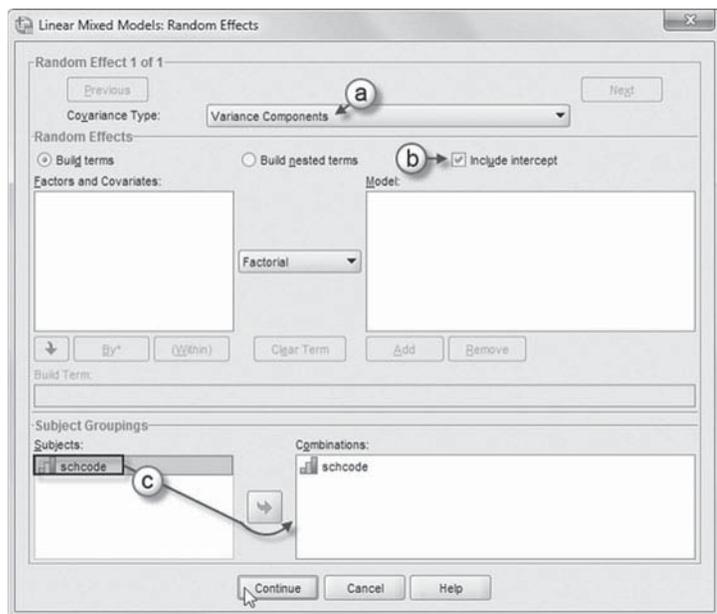
Since this first model (null) excludes predictor variables, as summarized by Equation 3.2, we will skip over the **FIXED** button, which enables adding fixed effects to a model.

$$Y_i = \beta_0 + \epsilon_i \quad (\text{Eq. 3.2})$$

Next, we will add random intercept effect at Level 2 to this model. Click the **RANDOM** button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen. This is the default screen when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.
- The default covariance structure is *Variance Components* (VC).

VC is the default covariance structure for random effects. This specifies a diagonal covariance matrix for the random effects; that is, it provides a separate variance estimate for each random effect, but not covariances between random effects.



In this case, there is only one random effect (the intercept), so we can use the default VC. For models with random intercepts and slopes, a common choice is an “unstructured” (UN), or a completely general, covariance matrix, which fits all variances and covariances between random effects.

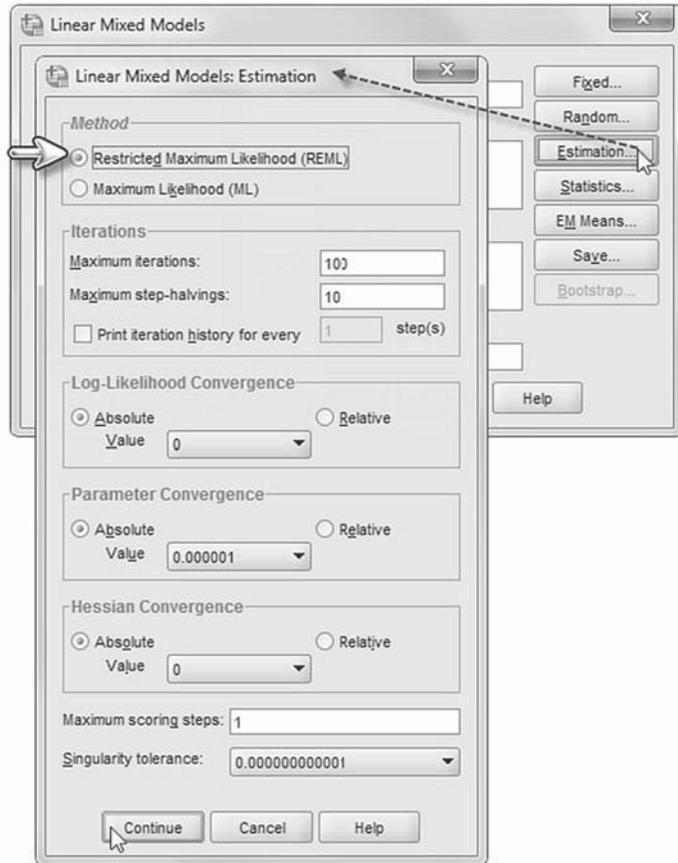
- We want the intercept to be included in the model, so click the *Include intercept* option.
- The *Subject Groupings* box displays the *schcode* variable that was specified as a subject variable in the *Specify Subjects and Repeated* dialog box shown in step 2. We will specify *schcode* as the subject for the random-effects Level 1 part of this model. Click to select *schcode* then click the right-arrow button to move the variable into the *Combinations* box.

Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

5. In the *Linear Mixed Models* dialog box, click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box. The estimation method choices are maximum likelihood (ML) or restricted maximum likelihood (REML). In ML, both regression coefficients and variance components are included in maximizing the likelihood function—that is, the process of minimizing the difference between the sample covariance matrix and the model-implied covariance matrix. In REML, only the variance components are included in estimating the likelihood function; thus, REML is a restricted solution.

Because in REML the regression coefficients are considered to be unknowns, taking the loss in degrees of freedom due to estimating  $P + 1$  regression coefficients in the model results in unbiased estimates of variance components when there are small numbers of groups (Snijders & Bosker, 1999). With sufficient numbers of groups, the differences in estimation methods are negligible. In this chapter and subsequent chapters, we will use REML to estimate the models when we are not making comparisons between successive models using modeling fit criteria involving both regression coefficients and variance components. In this latter situation, ML estimation should be used (Hox, 2010).

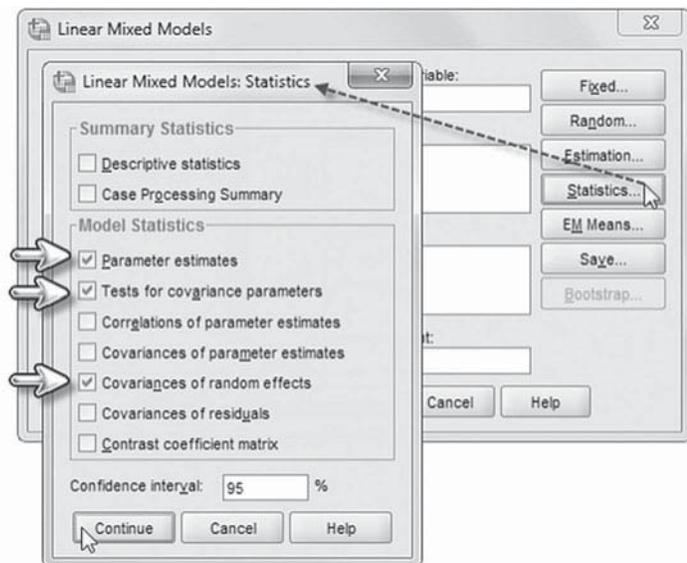
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



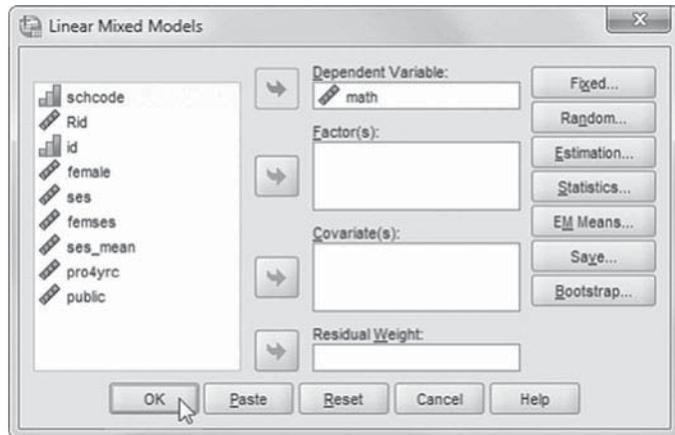
6. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 1 (Null)

The first table (Table 3.4) in the resulting output summarizes the total number of parameters being estimated (three). This is the same as we noted in Equation 3.4. The total parameters estimated include the fixed-effect value for the intercept, random Level 2 variance, and the Level 1 variance (referred to as “Residual” in the IBM SPSS output).

The column referred to as “Number of Levels” describes the fixed effects (1) and the number of random effects (1). There is one fixed effect to be estimated (the intercept) and one random effect (the randomly varying intercept). The column referred to as “Subject Variables” indicates the number of levels in the analysis (i.e., in this case, *schcode* [the school identifier] implies a two-level analysis). The covariance structure describes the way the covariance matrix of random effects is dimensionalized at the group level. In this case, we use the default (VC), which provides an estimate of the intercept variance ( $\sigma_I^2$ ). However, in this first example, at Level 2 there is no random slope variance ( $\sigma_S^2$ ) or covariance between the intercept and slope. In this case, the VC covariance matrix will be the same as an identity matrix.

The next MIXED output describes model-fitting criteria (Table 3.5). The  $-2 * \log$  likelihood can be useful in examining the improvement of model fit when comparing two successive (or

**TABLE 3.4 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
Random Effects	Intercept	1	Variance Components	1	schcode
Residual				1	
Total		2		3	

<sup>a</sup> Dependent variable: math.

**TABLE 3.5 Information Criteria<sup>a</sup>**

-2 Restricted Log Likelihood	48,877.256
Akaike's Information Criterion (AIC)	48,881.256
Hurvich and Tsai's Criterion (AICC)	48,881.257
Bozdogan's Criterion (CAIC)	48,896.925
Schwarz's Bayesian Criterion (BIC)	48,894.925

The information criteria are displayed in smaller-is-better forms.

<sup>a</sup> Dependent variable: math.

nested) models (i.e., by constructing a likelihood ratio test). Note that comparing nested models with this test should only be done when using ML estimation (and not REML, unless only the random parameters are compared). This output also includes other types of information about overall model fit. We will discuss model fit in further detail in Chapter 4.

Table 3.6 reports the estimates of fixed effects in the model. The intercept (or grand mean for school outcomes) is estimated as 57.674. This represents the average level of math outcomes in the 419 schools (and is close to the unadjusted mean of 57.734 for the 6,871 students in the sample). The variance component output (Table 3.7) indicates that the proportion of variance in achievement that lies between schools is 0.138. This can be calculated from Equation 3.5  $[10.642 / (10.642 + 66.551) = 10.642 / 77.193]$ , or 13.8%. The intraclass correlation provides a sense of the degree to which differences in the outcome  $Y$  exist between Level 2 units; that is, it helps answer the question of the existence or nonexistence of meaningful differences in outcomes between the Level 2 units. The results of the null or no-predictors model (basically a one-way ANOVA analysis) suggest that the development of a multilevel model is warranted. Because intercepts vary significantly across schools (Wald  $Z = 10.346, p < .001$ ), and the ICC suggests that about 13.8% of the total variability in math scores lies between schools, we can develop a multilevel model first to explain this variability in intercepts within and between schools.

The reliability of the sample mean for any unit as an estimate for its population mean can also be assessed with information drawn from the variance components. This can provide the analyst with a means by which the assumption of differences in outcomes across units can be checked. Because sample sizes are likely to differ within each unit  $j$ , this reliability will vary across the Level 2 units. Reliability within any particular unit can be estimated as

$$\lambda = \frac{\sigma_B^2}{[\sigma_B^2 + (\sigma_W^2/n_j)]} \tag{3.6}$$

In this example, the within-group sample sizes range from 12 to 37. Using the within-group sample sizes, we can estimate the reliability for the smallest unit as  $10.64 / [10.64 + (66.55/12)] = 0.657$ . This is contrasted with the school that has 37 students tested:  $10.64 / [10.64 + (66.55/37)] = 0.855$ .

Our first type of multilevel question was the following: Do the average math scores students receive vary across schools in the sample? We can answer that question from Table 3.7. The

**TABLE 3.6 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	57.674	0.188	416.066	306.344	.000	57.304	58.044

<sup>a</sup> Dependent variable: math.

**TABLE 3.7 Estimates of Covariance Parameters<sup>a</sup>**

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Residual	66.551	1.172	56.802	.000	64.293	68.887
Intercept [subject = schcode] Variance	10.642	1.029	10.346	.000	8.806	12.862

<sup>a</sup> Dependent variable: math.

Residual parameter describes the variance due to differences among individuals within their respective units. As the table suggests, there is significant variance to be explained within groups (Wald  $Z = 56.802, p < .001$ ). Similarly, the intercept parameter indicates that the intercepts vary significantly across the sample of schools (Wald  $Z = 10.346, p < .001$ ). The Wald  $Z$  test provides a  $Z$  statistic summarizing the ratio of the estimate to its standard error.

Researchers have noted several problems with this statistic, which analysts may want to keep in mind. First, the Wald  $Z$  test is a two-tailed test. Because variances cannot be smaller than 0 (i.e., the null hypothesis is that the parameter = 0), when using the Wald  $Z$  to test a variance component, the test can be conducted as a one-tailed test (Hox, 2010). This means that when testing variances (as in Table 3.7), the significance level can be divided by 2 to reflect a one-tailed probability level. Of course, in this instance this will not make a difference, since the two-tailed  $p$  value is very small. Note that for testing covariances between random effects (which can take on positive or negative values), the two-tailed significance test can be maintained. Second, for a large estimated variance coefficient, the standard error can be inflated, which lowers the Wald statistic value and, therefore, can make it overly conservative. Third, the Wald  $Z$  test can also perform poorly under conditions of extreme multicollinearity or where there are small sample sizes and a variance component close to 0 since the test is based on a normal distribution (Hox, 2010). In situations with small samples, the likelihood-ratio test (which can be constructed from IBM SPSS output) tends to be more reliable than the Wald test. This test, which is based on a chi-square distribution, can be constructed from the difference in model deviance (i.e., defined as  $-2 * \log$  likelihood) between two models (i.e., one with the parameter in question included versus one with it excluded). Models with lower deviance generally fit better than models with higher deviance. The chi-square difference test, however, is also a two-tailed test, so the significance level should be divided by 2 if a variance component is tested (Berkhof & Snijders, 2001). Where there may be a discrepancy between the equivalent Wald  $Z$  test and a likelihood ratio test, the latter is preferred (Hox, 2010).

## Step 2: Building the Individual-Level (or Level 1) Random Intercept Model

We will first build a model to examine variability in intercepts across schools. The individual-level model is represented as Equation 3.7, which suggests that students' SES background affects math achievement. In a multilevel analysis, we work primarily with three equations: a within-group (or individual-level) equation, a between-groups intercept equation, and a between-groups slope equation. For each individual  $i$  in school  $j$ , a proposed model similar to Equation 3.1 (summarizing the effect of student SES on math achievement) can be expressed as

$$Y_{ij} = \beta_{0j} + \beta_{1j}SES_{ij} + \epsilon_{ij}. \quad (3.7)$$

Equation 3.7 suggests that at the individual level, within-groups student SES is related to achievement levels.

Equation 3.8 (which is the same as Eq. 3.3) implies that variation in intercepts can be described by a school-level intercept ( $\gamma_{00}$ ), or grand mean, and a random parameter capturing variation in individual school means ( $u_{0j}$ ) from the grand mean:

$$\beta_{0j} = \gamma_{00} + u_{0j}. \quad (3.8)$$

Equation 3.9 indicates that a within-unit slope (e.g., SES-achievement) can also be examined as randomly varying across units in the sample:

$$\beta_{1j} = \gamma_{10} + u_{1j}. \quad (3.9)$$

More specifically, the equation suggests that variability in slopes can be described by a school-level average slope coefficient ( $\gamma_{10}$ ), or grand mean, and a random parameter capturing variation in individual school coefficients ( $u_{1j}$ ) from the grand mean. Because the slope is considered to be randomly varying across schools, the corresponding test of significance of the parameter will be based on the number of schools in the sample. Often, in building models, we may treat the within-group slopes as fixed in preliminary analyses (i.e., in situations where we are not testing a particular hypothesized relationship). In the case where we wish to treat the within-group slope as fixed (i.e., it does not vary across schools), Equation 3.9 would be rewritten as

$$\beta_{1j} = \gamma_{10} \quad (3.10)$$

As Equation 3.10 indicates, there is no random component ( $u_{1j}$ ), so the slope coefficient is fixed to one value for the sample. In the case where the Level 1 slope coefficient is fixed, the significance test for the slope will be based on the number of individuals in the sample. If it turns out that both intercepts and slopes vary randomly across schools, Equations 3.8 and 3.9 suggest that group-level models can subsequently be built to explain variation in the random intercept and slope across groups.

The school-level model is represented as Equations 3.8 and 3.10, suggesting that only the school-level intercepts vary randomly across schools. We discuss the school-level predictive model in a subsequent section. Through substitution of Equations 3.8 and 3.10 into Equation 3.7, we arrive at the combined equation describing Model 1:

$$Y_{ij} = \gamma_{00} + u_{0j} + \gamma_{10}SES_{ij} + \varepsilon_{ij}, \quad (3.11)$$

which reflects that the component representing the Level 1 slope ( $\gamma_{10}$ ) is multiplied by  $SES_{ij}$ , which suggests it is a cross-level interaction, but with variance fixed to 0 at Level 2. The terms can then be reorganized with fixed effects and variance components:

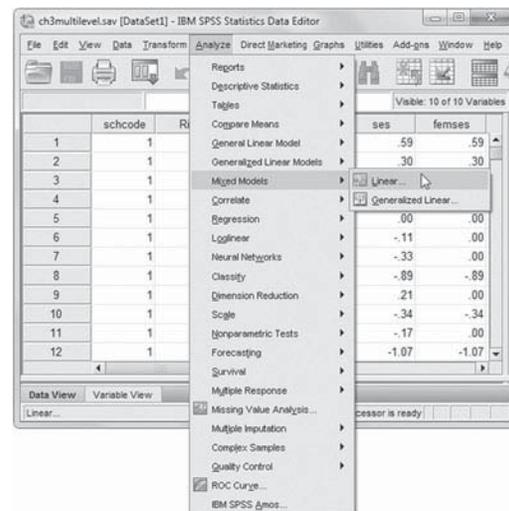
$$Y_{ij} = \gamma_{00} + \gamma_{10}SES_{ij} + u_{0j} + \varepsilon_{ij}. \quad (3.12)$$

### Defining Model 2 with IBM SPSS Menu Commands

Settings will default to those used in Model 1.

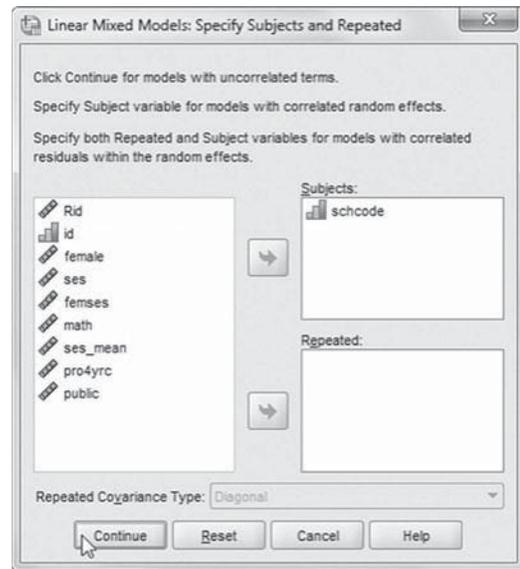
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

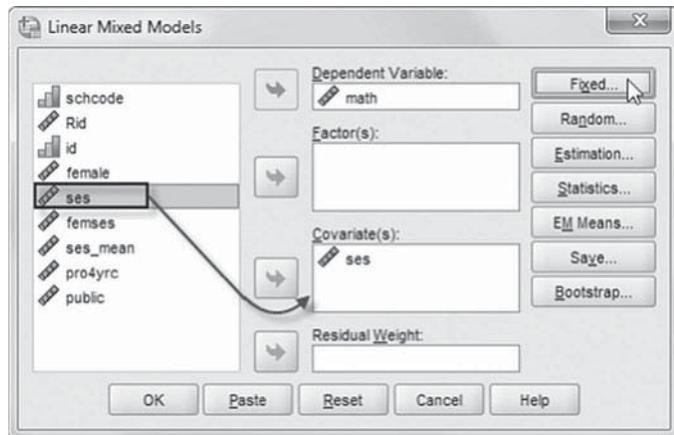
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



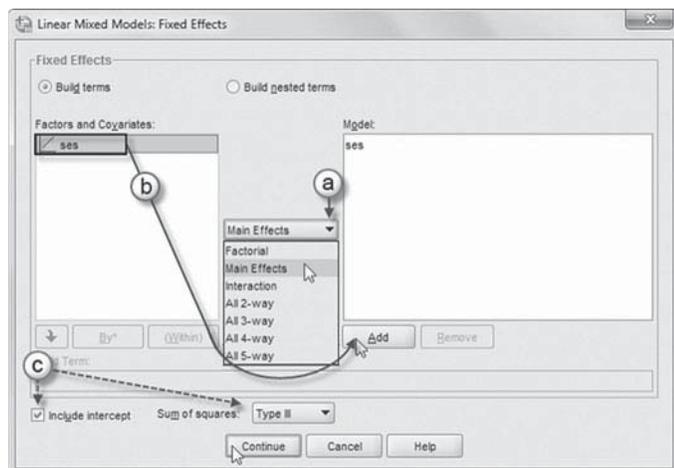
3. At Level 1, we will specify a predictor variable (*ses*) to be used in the model. Locate and click *ses* from the left column listing, and then click the right-arrow button to move the variable into the *Covariate(s)* box.

We may now proceed to define fixed effects for the variable.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
- b. Click to select *ses* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box. We now have the Level 1 model specified (Eq. 3.7).
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.

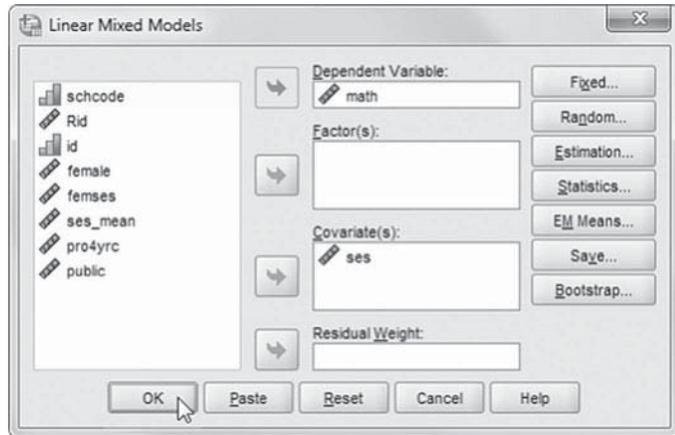


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

$$Y_{ij} = \beta_{0j} + \beta_{1j}SES + \varepsilon_{ij} \quad (\text{Eq. 3.7})$$

Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

**Note:** It is possible to save predicted estimates and residuals for random and fixed effects using the SAVE button. This would allow us to examine their normality and linearity.



**Interpreting the Output From Model 2**

Following is the IBM SPSS output generated from the Level 1 model. Table 3.8 summarizes the total number of parameters being estimated (four). This fits with Equation 3.11, suggesting two fixed-effect parameters to be estimated (i.e., the intercept and the within-school predictor SES) and two variance parameters (the random Level 2 variance and the Level 1 residual variance). The column referred to as “Number of Levels” in Table 3.8 describes the fixed effects (two) and the number of random effects (one). There are two fixed effects to be estimated (the intercept and SES) and one random effect (the Level 2 variance component describing variability in the intercept across schools in the sample). The covariance structure describes the way the covariance matrix of random effects is dimensionalized at the group level. In this case, we use the default (VC), which provides an estimate of the intercept variance but again no slope variance because we opted to fix the slope (nor will there be covariance between intercept and slope). This is the same as specifying an identity covariance matrix.

Table 3.9 presents the typical ANOVA table for examining the significance of the fixed-effect parameters in the model. The large *F*-ratio associated with SES in the table suggests that student SES is significantly related to student math scores. The test of the significance of the intercept is generally not of interest, as it is merely a test of whether the intercept is 0 in the model. As the table suggests, we can reject the null hypothesis that the intercept is 0 (i.e., we already know from the null model and descriptive analysis that it is approximately 57.6).

Table 3.10 provides the estimates of the fixed-effect coefficients. First, we can see that the intercept (adjusted for student SES) is 57.596. This represents the average school mean adjusted for student SES. The standard error is 0.133. Once again, the *t* test of the significance of the

**TABLE 3.8 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	ses	1		1	
Random Effects	Intercept	1	Variance Components	1	schcode
Residual				1	
Total		3		4	

<sup>a</sup> Dependent variable: math.

TABLE 3.9 Type III Tests of Fixed Effects<sup>a</sup>

Source	Numerator <i>df</i>	Denominator <i>df</i>	<i>F</i>	Sig.
Intercept	1	375.699	187,802.817	.000
ses	1	3,914.638	803.954	.000

<sup>a</sup> Dependent variable: math.

TABLE 3.10 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	<i>df</i>	<i>t</i>	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	57.596	0.133	375.699	433.362	.000	57.335	57.857
ses	3.874	0.137	3,914.638	28.354	.000	3.606	4.142

<sup>a</sup> Dependent variable: math.

intercept is not really interesting, since it is a test of whether the intercept is equal to 0. The degrees of freedom reported for each fixed effect, which reflect the Satterthwaite (1946) correction for approximating the denominator degrees of freedom for significance tests of fixed effects in models where there are unequal variances and group sizes, are useful in determining at what level each variable is measured in the model. For example, we know there are 419 schools in the sample. This is consistent with the 375.7 degrees of freedom reported in Table 3.10. In contrast, we know that SES is an individual-level variable. There are 6,871 individuals in the sample, so the degrees of freedom of 3,914.6 are consistent with a variable measured at Level 1. We note in passing that if we wish to estimate a model without using the Satterthwaite correction for degrees of freedom, we can use GENLINMIXED (instead of MIXED) and specify a continuous dependent variable with normal distribution and identity link function. In the *Build Options* dialog box, for estimating degrees of freedom it is possible to select *Residual method* instead of *Satterthwaite approximation*. It is also possible to select *robust* (rather than *model-based*) standard errors. In this case, for example, with robust standard errors, the standard error of the SES parameter would be more conservatively estimated at 0.145 (not tabled) as opposed to 0.137, as in Table 3.10. Analysts should keep in mind, however, that presently only REML estimation is available with that routine.

When we compare the intercepts between this model and the single-level linear regression analysis at the beginning of the chapter, we find that they are very similar (57.598 for the OLS model vs. 57.596 for the multilevel model). The difference in standard errors between these two estimates is larger, as we would expect given the clustering and associated intraclass correlation. More specifically, the estimated standard error (*SE*) from the OLS model was 0.098, and the standard error from the Level 1 multilevel model is 0.133, which is 36% larger. In that original single-level OLS analysis, the intercept described the average student achievement in the sample, without regard for students' school settings.

Second, in this case, we are more interested in the slope for SES ( $\beta = 3.874$ ) and the standard error (0.137). When we compare these against the single-level model, we find that they are considerably different (i.e., for the single-level analysis, the slope was 4.255 and the standard error was 0.126). When we test hypotheses about fixed-effect estimates (e.g., whether an unstandardized regression coefficient is significantly related to the dependent variable), the hypothesis test (generally a *t* test) is based on the ratio of the unstandardized estimate to its standard error (e.g., for SES in the single-level model, this is  $4.255/0.126 = 33.858$ ). If the *t*-ratio is significantly

large, given the sample under consideration, the parameter is considered statistically significant. In the multilevel analysis, the ratio of the unstandardized estimate of SES to its standard error is smaller ( $t$ -ratio = 28.344, but still is significant as we might expect).

These contrasting results in our example illustrate the general point that parameter estimates and standard errors can be different in single-level versus multilevel analyses. As this simple multilevel analysis suggests, standard errors are often underestimated in single-level analyses, which can lead to a greater number of significant  $t$ -values and, hence, support for a proposed model than would be observed if a proposed model were tested with multilevel techniques. Directly as a result, in a multilevel analysis, adjusting for clustering generally results in a reduction of Type I errors (false rejection of the null hypothesis).

The output also provides information about the model's random parameters. The output suggests that the addition of the within-group predictor, SES, reduces the residual (within-group) variability (i.e., from 66.551 in the null model to 62.807 in the Level 1 model). This reduction in variance between the one-way ANOVA (or null) model and the current model can be used to calculate a reduction in variance estimate (similar to  $R^2$ ) estimate for the within-school and between-school portions of the model. For each level, it is calculated as follows:

$$\left( \sigma_{M1}^2 - \sigma_{M2}^2 \right) / \sigma_{M1}^2, \quad (3.13)$$

where  $M1$  refers to the one-way ANOVA (no predictors) Level 1 or Level 2 variance components and  $M2$  refers to the current model's variance components. For the within-groups portion, this is calculated as 0.056 ( $66.551 - 62.807 = 3.744/66.551 = 0.056$ ). This suggests that student SES background accounts for about 6% of the within-school variability in student scores. Notice also that the within-school predictor also affects that residual variability in intercepts at the school level. In particular, the initial variance component for schools, from the null model, was 10.642. After SES is added, however, the between-school variance in math achievement shrinks to 3.469 (see Table 3.11). For the reduction in variance between schools, this would be calculated as 0.674 [ $(10.642 - 3.469)/10.642$ ]. This suggests that within-group SES accounts for almost two thirds (67.4%) of the between-groups variability in achievement. In other words, a full two thirds of the variation in means across schools can be attributed to differences in the socioeconomic status of students in those schools. Another way of looking at this is that the initial variability in math achievement observed between schools (i.e., the ICC) is reduced considerably after controlling for student SES. From Equation 3.2, we find that ICC is now a little more than 5% [ $3.469/(3.469 + 62.807) = 0.052$ ].

We note, however, this reduction-in-variance type of  $R^2$  statistic should be interpreted cautiously because it is sometimes possible to obtain negative values for  $R^2$ . This is because the variance components may be less accurately estimated when there are no predictors in the model. For example, when an individual variable is sampled through a multilevel-sampling process, it may show some between-group variability even if there is no between-group variability present in the population (Hox, 2010). In these cases, Hox notes that the reduction-in-variance procedures described previously may work as described in our example (i.e., where the within-group variable reduces between-group variability). In other situations, however, there may be variables that have almost no variation at one of the levels in the model. For example, this might occur if we had exactly the same percentage of males and females (50%) in each school. If there were no school-level variation in average gender composition, this would be less variance than expected by simple random sampling and could produce negative explained variance (for further discussion, see Hox, 2010; or Snijders & Bosker, 1999). Hox notes that another problem that can occur is that if random slopes are present, the estimated residual variances are related to the scale of measurement chosen for the explanatory variables (i.e., how they are centered and whether they might be standardized). When the interest is in the size of the variance components produced,

**TABLE 3.11 Estimates of Covariance Parameters<sup>a</sup>**

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Residual	62.807	1.109	56.640	.000	60.671	65.019
Intercept [subject = Variance schcode]	3.469	0.539	6.439	.000	2.559	4.704

<sup>a</sup> Dependent variable: math.

**TABLE 3.12 Random-Effect Covariance Structure (G)<sup>a</sup>**

Intercept   schcode	
Intercept   schcode	3.469

Variance Components

<sup>a</sup> Dependent variable: math.

it is therefore desirable to center the explanatory variables with random slopes on their grand means since this will provide estimates for an “average” sampling unit (Hox, 2002). We discuss centering decisions further in subsequent chapters.

The covariance parameters table (Table 3.11) also suggests that after the introduction of SES into the model, there is still significant variability to be explained both within schools (Wald  $Z = 56.640, p < .001$ ) and between schools (Wald  $Z = 6.439, p < .001$ ). The Wald  $Z$  test suggests that, even after controlling for student SES within schools, a statistically significant amount of variation in outcomes still remains both within and between schools. This suggests that we could add other predictors (e.g., gender, ethnicity, and motivation) within schools and between schools (e.g., student composition and school process indicators) that might explain this residual variability in intercepts.

The final output (Table 3.12) provides the random-effect covariance structure for Level 2. In this case, there is only one random effect. Because we specified the SES-achievement slope to be fixed within schools, there is no variance component describing variability in slopes.

### Step 3: Building the Group-Level (or Level 2) Random Intercept Model

Next, we will add school-level variables to explain the variability in intercepts across schools. In this case, our thesis is that school context variables (i.e., aggregated student SES composition and type of school [public = 1, other = 0]) and the focus of the school’s academic program (i.e., the aggregate percentage of students who intend to study at 4-year universities after high school) will impact the remaining variability in achievement between schools. It is important to note that when Level 2 predictors are added to the model, care should be taken that every individual in each unit has the same value on that variable. This will ensure that the estimates are calculated based on the number of Level 2 units, rather than the number of Level 1 units in the model. If there are different values or missing values for some individuals, they should be changed for consistency.

At the school level, we must modify the intercept equation (Eq. 3.8) to indicate the three predictors that we are adding to the between-group portion of the model:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + u_{0j} \tag{3.14}$$

Public school is a dummy-coded variable (1 = public, 0 = private). Because it is dichotomous, we can add it to the model either as a factor or as a covariate. Variables with three or more categories should be added as factors. When categorical variables are entered as factors, the reference group is the last category. If the user wishes to select a different category as the reference group, she or he could recode the variable. Substituting the intercept equation (Eq. 3.14) and the fixed slope equation (Eq. 3.10) into the Level 1 equation (Eq. 3.7) will result in the following combined equation with random Level 2 intercept:

$$Y_{ij} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + \gamma_{10}SES_{ij} + u_{0j} + \varepsilon_{ij}. \quad (3.15)$$

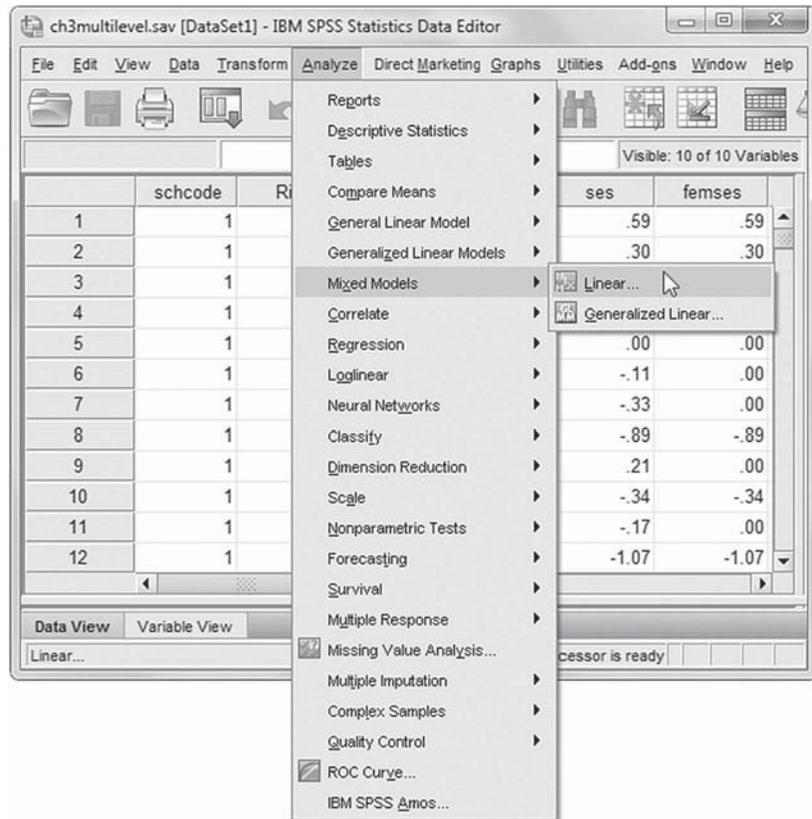
We will first demonstrate how to add a public school indicator to the model as a factor. The other two variables are continuous, so they are added as covariates. In all, we are adding three fixed effects, but the other parameters remain the same, so we are estimating a total of seven parameters in this model (i.e., five fixed effects, the random Level 2 residual variance, and the Level 1 residual variance).

### Defining Model 3 with IBM SPSS Menu Commands

Settings will default to those used in Model 2.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

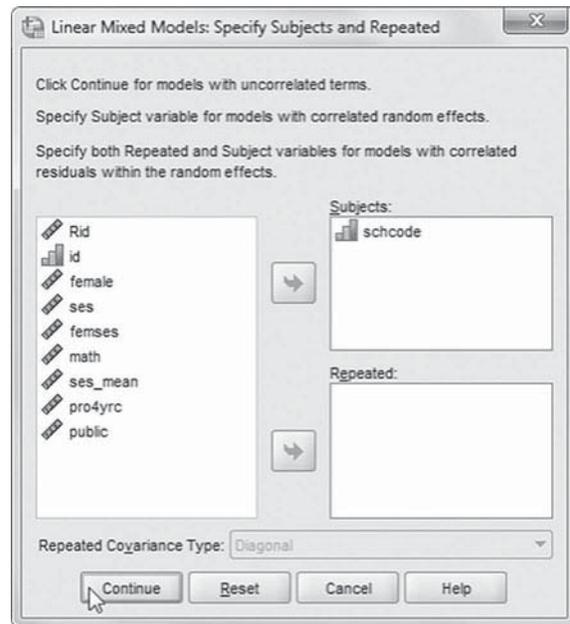


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

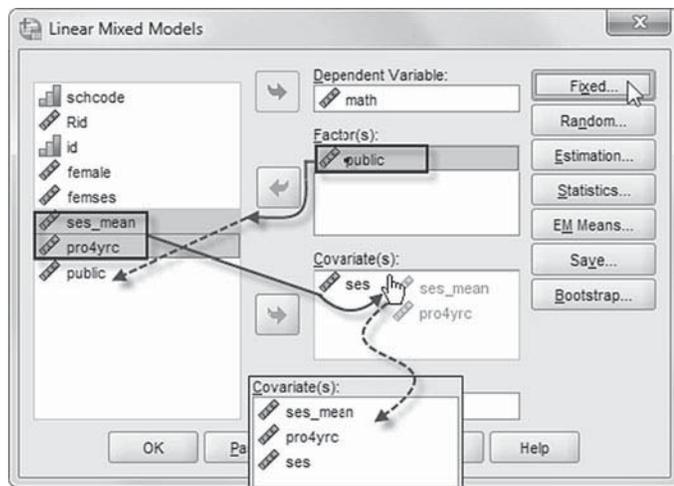
If there is a categorical indicator, it should be added as a factor. Each level of a factor can have a different linear effect on the value of the dependent variable. If a variable is dichotomous, however, it may be added as either a factor or a covariate. We will demonstrate by adding *public* as a factor first and then as a covariate. We note that in MIXED, the reference group for a variable entered as a factor is the last category. This means the estimate will be for the category coded 0 (i.e., in this case, private schools = 0). Often, we find it is convenient to add dichotomous variables as covariates, so that the estimate will be for the category coded 1 (i.e., in this case, public = 1) instead of for the category coded 0. This is consistent with dummy coding in multiple regression models. We note in passing that in some instances researchers might prefer other coding schemes such as effect coding (e.g., -1, +1).



We will now introduce three Level 2 predictors to the previous model (*public*, *ses\_mean*, and *pro4yrc*).

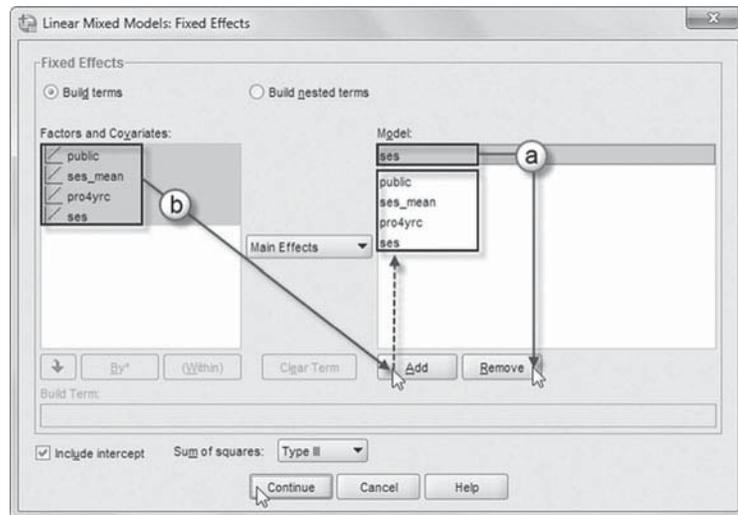
- a. First, click to select *public*, and then click the right-arrow button (or “drag”) the variable to the *Factor(s)* box.
- b. Now click to select *ses\_mean* and *pro4yrc*, and then “drag” both variables above *ses* in the *Covariate(s)* box.

The sequence of the variables is the following: *ses\_mean*, *pro4yrc*, and *ses* (see insert).



We will now designate fixed effects, so click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading the output tables, we will rearrange the sequence order of the variables. First, remove *ses* from the model by clicking to select the variable and then clicking the REMOVE button.

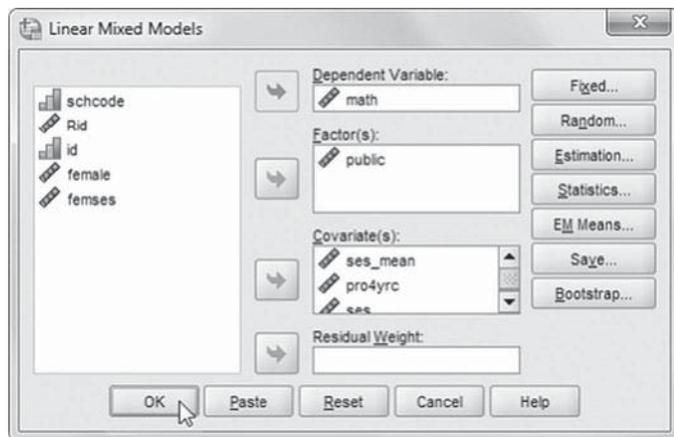


- b. Now click to select four variables (*public*, *ses\_mean*, *pro4yrc*, and *ses*) from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box. Through substitution, the two levels can be summarized as a single-equation model with fixed and random effects (Eq. 3.15).

$$Y_{ij} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + \gamma_{10}SES_{ij} + u_{0j} + \varepsilon_{ij} \quad (\text{Eq. 3.15})$$

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 3

The first IBM SPSS output table (Table 3.13) confirms that we are estimating seven total parameters in this model.

Table 3.14 describes the fixed-effect estimates. Level 2 predictive models describe how differences in school variables (e.g., context, resources, student composition, and educational processes) may influence individual processes within each school (Raudenbush & Bryk, 2002), for example, students' levels achievement outcomes or the relationships between students' background, motivation, and previous learning and their current outcomes. In order to interpret the coefficients, however, we need to know how the predictors are measured. More specifically, *ses\_mean* is defined

TABLE 3.13 Model Dimension<sup>a</sup>

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	public	2		1	
	ses_mean	1		1	
	pro4yrc	1		1	
	ses	1		1	
Random Effects	Intercept	1	Variance Components	1	schcode
Residual				1	
Total		7		7	

<sup>a</sup> Dependent variable: math.

TABLE 3.14 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	56.277	0.430	411.160	130.978	.000	55.433	57.122
[public = 0]	0.164	0.276	409.345	0.595	.552	-0.378	0.707
[public = 1]	0.000 <sup>b</sup>	0.000	–	–	–	–	–
ses_mean	2.473	0.307	709.247	8.059	.000	1.871	3.076
pro4yrc	1.420	0.471	413.879	3.012	.003	0.493	2.346
ses	3.191	0.158	6,448.937	20.220	.000	2.881	3.500

<sup>a</sup> Dependent variable: math.

<sup>b</sup> This parameter is set to 0 because it is redundant.

a standardized ( $z$ ) score (mean = 0,  $SD$  = 1). The measure of the school's academic focus (*pro4yrc*) is defined by the proportion of students planning to attend a 4-year university (ranging from = 0 to 1). School type is a dummy-coded variable (coded 1 = public high school, and 0 = private high school), and student SES is also a  $z$ -score. We can therefore interpret the intercept as the average school math achievement in *public* high schools (since the reference category in this case is the category coded 1), where the proportion of students planning to attend a 4-year institution is 0, and where school SES composition and student SES are at their respective grand means (0).

Regarding the school-level predictors, controlling for the other predictors in the model, we first note that school type does not affect achievement ( $\gamma_{03} = 0.164, p > .05$ ). Because school type has been defined as a factor in MIXED, the second category is the reference group. In this case, the variable was dummy coded as “public” (i.e., private = 0; public = 1), so the second category is redundant. The output in Table 3.14 therefore suggests that private schools have slightly higher math scores. If instead we wish the reference category to be “private” schools (since we coded it 0), we might choose to enter public as a covariate or else recode the variable. Regarding the type of academic environment (i.e., as defined by the proportion of students who say they plan to attend a 4-year institution after high school), we find a positive relationship with math achievement ( $\gamma_{02} = 1.420, p < .01$ ). Turning to SES, aggregate student SES composition affects math achievement ( $\gamma_{01} = 2.473, p < .001$ ), even after controlling for individual SES within schools.

We can write the estimated predictive score for individual  $i$  in school  $j$  for this model as follows:

$$\hat{Y}_{ij} = 56.28 + 2.47(\text{ses\_mean}_j) + 1.42(\text{pro4yrc}_j) + 0.16(\text{private}_j) + 3.19(\text{SES}_{ij}).$$

We can interpret the intercept as the adjusted school math score when the predictors in the model are all equal to 0 (i.e.,  $\text{ses\_mean} = 0$ ,  $\text{pro4yrc} = 0$ , and  $\text{student SES} = 0$ ). In the case of *public*, the reference group is actually public schools (coded 1) since when a dichotomous variable is entered in the model as a factor, the reference group is the second category. This suggests that the average school intercept of 56.28 is the predicted score for students in public schools (coded 1) when average school SES ( $\text{ses\_mean}$ ) is 0, where no students plan to attend 4-year institutions, and where individual SES is held at the grand-mean average (0).

If we hold the other variables in the predictive model constant, we can estimate the likely school mean if one of the predictors is increased by a unit. For example, if the proportion of students planning to go to a 4-year institution after high school changes from 0 to 1 (i.e., implying a strong academic focus), the relevant part of the equation is as follows, since the other variables are held constant at 0:

$$\hat{Y}_{ij} = 56.28 + 1.42(\text{pro4yrc}).$$

For schools where the proportion of students planning to attend 4-year universities after high school is 0, the expected achievement intercept is 56.28 [ $56.28 + 1.42(0.0) = 56.28$ ]. In contrast, where all students are planning to attend a 4-year institution the expected math level would be 57.69 [ $56.28 + 1.42(1.0) = 57.70$ ]. Readers should keep in mind that because of the coding of this variable as a proportion (i.e., ranging from 0 to 1), a change from 0 to 1 represents a 1-unit change. This illustrates why it is important to know what the unit of measurement is for each predictor. For the school where 0.75 plan to attend this type of institution, the estimated intercept would be a little lower at 57.35 [ $56.28 + 1.42(0.75) = 57.35$ ].

For aggregate SES composition, controlling the other variables in the model, a 1-*SD* increase in aggregate student SES composition above the grand mean would be expected to increase achievement from 56.28 to 58.75 [ $56.28 + 2.47(1) = 58.75$ ]. We also note in passing that the addition of aggregate student SES background as a school predictor reduces the size of the SES effect on math achievement observed within schools (i.e., the  $\beta_1$  slope for SES has an average effect of  $\gamma_{10} = 3.19$  vs. 3.87 in the previous model). The model indicates that the effects of student SES background are significant at both the individual and school levels. This is an example of a compositional effect, or an effect that tends to compound across levels of the data hierarchy. The compositional effect is defined as the extent to which the size of the organizational effect differs from the person-level effect. More specifically, compositional effects occur when the aggregate of a person-level characteristic (i.e.,  $\text{ses\_mean}$ ) is related to the outcome, even after controlling for the effect of the individual-level characteristic ( $\text{SES}$ ) in the Level 1 model (Raudenbush & Bryk, 2002). Compositional effects often represent proxies for other types of organizational processes that occur (e.g., curricular organization, expectations for students, etc.). We take up estimating these types of models further in Chapter 4.

We also might want to think about how the Level 2 continuous predictors are centered. Let's consider the proportion of students who are planning on attending a 4-year university. Since the intercept is the level of the outcome when the predictors are all 0, the intercept is the outcome when there are no students in the school planning on attending a 4-year university. In the sample, there may not be any schools where no students are planning to attend a 4-year

university. Instead, we may wish to center this variable on its grand mean, so the intercept would then be defined as the level of math outcomes for schools at the sample average in terms of students planning to attend 4-year universities. Centering continuous predictors on their grand means (which provides a mean of 0 but maintains the original standard deviation) can provide some advantages in multilevel modeling for interpreting intercepts, interaction terms, and variances (Hox, 2010). Standardizing the predictor, which is a type of centering, produces the same mean (0) as grand-mean centering but requires an additional calculation to make the standard deviation equal to 1.0 which, unlike grand-mean centering, also affects the slope and variance estimates. For now, however, we will leave *pro4yrc* in its raw proportion metric.

In Table 3.15, we present results where we entered public as a covariate instead of a factor. We provide MIXED menu commands for treating public school as a covariate (Model 3A) at the end of this section. Readers may also notice that this change results in a slightly different intercept ( $\gamma_{00} = 56.44$  vs. 56.28 in the previous model). This is because the intercept in this current model now represents the mean for *private* schools (i.e., with public schools [coded 1] now being 0.16 of a point lower). Note that if we add the public school intercept and private school estimate ( $56.28 + 0.16$ ), we obtain 56.44, which is the intercept in Table 3.14 when *public* school is entered in the model as a factor instead of a covariate. In Table 3.15, we can see that public schools are 0.16 of point lower than this intercept. This suggests the two model formulations are the same, as we would expect. Since the estimates are the same, we will continue to define school type as a covariate rather than a factor for the rest of the analysis.

The estimates of the variance components in Table 3.16 suggest that student SES at Level 1 and the three predictors at Level 2 (*public*, *ses\_mean*, and *pro4yrc*) reduce the variance component at the school level substantially (i.e., from 10.64 in the one-way ANOVA model to about 2.40 in Table 3.16). We also point out that it is often the case that adjusting Level 2 unit means by characteristics of individuals within the units will reduce the variance at Level 2, even before the Level 2 predictors are added to the model.

From Equation 3.13, the reduction in variance observed at Level 2 between Models 1 and 3A can be used to calculate the amount of variance accounted for ( $R^2$ ) at Level 2, which is

**TABLE 3.15 (Public School Defined as a Covariate) Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	56.442	0.474	421.055	118.966	.000	55.509	57.374
public	-0.164	0.276	409.345	-0.595	.552	-0.707	0.378
ses_mean	2.473	0.307	709.247	8.059	.000	1.871	3.076
pro4yrc	1.420	0.471	413.879	3.012	.003	0.493	2.346
ses	3.191	0.158	6,448.937	20.220	.000	2.881	3.500

<sup>a</sup> Dependent variable: math.

**TABLE 3.16 Estimates of Covariance Parameters<sup>a</sup>**

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Residual	62.630	1.103	56.784	.000	60.505	64.830
Intercept [subject = Variance schcode]	2.395	0.444	5.399	.000	1.666	3.444

<sup>a</sup> Dependent variable: math.

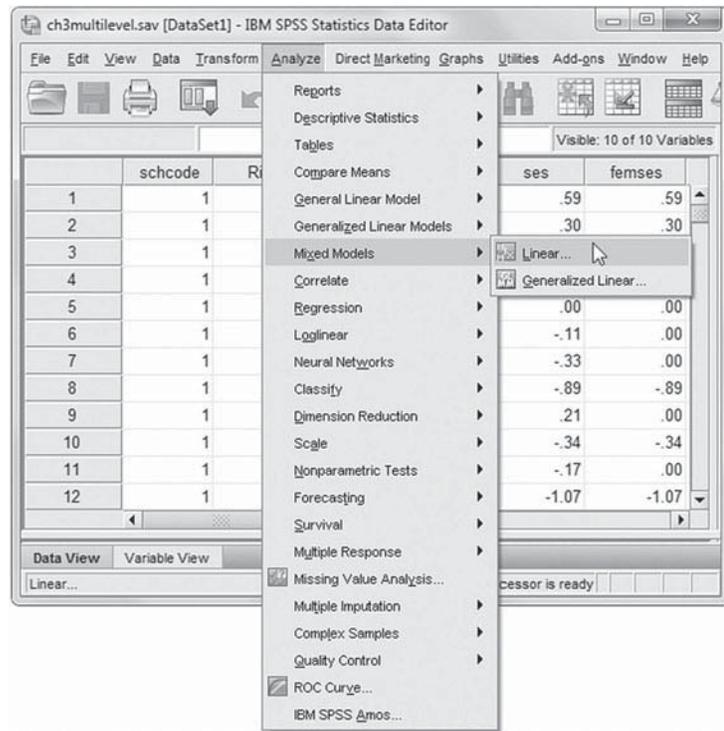
0.774 [(10.64 – 2.40)/10.64], or 77.4%. We could also calculate an  $R^2$  coefficient for Level 1 using the Level 1 one-way ANOVA and current Level 1 variance components in the same manner [(66.55 – 62.63)/66.55 = 0.059]. Student SES therefore accounts for about 6% of the Level 1 variance in individual math achievement (about the same as the estimate in Table 3.11).

### Defining Model 3A (Public as Covariate) with IBM SPSS Menu Commands

Settings will default to those used in Model 2. The following instructions generate results shown in Table 3.15.

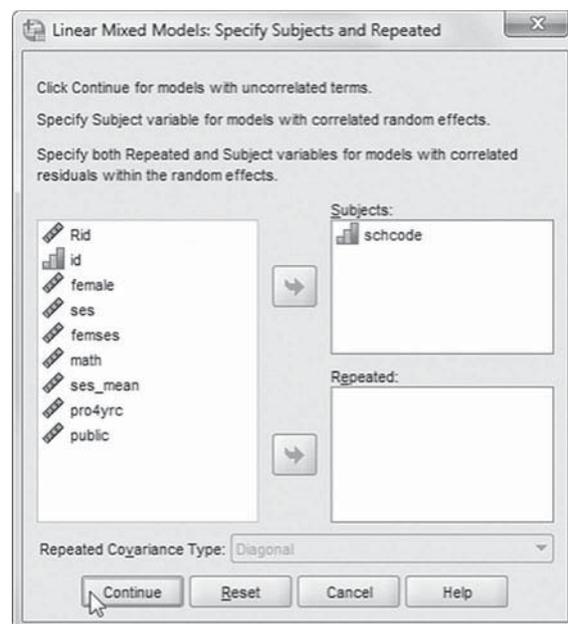
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

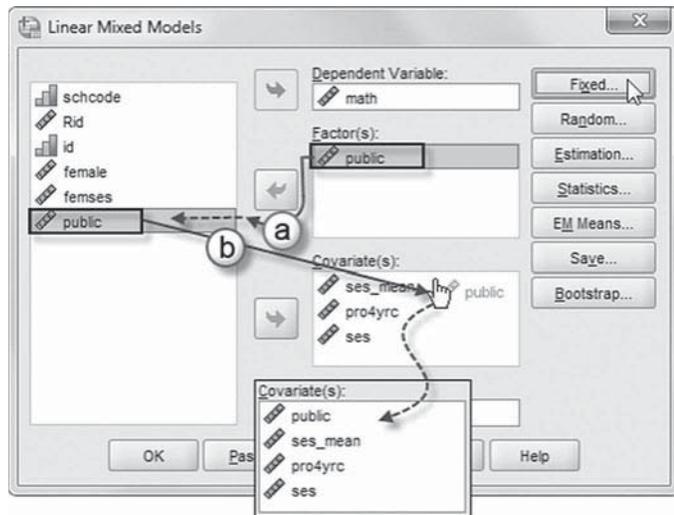
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box settings default to those used in Model 3.
  - a. We will change *public* from a factor to a covariate. First, click to select *public* then click or drag to remove the variable from the *Factor(s)* box.
  - b. Click to select *public*, and then drag the variable above *ses\_mean* in the *Covariate(s)* box.

The sequence of the variables is: *public*, *ses\_mean*, *pro4yrc*, and *ses* (see insert).

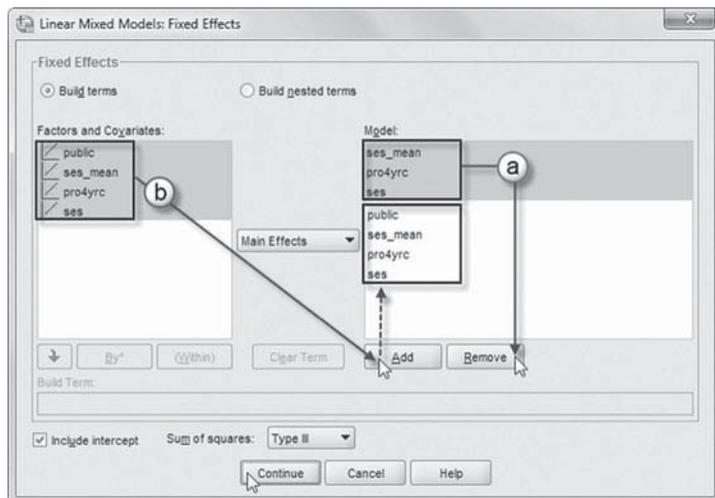
We will now designate fixed effects, so click the **FIXED** button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading of the output tables, we will rearrange the sequence order of the variables. First, remove *ses\_mean*, *pro4yrc*, and *ses* from the model by clicking to select the variables and then clicking the **REMOVE** button.

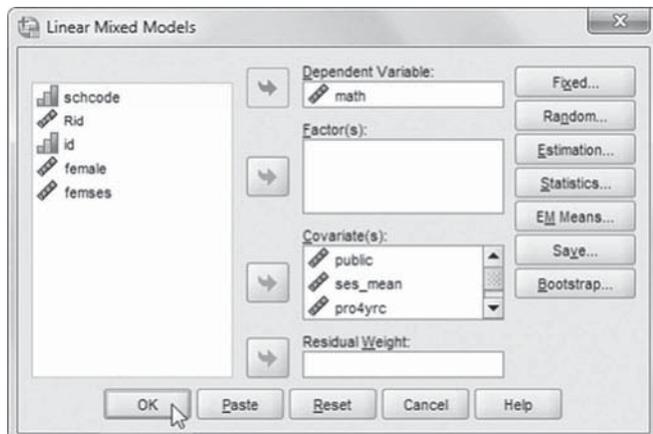
**Note:** *Main Effects* is the default setting from the prior model.

- b. Now click to select the four variables (*public*, *ses\_mean*, *pro4yrc*, and *ses*) from the *Factors and Covariates* box, and then click the **ADD** button to move the variables into the *Model* box.



Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the **OK** button to run the model.



#### Step 4: Adding a Randomly Varying Slope (the Random Slope and Intercept Model)

Since individual SES is significantly related to achievement, we might want to see if the slope varies across schools. This could help us identify schools that are more (or less) equitable in producing outcomes for students of varying SES backgrounds. It may be that school composition, school type, or school academic environment interact with the size of the within-school slope. We would like to find settings where the effect of individual SES on achievement is reduced. In this case, we might ask: Do the slopes vary randomly across schools, and if they do, is there a relationship between features of schools' environments and the strength of the slope relationship?

The important equation in this step is Equation 3.10 ( $\beta_{1j} = \gamma_{10}$ ). First, we will examine whether the slope varies, so we need to specify the SES-achievement slope as a randomly varying parameter in the model as it was originally defined in Equation 3.9 ( $\beta_{1j} = \gamma_{10} + u_{1j}$ ). When we substitute Equation 3.12 (i.e., the random intercept equation) and Equation 3.9 (i.e., the random slope equation) into the Level 1 equation (Eq. 3.7), we obtain the combined equation in Equation 3.16. In creating the random slope portion of the combined equation, we note that  $\gamma_{10}SES_{ij} + u_{1j}SES_{ij}$  is substituted for  $\beta_{1j}$ , as a result of multiplying  $\gamma_{10} + u_{1j}$  in Equation 3.9 by  $SES_{ij}$  in Equation 3.7:

$$\beta_{0j} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + u_{0j} + \gamma_{10}SES_{ij} + u_{1j}SES_{ij} + \epsilon_{ij}. \quad (3.16)$$

We can then reorder the terms, with the fixed effects and then the random effects and residual:

$$Y_{ij} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + \gamma_{10}SES_{ij} + u_{1j}SES_{ij} + u_{0j} + \epsilon_{ij}. \quad (3.17)$$

We note that the substitution results in the more complex error term ( $u_{1j}SES_{ij}$ ), which represents the Level 2 variance for the Level 1 SES-achievement slope ( $\beta_{1j}$ ).

Second, we need to change the covariance matrix of random effects to accommodate the randomly varying slope. The other parameters in the model will remain the same. Adding the randomly varying slope will change the number of random effects in the model from one (i.e., the intercept) to two (i.e., the intercept and the SES-achievement slope). If we only add the randomly varying slope, we will have a diagonal covariance structure. We can estimate this structure using the default VC matrix or by specifying a diagonal covariance matrix (DIAG) in IBM SPSS. As Equation 3.18 suggests, this provides estimates of the intercept and slope variances:

$$\begin{bmatrix} \sigma_I^2 & 0 \\ 0 & \sigma_S^2 \end{bmatrix}. \quad (3.18)$$

In contrast, an UN covariance matrix implies that the intercept and slope variances are estimated, as well as the covariance between the intercept and slope. If we also estimate the covariance ( $\sigma_{IS}$ ) between the intercept and slope, an additional parameter is added to the model:

$$\begin{bmatrix} \sigma_I^2 & \sigma_{IS} \\ \sigma_{IS} & \sigma_S^2 \end{bmatrix}. \quad (3.19)$$

Notice that because the covariance matrix is a square matrix, the covariance appears both above and below the diagonal. We can also specify an unstructured covariance-correlation matrix (abbreviated as UNR in MIXED). This would provide a correlation between the intercept and slope, which is often easier to interpret. Where we estimated seven parameters before (i.e., five fixed

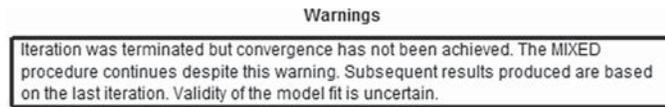


FIGURE 3.3 Warning message.

effects, the random Level 2 variance, and the Level 1 residual variance), we will be estimating nine parameters now (i.e., five fixed effects, three random Level 2 effects, and the Level 1 residual variance).

Although it is usually desirable to obtain an estimate of the covariance between the intercept and slope, we note that sometimes a warning is issued when an unstructured covariance structure is used. For example, if the covariance or slope component is small, the estimation procedure may not be able to estimate the parameter adequately. In this case, when we ran the model with an unstructured matrix (i.e., Eq. 3.19), we received an error message (Figure 3.3).

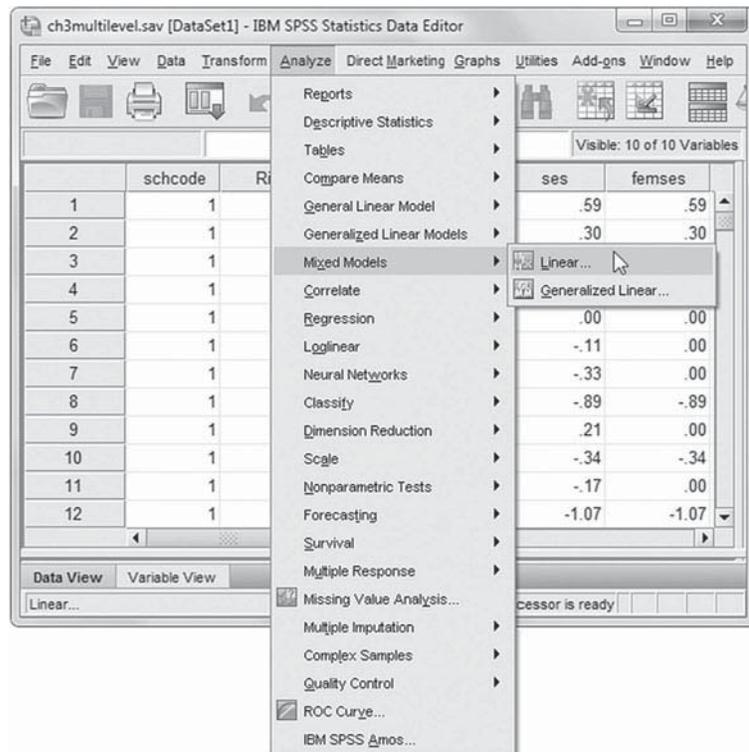
In such cases, the user can try specifying a diagonal covariance structure for the random parameters. The VC specification used in previous models will accomplish the needed change (see Eq. 3.18). This means that we will estimate a total of eight parameters as suggested by Equation 3.17 (i.e., five fixed effects, two random effects at Level 2, and the Level 1 residual) since we are eliminating the covariance between the intercept and slope at Level 2 (as would be specified in Eq. 3.19).

#### Defining Model 4 with IBM SPSS Menu Commands

Settings will default to those used in Model 3A.

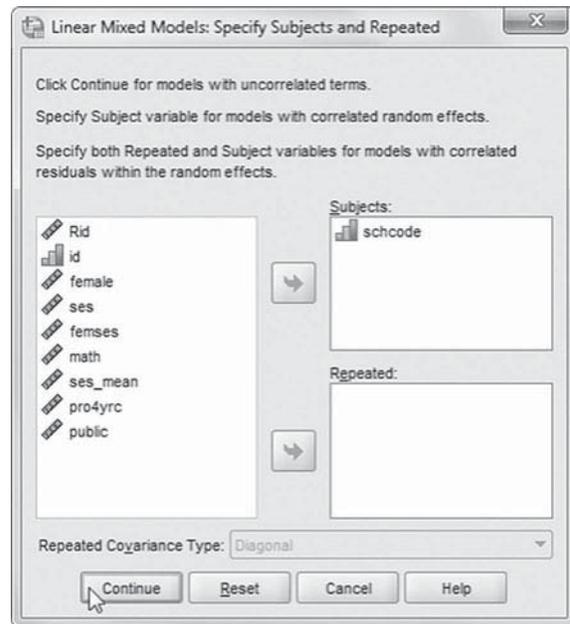
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



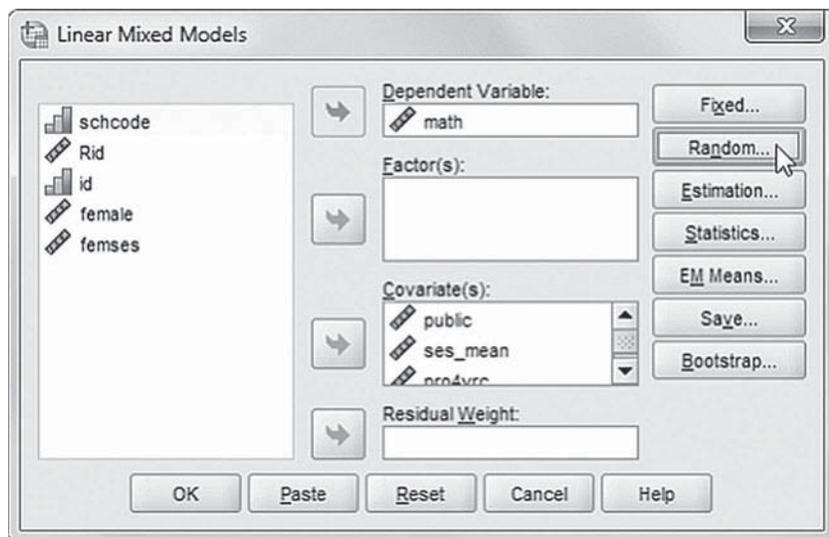
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

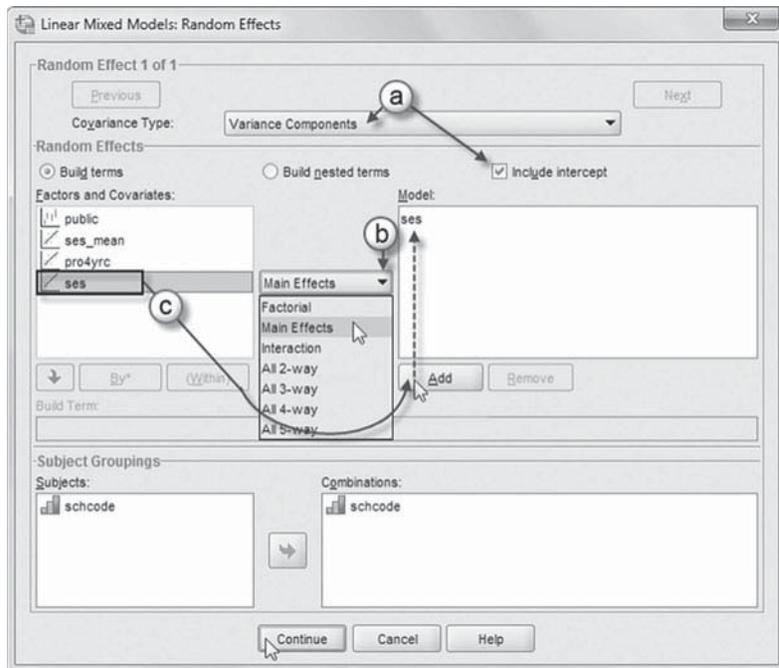


3. The *Linear Mixed Models* dialog box settings default to those used in Model 3A.

We will change the model's random effects, so click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



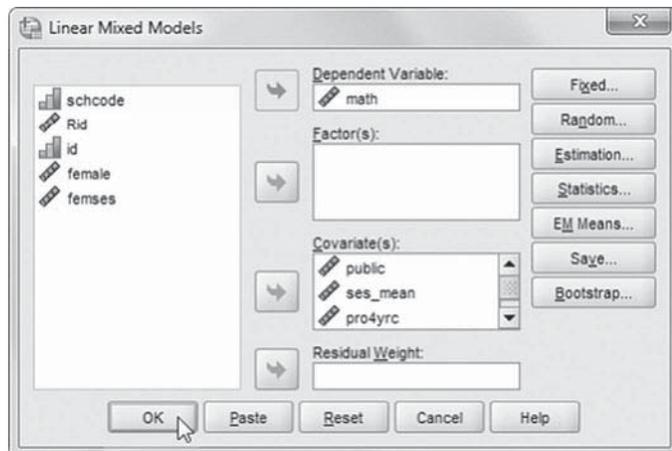
- 4a. The *Linear Mixed Models: Random Effects*: *Random Effects* displays the *Random Effect 1 of 1* screen with the default model settings of a *Variance Components* structure and intercept.
- b. We will add a random slope (*ses*) to the random-effects model, so first click the pull-down menu and change the setting to *Main Effects*.
- c. Now click to select *ses*, and then click the ADD button to move the variable into the random-effects *Model* box (see Eq. 3.17).



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

$$Y_{ij} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + \gamma_{10}SES_{ij} + u_{1j}SES_{ij} + u_{0j} + \varepsilon_{ij}. \quad (\text{Eq. 3.17})$$

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 4

We can first confirm that we are estimating the eight parameters we anticipated (see Table 3.17 on page 114).

Similar to the last model, the fixed-effects table (Table 3.18) indicates that school aggregate SES, percentage of students attending 4-year universities, and individual SES all impact math achievement. We can observe that defining individual SES as randomly varying changes the estimates slightly ( $\gamma_{10} = 3.16$ ) from the previous model with a fixed SES-achievement slope ( $\gamma_{10} = 3.19$ ). For example, the intercept changes slightly from 56.44 in Table 3.15 to 56.47, and the coefficient for the proportion of students planning to attend a 4-year university changes from 1.42 to 1.36.

**TABLE 3.17 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	public	1		1	
	ses_mean	1		1	
	pro4yrc	1		1	
	ses	1		1	
Random Effects	Intercept + sesa	2	Variance Components	2	schcode
Residual				1	
Total		7		8	

<sup>a</sup> Dependent variable: math.

**TABLE 3.18 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	56.470	0.472	419.501	119.749	.000	55.543	57.397
public	-0.120	0.274	407.915	-0.437	.662	-0.659	0.419
ses_mean	2.660	0.314	698.064	8.480	.000	2.044	3.275
pro4yrc	1.360	0.468	410.212	2.907	.004	0.440	2.280
ses	3.164	0.169	635.541	18.734	.000	2.832	3.496

<sup>a</sup> Dependent variable: math.

For this model, our primary interest is with the estimates of the variance components (see Table 3.19). Defining the SES-math slope as randomly varying slightly changes the variance remaining at Level 1 (i.e., from 62.63 to 62.11). The intercept variance is slightly different also (i.e., from 2.40 to 2.11). The remaining intercept variance is still significant (Wald  $Z = 4.74$ ,  $p < .001$ ), which indicates that even after adding the three predictors to the model, there is still variance in intercepts that could be explained across schools by adding additional school-level variables.

Most important for our purposes, the slope variance (1.31) is significant (Wald  $Z = 2.32$ , one-tailed  $p = .01$ ). This suggests that the slopes vary across schools in the sample. We note that it can be more challenging to identify random slopes that vary significantly at higher levels in multi-level models and to explain this variation because slopes are generally less reliably estimated than intercepts are estimated. Because the Level 1 slope depends on the distribution of the outcome within each unit and the strength of correlation between the two, as well the sample size of the unit (i.e., with smaller within-unit samples increasing error variance in estimating slopes), this lower reliability can also weaken the power to detect Level 2 relationships that might explain variation in slopes (Raudenbush & Bryk, 2002).

Because we could not fit the model with a proposed covariance between the intercept and slope, we cannot examine their relationship with information from this model. It turns out in this case that the “public” school variable is the problem. If we remove that variable from the model, we can estimate the covariance as  $-1.59$  ( $p < .01$ ).

TABLE 3.19 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		62.115	1.111	55.893	.000	59.974	64.331
Intercept [subject = schcode]	Variance	2.112	0.445	4.741	.000	1.397	3.194
ses [subject = schcode]	Variance	1.314	0.566	2.320	.020	0.565	3.059

<sup>a</sup> Dependent variable: math.

### Step 5: Explaining Variability in the Random Slope (More Complex Random Slopes and Intercept Models)

The results in the variance components table (Table 3.19) suggest that a model could be developed to explain the variability in the SES-achievement slope across schools (instructions provided in Model 5). Building such a model requires the introduction of cross-level interactions, or school-level variables that moderate (i.e., enhance or diminish) the size of the within-school SES-math achievement slopes. In simple models, the effects of predictors on the outcome do not depend on the values of other predictors in the model. Interactions indicate that the relationship between a predictor and the outcome depends on the value of a third variable. MIXED allows the analyst to build a variety of different interactions within the fixed-effects syntax statements or within the model-building menu statements.

Interactions are built as product terms from two predictors in the model ( $A*B$ ), so there are three relevant parameters in the model ( $A$ ,  $B$ , and  $A*B$ ). More specifically, interactions estimate the linear impact of  $A$  on  $Y$  when  $A$  increases by 1 unit and the slope of  $B$  remains constant. This implies that the slope coefficient of  $A$  added to the slope coefficient of  $A*B$  should be the complete coefficient for  $A$  (Hamilton, 1992). The same would apply for predicting the complete impact of  $B$  on  $Y$  with the interaction.

Cross-level interactions indicate similar types of interactions but across levels of the data hierarchy. For example, assume we wish to estimate the cross-level interaction of student SES composition (*ses\_mean*) on the individual SES-achievement slope within schools in our data set. This is an example of a covariate-covariate interaction. Table 3.20 suggests that if the impact of individual SES ( $A$ ) on math achievement ( $Y$ ) is 3.18 and the cross-level interaction term ( $ses*ses\_mean$ ) is  $-0.25$ , then the impact of a 1-*SD* increase in individual SES on math achievement is actually about 2.93 [ $3.18 + (-0.25) = 2.93$ ] when the slope of *ses\_mean* is held constant (2.78). This result implies that math achievement advantages due to individual SES become less pronounced in schools where the student SES composition is higher. (Refer to Appendix A for Table 3.20 syntax.)

TABLE 3.20 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	57.606	0.138	495.962	418.600	.000	57.335	57.876
ses_mean	2.782	0.309	794.877	8.997	.000	2.175	3.389
ses	3.179	0.158	6,483.723	20.088	.000	2.869	3.489
ses_mean * ses	-0.251	0.257	1,607.930	-0.976	.329	-0.756	0.254

<sup>a</sup> Dependent variable: math.

A factor-covariate cross-level interaction ( $public * ses$ ) implies that the linear relationship between SES ( $A$ ) and math achievement ( $Y$ ) changes for different levels of factor ( $B$ )—that is, types of schools. The output will provide an estimate for each level of the factor except the last category (which is the reference group). A factor-factor cross-level interaction (e.g.,  $public * female$ ) implies that each combination of factor levels may have a different linear effect on the outcome. For this interaction, the output will provide coefficients for each nonredundant combination using the last category of each factor as the reference group. We will discuss how to build these types of models in further detail in the next chapter.

For our purposes here, we will simply note that we add the three school-level predictors to the slope model to determine whether they might account for variability in the size of the SES–math achievement relationships across schools. The model can be written as

$$\beta_{1j} = \gamma_{10} + \gamma_{11}ses\_mean_j + \gamma_{12}pro4yrc_j + \gamma_{13}public_j + u_{1j}. \quad (3.20)$$

When we substitute Equation 3.20 and Equation 3.14 into the Level 1 model (Eq. 3.7), we obtain the following combined equation:

$$Y_{ij} = \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \gamma_{03}public_j + \gamma_{10}SES_{ij} + \gamma_{11}ses\_mean_j * SES_{ij} + \gamma_{12}pro4yrc_j * SES_{ij} + \gamma_{13}public_j * SES_{ij} + u_{1j}SES_{ij} + u_{0j} + \varepsilon_{ij}. \quad (3.21)$$

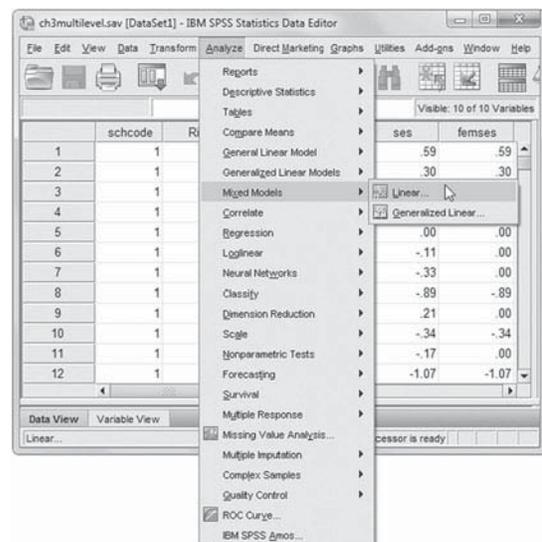
After substitution, in Equation 3.21 we can see that the first and second lines specify the three cross-level interactions explaining ( $\beta_{1j}$ ). The cross-level effects (referred to as  $\gamma_{11}$  to  $\gamma_{13}$ ) appear in the output as interactions between aggregate school SES composition and the within-school SES–math achievement slope (i.e.,  $ses\_mean * SES$ ), between aggregate proportion of students planning to attend 4-year colleges and the within-school SES–math achievement slope (i.e.,  $pro4yrc * SES$ ), and between school type and SES ( $public * SES$ ). The random coefficient for the slope  $u_{1j}$  takes into consideration reduction in variance due to the cross-level interactions on the SES–math slope ( $\beta_{1j}$ ). This residual variance for the slope at Level 2 will reflect a difference from the previous model with random slope (but no cross-level predictors) to the current model. Equation 3.21 will add 3 extra parameters to be estimated, making a total of 11 parameters to estimate.

### Defining Model 5 with IBM SPSS Menu Commands

Settings will default to those used in Model 4.

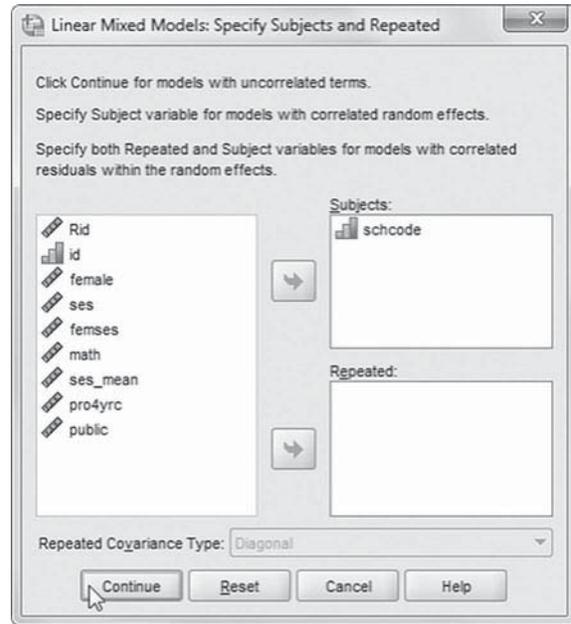
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

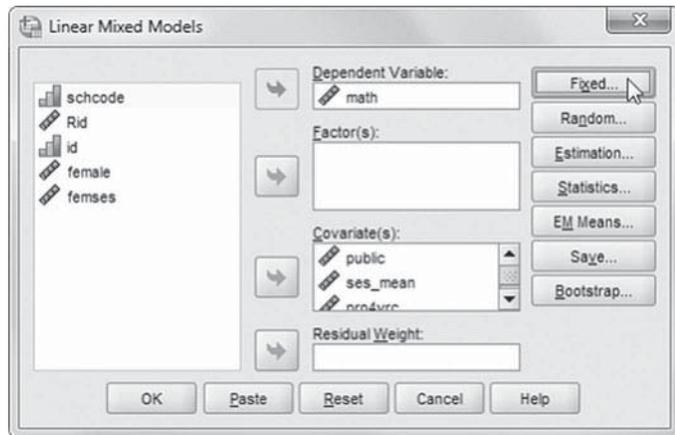
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* screen displays the default settings from the prior model.

We will add cross-level interactions to the model, so click the FIXED button to display the *Linear Mixed Models: Fixed Effects* dialog box.

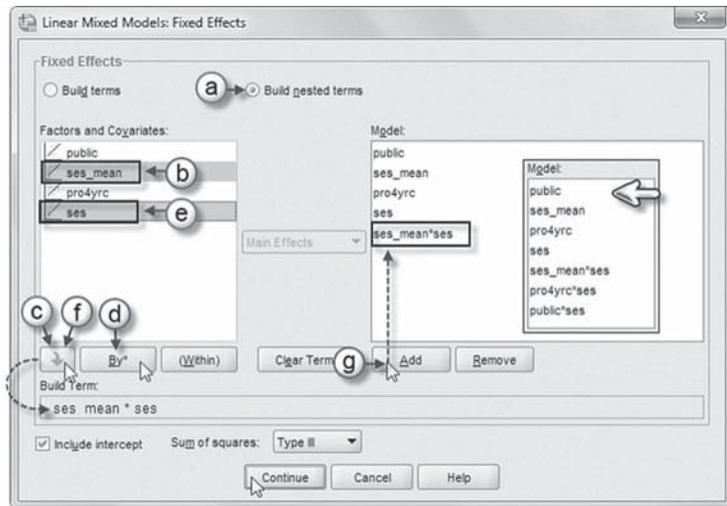
Three cross-level interactions (or nested terms) will be created and added to the model:  $ses\_mean * ses$ ,  $pro4yrc * ses$ , and  $public * ses$ . These interactions will tell us if individual student socioeconomic status (*ses*) affects math achievement for (a) school aggregate student socioeconomic status within schools (*ses\_mean*); (b) percentage of students attending 4-year universities (*pro4yrc*); and (c) school type (*public*).



**Note:** To reproduce Table 3.19, perform steps 4a to 4g, and then generate the model results.

Add First Interaction to Model 5: *ses\_mean \* ses*

- 4a. Click to select *Build nested terms*.
- b. Now click to select the variable *ses\_mean* from the *Factors and Covariates* box.
- c. Click the arrow button below the *Factors and Covariates* box. This moves *ses\_mean* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- d. Next, click the BY\* button, which will insert the computation command symbol: *ses\_mean\**
- e. Click to select *ses* from the *Factors and Covariates* box.
- f. Click the arrow button below the *Factors and Covariates* box to move *ses* into the *Build Term* box and complete the interaction term: *ses\_mean\*ses*.
- g. Click the ADD button to transfer the interaction into the *Model* box.



$$\begin{aligned}
 Y_{ij} = & \gamma_{00} + \gamma_{01}ses\_mean_j + \gamma_{02}pro4yrc_j + \\
 & \gamma_{03}public_j + \gamma_{10}SES_{ij} + \gamma_{11}ses\_mean_j * SES_{ij} + \\
 & \gamma_{12}pro4yrc_j * SES_{ij} + \gamma_{13}public_j * SES_{ij} + \\
 & u_{1j}SES_{ij} + u_{0j} + \epsilon_{ij}
 \end{aligned}
 \quad (\text{Eq. 3.21})$$

Add Second Interaction to Model 5: *pro4yrc \* ses*

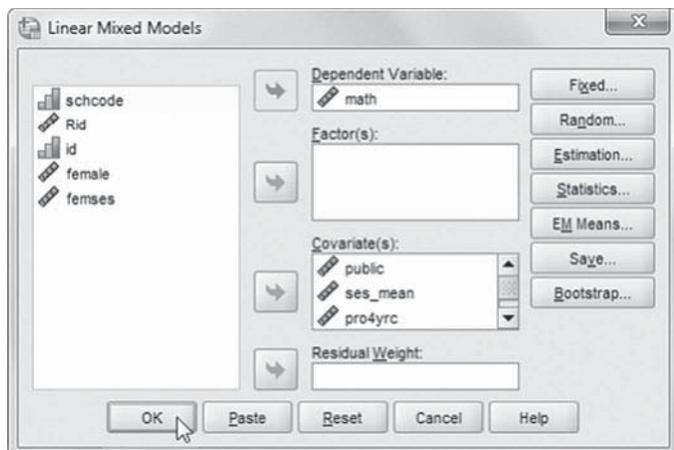
Repeat steps 4b to 4g using *pro4yrc* and *ses* for the interaction.

Add Third Interaction to Model 5: *public \* ses*

Repeat steps 4b to 4g using *public* and *ses* for the interaction.

The completed model (Eq. 3.21) is shown in the insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 5

We can confirm in the model dimension table (Table 3.21) that we estimated 11 parameters. Notice also that the Level 2 variance components table is designated as Variance Components (i.e., a diagonal matrix of random effects). The table specifies the three cross-level interactions (*ses\_mean \* ses*, *public \* ses*, and *pro4yrc \* ses*) that were added to the fixed-effect portion of the model.

Once again, the fixed-effects table (Table 3.22) indicates that aggregate SES and proportion of students planning on attending 4-year postsecondary institutions affect achievement in math. School type is not related to math achievement. Within schools, individual SES remains significantly related to math achievement.

Regarding the slope model, the cross-level interactions suggest that the within-school SES-achievement slope is different for public and private schools. As we noted previously, this is an example of a factor by covariate interaction, which suggests that in public schools the relationship between student SES and math achievement is weaker than in private schools

**TABLE 3.21 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	public	1		1	
	ses_mean	1		1	
	pro4yrc	1		1	
	ses	1		1	
	ses_mean * ses	1		1	
	pro4yrc * ses	1		1	
	public * ses	1		1	
Random Effects	Intercept + ses <sup>a</sup>	2	Variance Components	2	schcode
Residual				1	
Total		10		11	

<sup>a</sup> Dependent variable: math.

**TABLE 3.22 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	56.505	0.485	450.229	116.427	.000	55.551	57.459
public	-0.120	0.274	405.467	-0.437	.662	-0.659	0.419
ses_mean	2.706	0.324	759.554	8.351	.000	2.070	3.343
pro4yrc	1.362	0.479	439.899	2.841	.005	0.420	2.304
ses	3.757	0.606	518.400	6.203	.000	2.567	4.947
ses_mean * ses	-0.137	0.299	303.798	-0.457	.648	-0.724	0.451
pro4yrc * ses	-0.130	0.592	479.307	-0.220	.826	-1.294	1.033
public * ses	-0.668	0.331	404.838	-2.018	.044	-1.319	-0.017

<sup>a</sup> Dependent variable: math.

( $\gamma_{13} = -0.668, p < .05$ ). More specifically, in public schools (coded 1), the combined effect of individual SES on math achievement would be 3.09 [ $3.76 + (-0.67)(1) = 3.09$ ]. In contrast, in private schools (coded 0), the SES-achievement slope would be 3.76 [ $3.76 + (-0.67)(0) = 3.76$ ]. Since the slopes are different, it implies that the social distribution effects due to individual SES on math learning are less consequential for students in public schools (i.e., indicated by a flatter slope) than for students in private schools. Table 3.22 also suggests that aggregate school SES composition did not moderate the within-school SES–math achievement slope ( $\gamma = -0.14, p > .05$ ). Similarly, the proportion of students indicating that they plan to attend a 4-year college did not moderate the within-school SES-achievement slope ( $\gamma = -0.13, p > .05$ ). These findings provide partial support for our initial research goal of identifying school settings where the within-school relationship between SES and math achievement (i.e., our measure of the social distribution of learning) is diminished.

The variance components table (Table 3.23) suggests that there is still significant variance in the intercept and the SES-achievement slope to be explained across schools, even after adding this set of school variables to the model. For example, the SES-achievement slope still varies across schools (Wald  $Z = 2.375$ , one-tailed  $p = .009$ ).

We can remove the interaction terms that are not statistically significant if we wish to obtain a more parsimonious model (see Table 3.24 and instructions for Model 5A). The variables that comprise the significant interaction term in the model, however, should be retained in the model even if they are not significant (e.g., public). In a subsequent chapter, we describe procedures for examining the fit of a comparison model against a more restricted model. We exclude the presentation of the variance components for this reduced model, as they are similar to the previous model.

**TABLE 3.23** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		62.101	1.111	55.882	.000	59.961	64.318
Intercept [subject = schcode]	Variance	2.087	0.445	4.687	.000	1.374	3.171
ses [subject = schcode]	Variance	1.345	0.566	2.375	.018	0.589	3.070

<sup>a</sup> Dependent variable: math.

**TABLE 3.24** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	56.440	0.471	418.862	119.855	.000	55.515	57.366
public	-0.123	0.274	406.624	-0.449	.653	-0.662	0.415
ses_mean	2.659	0.313	697.326	8.491	.000	2.044	3.274
pro4yrc	1.404	0.468	410.165	3.003	.003	0.485	2.323
ses	3.659	0.292	461.438	12.527	.000	3.085	4.234
public * ses	-0.682	0.328	395.850	-2.078	.038	-1.328	-0.037

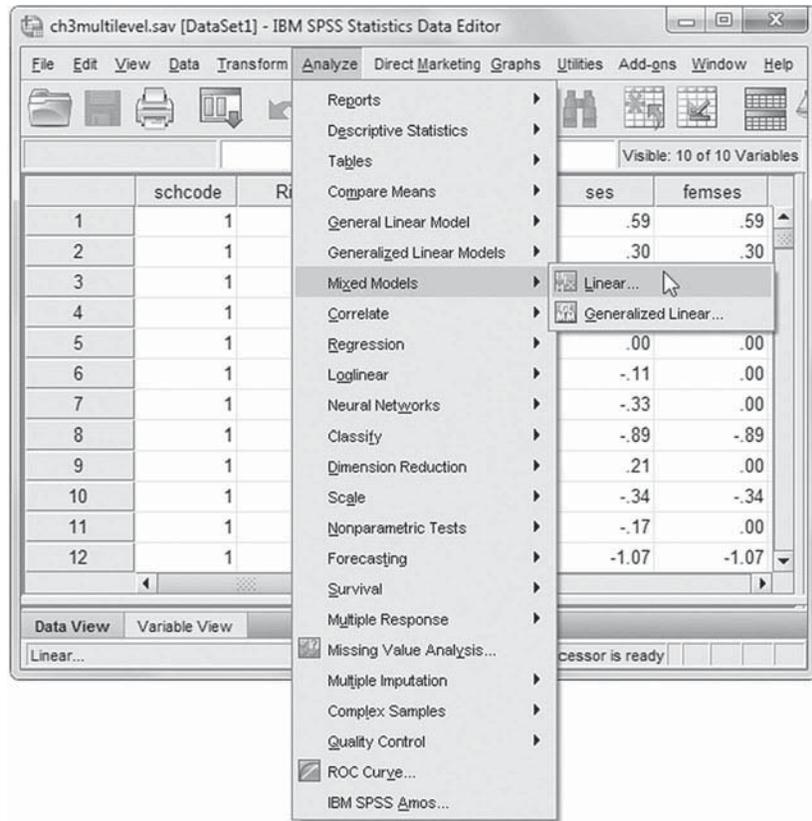
<sup>a</sup> Dependent variable: math.

### Defining Model 5A with IBM SPSS Menu Commands

Settings will default to those used in Model 5.

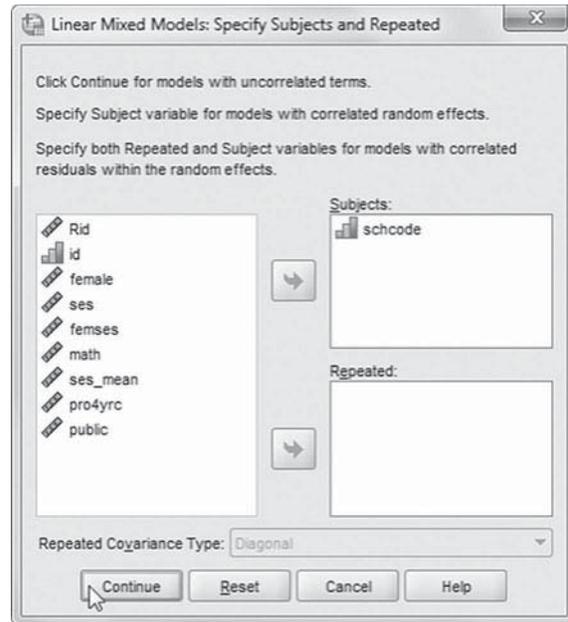
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

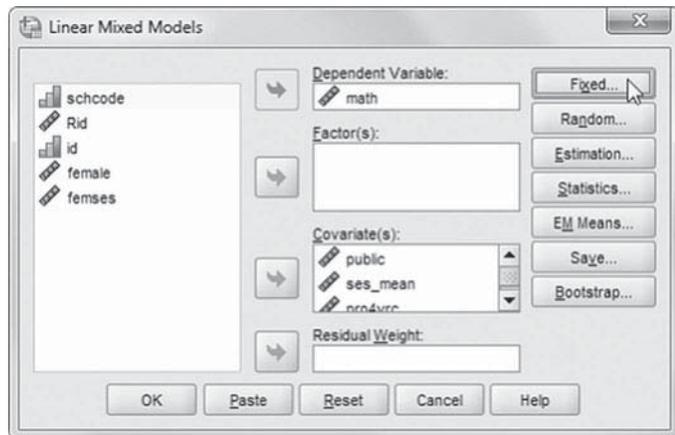


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



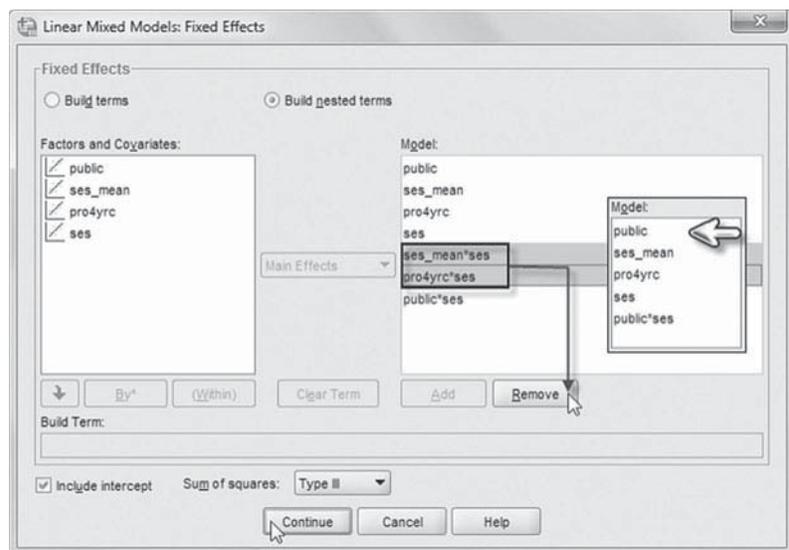
3. We will modify the model's fixed effects, so click the **FIXED** button to access the *Linear Mixed Models: Fixed Effects* dialog box.



4. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. We will remove two cross-level interactions ( $ses\_mean * ses$  and  $pro4yrc * ses$ ) from the model.

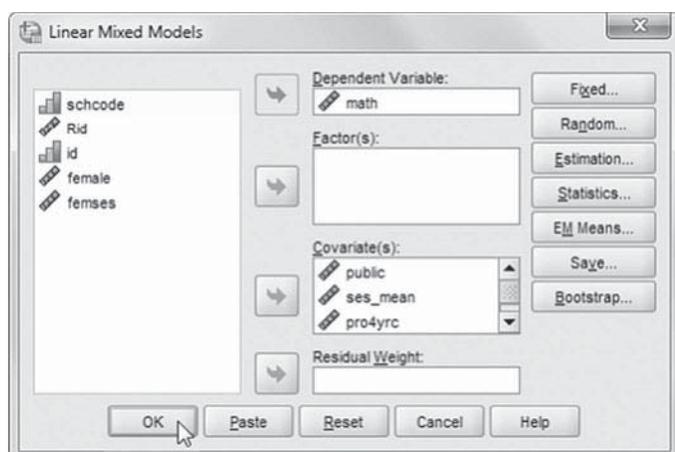
Click to select  $ses\_mean * ses$  and  $pro4yrc * ses$ , and then click the **REMOVE** button. The final model is shown in the insert.

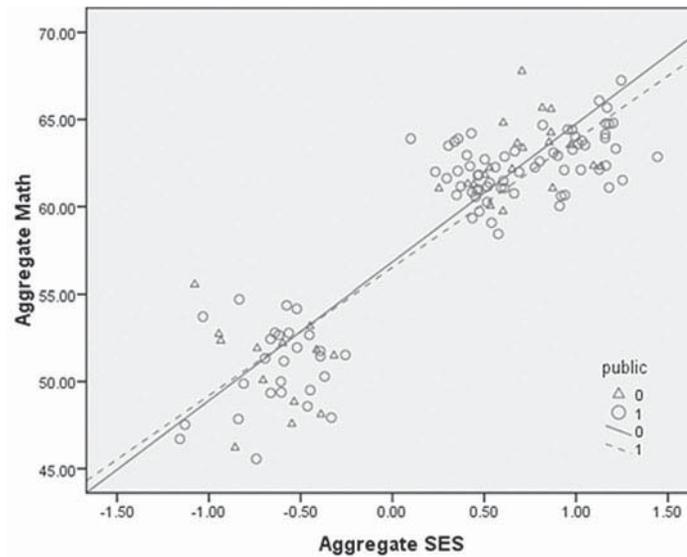
Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.



5. Finally, in the *Linear Mixed Models* dialog box, click the **OK** button to run the model.

We next provide a simple graph to illustrate what the effect of a cross-level interaction looks like (Figure 3.4). We include the menu commands for creating the graph at the end of this section. For a factor \* covariate interaction, the relationship implies that the group regression lines are not parallel. A cross-level interaction represents the effect of a unit-level variable on a within-unit relationship.





**FIGURE 3.4** SES-achievement relationships in high- and low-achieving public and private schools.

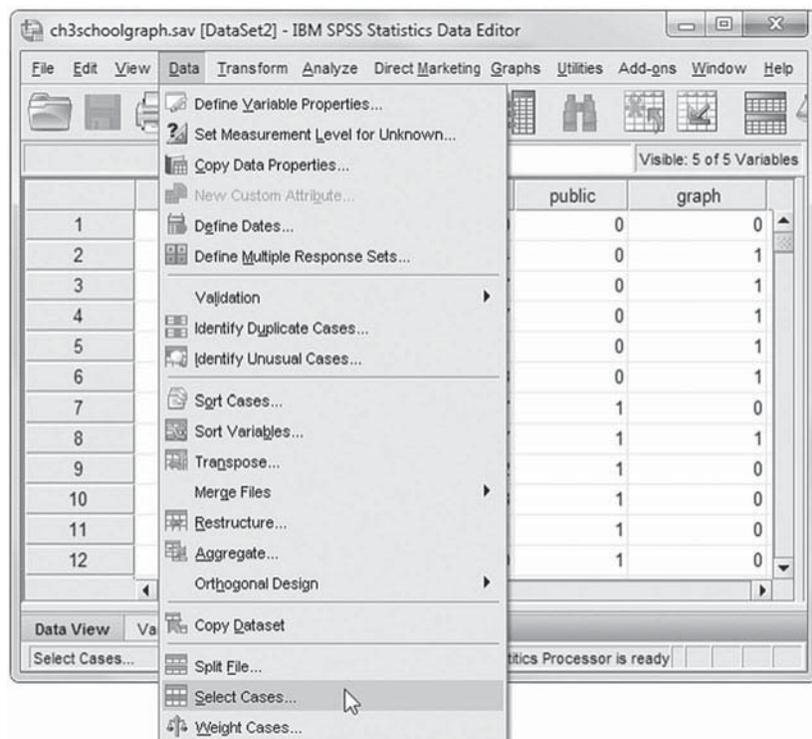
To illustrate this relationship in a rough way, we first aggregated individual math achievement and SES to the school level. We then created a subset of schools roughly 1 *SD* above and below the grand mean of achievement. This illustrates the approximate 0.7-point steeper SES-achievement slope associated with private schools. Because we focused on groups of schools some distance from the grand mean, the graph also provides a sense of where the regression lines for public and private school SES-achievement slopes cross.

### **Graphing a Cross-Level Interaction (SES-Achievement Relationships in High- and Low-Achieving Schools) with IBM SPSS Menu Commands**

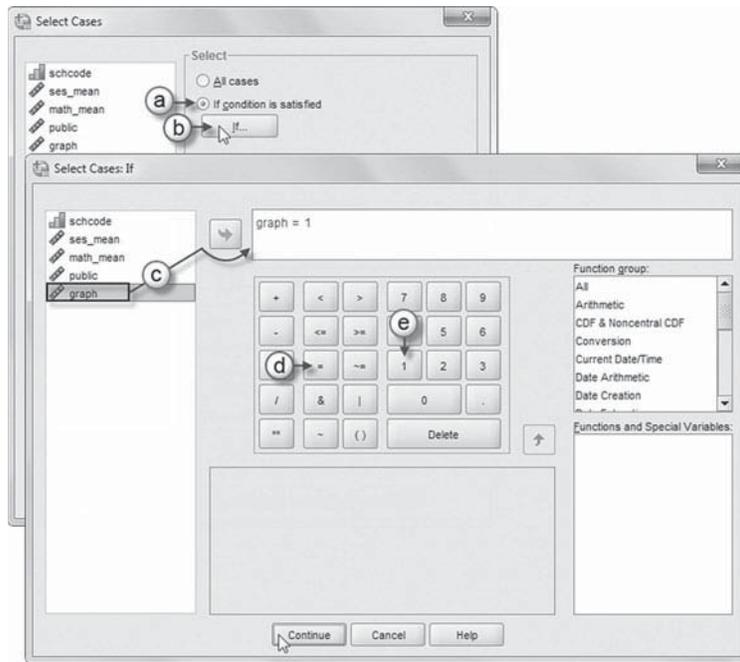
Launch the IBM SPSS application and select the data file: *ch3schoolgraph.sav*.

1. Go to the toolbar and select DATA, SELECT CASES.

This command will open the *Select Cases* dialog box.



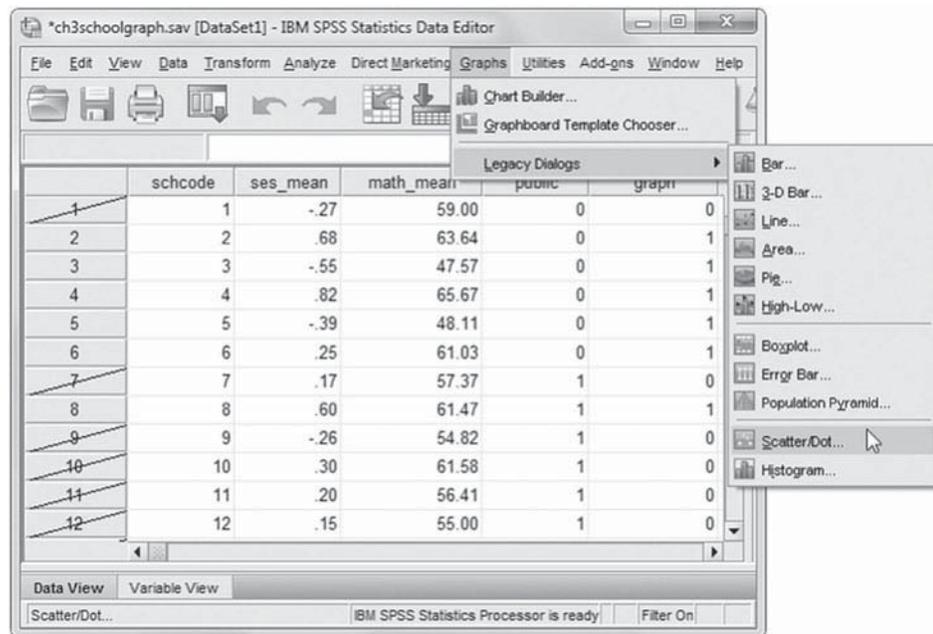
- 2a. Within the *Select Cases* dialog box, select the option *If condition is satisfied*.
- b. Click the IF button, which will open the *Select Cases: If* dialog box.
- c. Begin defining the conditional term by clicking to select *graph*, and then click the right-arrow button to move the variable into the box.
- d. Next, click the equal sign (=) to add the term to the box.
- e. Finally, click “1” to complete the condition: *graph = 1*.



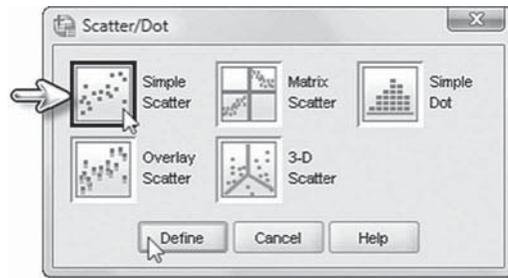
Click CONTINUE then OK to close the main *Select Cases* dialog box and return to the IBM SPSS main screen.

3. In the IBM SPSS toolbar, select **GRAPHS**, **LEGACY DIALOGS**, **SCATTER/DOT**.

This command will open the *Scatter/Dot* box.

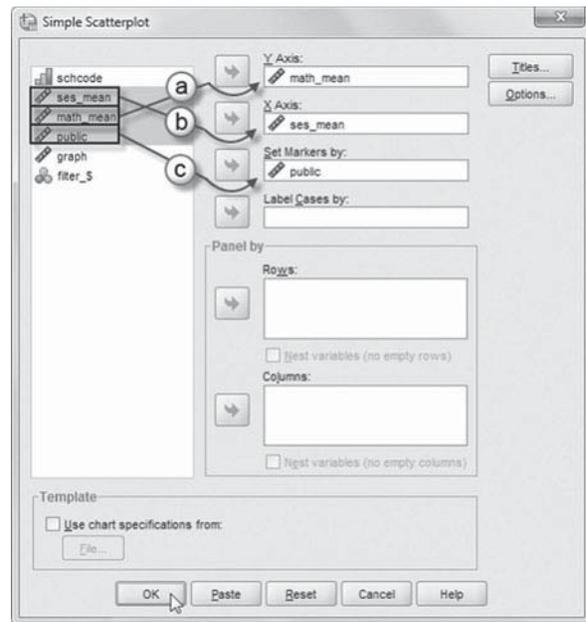


4. Click the SIMPLE SCATTER option in the *Scatter/Dot* box. Then click the DEFINE button to open the *Simple Scatterplot* box.

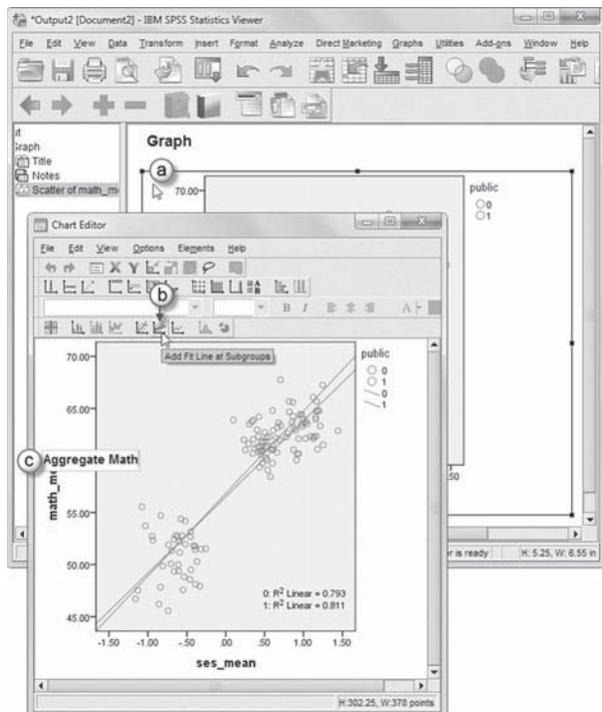


- 5a. Within the *Simple Scatterplot* box, click to select the *math\_mean* variable from the left column. Then click the right-arrow button to move *math* into the *Y Axis* box.
- b. Next, click *ses\_mean* to select the variable from the left column, and then click the right-arrow button to move the variable into the *X Axis* box.
- c. Finally, click *public* to select the variable from the left column. Then click the right-arrow button to move the variable into the *Set Markers by* box.

Click OK to generate the scatterplot.



6. The IBM SPSS output will display the scatterplot.
  - a. Place the cursor on the image and double-click to open the *Chart Editor*.
  - b. On the *Chart Editor* short-cut icon bar, click the ADD FIT LINE AT SUBGROUPS icon that has the *X* and *Y* axes. This will generate the fit line and the linear *R*-square of 0.387 for this subset of the sample.
  - c. Variable names appearing in the chart may be changed by double-clicking the text and typing in the preferred label or title.



## Centering Predictors

In this last section of the chapter, we discuss centering the predictors in a little more detail. In multilevel models, it is important that the fixed effects can be readily interpreted in relation to the proposed goals of the research. Centering decisions concern how to rescale the explanatory variables so that the intercept can be defined most advantageously. Although there is no one correct decision in every instance, centering decisions should be made primarily on the basis of the conceptual questions under investigation (Hofmann & Gavin, 1998).

Centering provides the expected value of an outcome  $Y$  when the covariate is equal to some designated value of theoretical interest (e.g., the unit mean or the overall sample mean). Mean centering is most commonly used in multilevel studies (e.g., grand mean and group mean). There are, however, other options (e.g., the median and the within-group standard deviation of the within-group coefficient of variation) for centering variables (Plewis, 1989). We note that centering is also required in some situations in order to achieve a solution that converges (i.e., does not produce an error message). Where there are widely different means and variances across groups, it may be necessary to center the raw estimates in order to achieve model convergence (Hox, 2010). In the examples in this workbook, for example, we encountered several situations when investigating random Level 1 slopes where the raw metric results would not converge, whereas centered results did converge.

As we have noted, alternative centering strategies can change the values of some parameters in a model. In Table 3.25, we illustrate the effects of different centering strategies where there is a random intercept only (Model 1) and where there is a random intercept and random slope (Model 2). We used a similar data set from the extended example in this chapter with a continuous Level 1 predictor (student SES) to illustrate the effects of the centering strategies on model parameters. In this case, we created a natural metric for SES, which ranges from 0 to 100, with a mean of 50.00 and standard deviation of 28.868. Model 1 illustrates the effects of different centering strategies for a random intercept and *fixed* Level 1 slope. As the table suggests, the natural metric estimate of SES and grand-mean-centered estimate (GMSES) are the same ( $\gamma = 0.082$ ,  $p < .05$ ); however, the location of the respective  $Y$  intercepts differs. The raw metric produces a  $Y$  intercept, which represents family SES for the lowest individual in the sample (which was coded SES = 0). For this individual, the expected math score would be 56.847. Keep in mind that in our example SES = 0 represents the individual with the lowest family income in the sample, but not an individual with family income of \$0.00.

Grand-mean centering (i.e., where the grand mean is subtracted from individuals' levels of family income) often facilitates the interpretation of a multilevel model since it results in an intercept that can be interpreted as the expected value of  $Y$  when the predictor is at its grand-mean value (0). The standard deviation, however, remains in the original metric. In this case, the original grand mean for SES (50.00) is rescaled to be 0, but the standard deviation remains 28.868. Where Level 1 predictors are all grand-mean centered, the solution provides a Level 2 intercept that has been adjusted for the Level 1 predictors. One related advantage of grand-mean centering is that it helps the analyst interpret the variances for the intercept and slopes as the expected variance when all explanatory variables are equal to 0—that is, the expected variances for the “average” individual (Hox, 2010).

In contrast, standardizing a continuous predictor ( $\bar{X} = 0$ ,  $SD = 1$ ) has the same effect on the intercept as grand-mean centering, but it also changes the metric of the predictor by transforming the standard deviation from its original metric (in this case, 28.868) to be equal to 1.0. More specifically, when the continuous predictor is standardized (i.e., a  $z$ -score), it is in standard deviation units. Therefore, as in Model 1, the predictor's metric will be different from the grand-mean-centered estimate ( $\gamma = 0.082$ ). In contrast, the standardized estimate of SES is 2.366, suggesting that an increase in student SES of 1  $SD$  would produce a 2.344 increase in the math outcome.

In many cases, the uncentered or natural metric may be a logical choice since for the natural metric the intercept is defined as the level of  $Y$  when  $X$  is 0 (0.0). Some natural metric solutions, however, may have little practical importance in organizational studies (Kreft, de Leeuw, & Aiken, 1995). In the example in Table 3.25, there might be occasions where we prefer centering on the lowest individual in the sample (i.e., where SES is scaled to be equal to 0), but most often it is convenient to center on the average level of the predictor in the sample. Importantly, we draw attention to the fact that for models with a random intercept only, solutions where the Level 1 predictors are in their natural metrics, grand-mean centered, or standardized will produce equivalent models (i.e., in terms of deviance, or  $-2$  log likelihood  $[-2LL]$ , and residual variances) as shown in Table 3.25 (for further discussion, see Hox, 2010; or Raudenbush & Bryk, 2002).

Group-mean centering produces an intercept equal to the expected value of  $Y$  for an individual when  $X$  is equal to the group's mean. Unlike grand-mean centering, with group-mean centering, the unit means are *unadjusted* for differences among their members. Group-mean centering puts the attention more on relational advantages that some individuals may enjoy within their particular social group. It is important to emphasize that group-mean-centered solutions are not the same as grand-mean or natural metric solutions. In contrast to the previous centering strategies, when we use group-mean centering of explanatory variables, the meaning of the

**TABLE 3.25** The Effects of Different Centering Strategies

Variables	SES	ZSES	GMSSES	GRPSES	GRPSES(1)
<b>Model 1: Random Intercept</b>					
<i>Between Schools</i>					
Intercept	56.847*	60.945*	60.945*	60.896*	53.567*
Mean SES					0.148*
<i>Within Schools</i>					
Estimate	0.082*	2.366*	0.082*	0.065*	0.065*
Variance Components					
Level 2 Variance	23.515*	23.515*	23.515*	29.957*	21.667*
Level 1 Variance	215.435*	215.435*	215.435*	215.125*	215.416*
Parameters	4.000	4.000	4.000	4.000	5.000
-2 Log Likelihood	104,289.430	104,289.430	104,289.430	104,403.419	104,246.714
<b>Model 2: Random Slope</b>					
<i>Between Schools</i>					
Intercept	56.862*	61.079*	61.079*	60.904*	53.587*
Mean SES					0.148*
<i>Within Schools</i>					
Estimate	0.084*	2.434*	0.084*	0.063*	0.036
Mean SES*Estimate					0.001
Variance Components					
Level 2 Slope	0.003*	2.459*	0.003*	0.003*	0.003*
Covariance	-0.375*	-6.571*	-0.228*	-0.243*	-0.249*
Level 2 Variance	53.062*	22.924*	22.924*	30.186*	21.915*
Level 1 Variance	213.682*	213.682*	213.682*	213.415*	213.660*
Parameters	6.000	6.000	6.000	6.000	8.000
-2 Log Likelihood	104,229.679	104,229.679	104,229.679	104,365.540	104,196.881

\* $p < .05$ .

model is changed (Hox, 2010). This is because group-mean centering separates the within-group and between-group portion of the predictor estimate; that is, the information concerning possible differences in the predictor across Level 2 units is removed.

As Raudenbush and Bryk (2002) suggest, it is often desirable to group-mean center a predictor if the focus of the analysis is on producing an unbiased estimate of the within-group (Level 1) effect, since group-mean centering results in a Level 2 mean that is unadjusted for the Level 1 predictor. If the analyst is interested in person-level variables (but where individuals are nested in groups), ignoring the nested structure of the data can lead to misleading results at Level 1 (Cronbach, 1976), since slope coefficients derived from the total covariance matrix (i.e., the covariance matrix based on the number of individuals in the study) will generally represent uninterpretable blends of their within-group and between-group estimates. Similarly, in this situation, the grand-mean-centered estimates also represent a mix of the within-group slope and any difference in the slopes that might exist across groups (i.e., in the case where the slope varies randomly across groups) since the model with grand-mean centering actually involves both the individual estimates of  $X_{ij}$  and the Level 2 mean of  $X$  as adjusted (Raudenbush & Bryk, 2002).

In Table 3.25, for the model with random intercept only (Model 1), we can observe that the group-mean-centered solution (GRPSES) results in a different intercept estimate (60.896), a different estimate of the Level 1 predictor ( $\gamma = 0.065$ ), as well as different estimates of the Level 2 intercept variance (29.957) and model deviance, or  $-2LL$ . The Level 2 intercept variance will typically be larger than in the previous solutions. This is because the Level 2 intercepts in the group-centered solution are unadjusted, whereas the intercepts in the previous solutions are adjusted for the Level 1 predictor (Raudenbush & Bryk, 2002). As Raudenbush and Bryk note, because *Mean SES* is likely related to the intercept (and also the slope, as in Model 2), we might attempt to add *Mean SES* to the Level 2 model [as we do with GRPSES(1) in the last column] in an effort to resolve the discrepancy in Level 2 intercept variance between the previous centering solutions and the group-centered solution. Because group-mean centering results in a different type of model (i.e., by isolating the within-group portion of the estimate and removing the between-group differences in that portion of the estimate), however, one cannot simply add the mean for the variable back at the group level [as in the GRPSES(1) column], since that results in adding more information that is not present in the raw scores (i.e., by adding one additional parameter to estimate). By adding another parameter, the model deviance will be smaller than the previous solutions for Model 1 (Raudenbush & Bryk, 2002). For example, in Table 3.25 we can observe for GRPSES(1) that the  $-2LL$  estimate is the lowest of the previous four centering decisions, reflecting the addition of the *Mean SES* parameter at Level 2.

### Centering Predictors in Models with Random Slopes

Centering decisions can also be important in multilevel models where there are anticipated cross-level interactions—that is, where the analyst wishes to examine variability in slopes across groups by building a model with Level 2 predictors to explain this variability. Where the focus is on random slopes, grand-mean-centering and group-mean-centering approaches generally address different research questions since grand-mean centering puts the focus on Level 2 relationships after adjustment for Level 1 relationships. Moreover, the effects of Level 2 predictors will be adjusted for differences between organizations in the mean of the  $X$  predictor (Raudenbush & Bryk, 2002). In contrast, group-mean centering is the better approach if the focus is on optimal estimation of Level 1 effects that consider nesting since group-mean centering removes confounding between-group effects in the Level 1 predictors.

It is important to note that modeling random slopes at Level 2 introduces modeling cross-level interactions. Cross-level interactions refer to situations where Level 2 variables are proposed to moderate the strength of Level 1 relationships, such as the effect of student SES on students' achievement. Cross-level interactions can actually be proposed as moderators of randomly varying Level 1 slopes or as moderators of fixed within-group relationships. In this latter case, the

slopes do vary from group to group, but rather than their variation being random, they vary as a function of the Level 2 moderator (Raudenbush & Bryk, 2002). Cross-level interactions introduce another aspect to be considered—that is, the interpretation of the interaction. In regression models, the interpretation is typically the expected value of one predictor when the other one is equal to 0 and vice versa (Hox, 2010). Therefore, we want to exercise some care in selecting a value for 0 that is meaningful and actually occurs in the data (Hox, 2010).

Given this, natural (or raw) metric solutions are often difficult to interpret since the level of  $Y$  (i.e., the  $Y$  intercept) may be difficult to interpret when  $X$  is 0. In contrast, centering on the grand mean will produce an interaction coefficient interpreted as the effect of the first predictor on  $Y$  for individuals who are at the grand mean (0) on the other predictor. Grand-mean centering is one straightforward way to facilitate the interpretation of interactions. Group-mean centering is also appropriate where there are hypotheses involving interactions among Level 1 variables, as in the case with cross-level interactions, since the Level 2 variables proposed to moderate the Level 1 relationship will be unaffected by adjustment within groups (as takes place in grand-mean centering).

In models where there is a randomly varying Level 1 slope, different centering strategies will also produce different results. In Table 3.25, Model 2 indicates that the choice of centering will make a difference in estimating the Level 2 intercept variance, as well as the slope variance and covariance between the intercept and slope. As we noted previously, the variance of  $Y$  at Level 2 will typically be larger in group-mean models than in grand-mean or raw metric models since the group means in the group-centered solution are unadjusted. The slope variance will also be different (i.e., generally larger in group-mean solutions), although that is not the case in our example (except for the standardized solution). This is likely because our sample sizes are fairly large for any given Level 2 unit since we have 12,600 students nested in some 400 schools. In such cases, centering generally does not make much difference (Raudenbush & Bryk, 2002). We again note in the last column of Model 2 that incorporating mean SES as a predictor in the Level 2 intercept model and also in the Level 2 random-slope model does not solve the problem of the different estimates of slope variability produced by the different centering approaches. This actually introduces two extra parameters to be estimated, which again results in a better-fitting model.

In some situations, group-mean centering may be preferable to estimate slope heterogeneity properly (Raudenbush & Bryk, 2002). For example, when the Level 1 sample size is small or moderate, or if the group mean of  $X$  varies across units, group-mean centering should be considered as a viable alternative if the focus of the analysis is to produce more robust estimates of unit-specific regression equations (Raudenbush & Bryk, 2002). Because grand-mean-centered models produce an adjusted intercept for the Level 1 predictors, in some cases (e.g., schools that are very high or very low in mean SES), the adjusted means can be estimated with little accuracy. This happens because the adjusted mean for school  $j$  represents the expected outcome for a child in the school who is at the grand mean of SES, and if there are few children like this in a particular school, the school mean will be less reliably measured than for some other schools where a larger number of students have family income levels that are at the grand mean for SES in the sample (Raudenbush & Bryk, 2002). This can result in making the slope variability more homogeneous than it actually is and, therefore, results in the underestimation of the slope variability at Level 2. As we noted, where sample sizes are large for any given Level 2 unit, centering will not make much difference, as well as when the slope does not vary across Level 2 units.

At the top of the data hierarchy (e.g., schools), the centering choices are not as critical as for predictors at lower levels (Raudenbush & Bryk, 2002). Usually it is convenient to center continuous variables around their grand means, as opposed to leaving them uncentered. We note that variables cannot be group centered at the highest level, since their “group” at that level is the Level 2 sample of schools. Centering strategies are the same for dichotomous variables.

Dummy-coded variables (0, 1) are often left in their natural metric, depending on the analyst's choice for the meaning of the reference group (coded 0) in the unit-level model. They can also be grand-mean or group centered. Dichotomous variables can also be effect coded (e.g., -1, +1), which results in a type of grand-mean centering. We discuss interpreting results with dichotomous outcomes in the next chapter.

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## Summary

We have shown in this chapter how multilevel analyses allow the analyst to investigate a wider range of research questions than can be addressed in single-level analyses. In Chapter 4, we continue to illustrate the flexibility of multilevel techniques by extending the basic two-level model to a three-level model. We also address issues regarding building a variety of within-level and cross-level interactions.

# CHAPTER 4

## Three-Level Univariate Regression Models

The examples presented in the previous chapter demonstrated the basic multilevel regression model for examining hierarchical data structures. The basic two-level model can be readily extended to cross-sectional models involving several levels in a data hierarchy, regression discontinuity designs, and multilevel longitudinal models involving individuals and groups. In this chapter, we provide an overview of three-level, cross-sectional modeling. We also introduce a strategy for centering predictors in multilevel models to facilitate the interpretation of effects and a strategy for comparing the fit of successive proposed models to the data.

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### Three-Level Univariate Model

As a first example, consider a three-level model with a univariate outcome, *math achievement*. In this example, we primarily wish to examine whether classroom teaching effectiveness (i.e., a characteristic of teachers at Level 2) and aggregate teaching effectiveness (i.e., as a characteristic of schools at Level 3) affect student outcomes. At the classroom level, we also focus on whether teacher effectiveness may be related to student composition. Finally, at the school level, we wish to examine whether the size of individual teacher effects varies across schools.

### Research Questions

We investigate three research questions in this example. The first question concerns whether a key organizational process, in this case the effectiveness of students' teachers, has an individual and an organizational effect on their academic outcomes, after relevant background and organizational context indicators have been controlled. This involves examining whether effects at lower levels (i.e., the classroom) of the data hierarchy tend to compound at a higher level (i.e., the school). As we mentioned in the previous chapter, this type of relationship is often referred to as a compositional effect. More specifically, we might ask: At the classroom level, does having a more effective teacher confer an academic advantage to those students compared with their peers having a teacher of average effectiveness? Moreover, does being in a school with more effective teachers on average confer any additional advantage compared with students in schools with less effective teachers?

The second research question concerns whether the teacher effectiveness slope varies across schools. We can address this question by examining the variance components for the slope at Level 3 of the model. If the slope does vary across schools, we can focus on building a model that explains variability in the random slope at the school level. This type of model concerns the presence of a cross-level interaction—that is, the potential effect a variable at a higher level of the data hierarchy may have on a relationship at a lower level.

The third question focuses on whether teacher effectiveness at the classroom level is contingent on student composition. Addressing this question allows us to demonstrate how to investigate an interaction between two variables at the same level of the data hierarchy. This is also

a common type of specification in a multilevel model. Interactions can be interpreted as the amount of change in the slope of  $Y$  (achievement) with respect to  $X$  (teacher effectiveness) when  $Z$  (student classroom socioeconomic status [SES] composition) changes by 1 unit. More specifically, we ask: Does the dependence of a student's achievement score on the effectiveness of her teacher also depend on differing levels of socioeconomic status in the classroom? Keep in mind that interactions are not simply "additive," as are the main effects of variables in a model (i.e., the additional effect of teacher effectiveness in explaining achievement while holding SES composition constant), but, rather, they depend on the specific levels of the two variables from which they are produced (effectiveness and classroom SES). We can then test whether the interaction term added to the model produces a better fit than the main-effects-only model.

## The Data

For this example, we will use a random sample of 9,196 students nested in 516 classrooms in 160 schools.

Table 4.1 provides a summary of the variables used in the example. The outcome is student scaled scores on a standardized math test. The sample data are found in the file *ch4threelevelURM.sav*. The data have an identification code for the school (*schcode*) and teacher (*teachid*),

**TABLE 4.1** Data Definition of *ch4threelevelURM.sav* ( $N = 9,196$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>schcode</i>	School	School identifier (160 public schools).	Integer	Ordinal
<i>teachid</i>	Class	Teacher identifier (516 teachers in their classrooms).	(1, 2, 3, . . . 516)	Ordinal
<i>Rteachid</i>	Class	Recoded <i>teachid</i> identifying individual teachers within schools ( <i>schcode</i> ).	(1, 2, 3, . . . 29)	Ordinal
<i>math</i>	Individual	Dependent variable measuring student math achievement score.	481 to 775	Scale
<i>lowses</i>	Individual	Dichotomous variable representing student socioeconomic status.	0 = Not Low Socioeconomic Status 1 = Low Socioeconomic Status	Scale
<i>teacheffect</i>	Class	A standardized measure of each teacher's classroom teaching effectiveness.	-2.64 to 3.05	Scale
<i>classlowses_mean</i>	Class	Predictor variable measuring student socioeconomic composition within school classrooms.	.00 to 1.00	Scale
<i>schlowses_mean</i>	School	Predictor variable measuring student economic status.	0.00 to 1.00	Scale
<i>gmlowses</i>	Individual	Predictor variable (grand-mean centered) measuring socioeconomic status.	-0.41 = Not Low Socioeconomic Status 0.59 = Low Socioeconomic Status	Scale
<i>gmclasslowses_mean</i>	Class	Predictor variable (grand-mean centered) measuring student socioeconomic status within classrooms.	-0.41 to 0.59	Scale

(Continued)

**Table 4.1** (Continued)

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>gmteacheffect</i>	Class	Predictor variable (grand-mean centered) measuring teacher effectiveness by their students' math scores.	-2.62 to 3.06	Scale
<i>gmschlowSES_mean</i>	School	Predictor variable (grand-mean centered) measuring school socioeconomic status.	-0.41 to 0.59	Scale
<i>gmaggtcheffect</i>	School	Predictor variable (grand-mean centered) measuring teacher effectiveness.	-0.97 to 1.57	Scale
<i>grouplowses</i>	Individual	Predictor variable (group-mean centered) measuring socioeconomic status.	-0.95 to 0.96	Scale
<i>groupclasslowses_mean</i>	Class	Predictor variable (group-mean centered) measuring student socioeconomic status within classrooms.	-0.72 to 0.80	Scale
<i>groupteacheffect</i>	Class	Predictor variable (group-mean centered) measuring teacher effectiveness by their students' math scores.	-3.49 to 3.41	Scale

<sup>a</sup> Individual = Level 1; Class = Level 2; School = Level 3.

as well as a recoded teacher identity (*Rteachid*). The recoded Level 2 identification code (i.e., individual teachers within schools) substantially reduces the amount of time it takes to run the model. Next is a math score, a measure of individual student socioeconomic status (coded 1 = low SES, 0 = else), a standardized (mean [*M*] = 0, standard deviation [*SD*] = 1) assessment of each teacher's classroom teaching effectiveness (*teacheffect*), a classroom SES composition variable (*classlowses\_mean*), and an aggregate (school-level) measure of student SES composition (*schlowses\_mean*). These latter two variables were created from the individual-level SES variable (*lowses*) by using the "aggregate" command within IBM SPSS (DATA, AGGREGATE) and *teachid* and *schcode* as the "break" variables, respectively.

### Defining the Three-Level Multilevel Model

Our notation follows that used by Raudenbush and Bryk (2002). For three-level models, fixed-effect coefficients at Level 1 are defined as  $\pi$ , so that Level 2 coefficients can be defined as  $\beta$  and Level 3 as  $\gamma$ . For individual  $i$  in class  $j$  in school  $k$ , the general Level 1 model can be defined as

$$Y_{ijk} = \pi_{0jk} + \sum_{p=1}^P \pi_{pjk} a_{pjk} + \varepsilon_{ijk}, \quad (4.1)$$

where  $\pi_{0jk}$  is an intercept;  $a_{pjk}$  represents Level 1 predictors ( $p = 1, \dots, P$ ), such as socioeconomic status, for individual  $i$  in Level 2 unit  $j$  and Level 3 unit  $k$ ;  $\pi_{pjk}$  represents corresponding Level 1 coefficients; and  $\varepsilon_{ijk}$  is the Level 1 residual. The Level 1 residual  $\varepsilon_{ijk}$  is assumed to be normally distributed with the mean equal to 0 and variance  $\sigma^2$ .

At Level 2, the general classroom model can be specified as

$$\pi_{pjk} = \beta_{p0k} + \sum_{q=1}^{Q_p} \beta_{pqk} X_{qjk} + r_{pjk} \quad (4.2)$$

where  $\beta_{p0k}$  is the intercept for school  $k$  in modeling the teacher effect;  $X_{qjk}$  represents Level 2 predictors ( $q = 1, \dots, Q_p$ ), such as teaching effectiveness;  $\beta_{pqk}$  represents corresponding Level 2 coefficients; and  $r_{pjik}$  represents Level 2 random effects. Level 1 coefficients can be modeled at Level 2 as fixed at the same value for all Level 2 units (i.e.,  $\pi_{pjik} = \beta_{p0k}$ ), which indicates that no predictors or random  $r_{pjik}$  component in Equation 4.2 is included in the model; as nonrandomly varying among Level 2 units (i.e., Eq. 4.2 without the random  $r_{pjik}$  component); or as randomly varying (Eq. 4.2 with the random component  $r_{pjik}$  included). The Level 2 random effects are collected in a covariance matrix whose dimensions depend on the number of random effects specified in the Level 2 model (see Raudenbush, Bryk, Cheong, & Congdon, 2004, for further discussion).

Between schools (Level 3), a general model can be defined as

$$\beta_{pqk} = \gamma_{pq0} + \sum_{s=1}^{S_{pq}} \gamma_{pqs} W_{sk} + u_{pqk}, \quad (4.3)$$

where  $\gamma_{pq0}$  is an intercept;  $W_{sk}$  represents Level 3 predictors ( $s = 1, \dots, S_{pq}$ ), such as student composition;  $\gamma_{pqs}$  represents corresponding Level 3 coefficients; and  $u_{pqk}$  represents Level 3 random effects. The dimensions of the Level 3 covariance matrix of random effects depend on the number of randomly varying effects in the model. As Raudenbush et al. (2004) suggest, Level 2 coefficients can be modeled at Level 3 as fixed ( $\beta_{pqk} = \gamma_{pq0}$ ), as nonrandomly varying (i.e., Eq. 4.3 without the random component  $u_{pqk}$ ), or as randomly varying (as specified in Eq. 4.3).

### The Null Model (No Predictors)

For this first no-predictors, or null, model, we can examine the decomposition of variance in math achievement associated with students, classrooms, and schools. For a three-level model, the proportion of variability (intraclass correlation) in outcomes at Level 3 can be defined as

$$\rho = \sigma_{Level\ 3}^2 / (\sigma_{Level\ 1}^2 + \sigma_{Level\ 2}^2 + \sigma_{Level\ 3}^2) \quad (4.4)$$

For Level 2, the intraclass correlation (ICC) would be

$$\rho = \sigma_{Level\ 2}^2 / (\sigma_{Level\ 1}^2 + \sigma_{Level\ 2}^2 + \sigma_{Level\ 3}^2) \quad (4.5)$$

and for Level 1, the ICC would be

$$\rho = \sigma_{Level\ 1}^2 / (\sigma_{Level\ 1}^2 + \sigma_{Level\ 2}^2 + \sigma_{Level\ 3}^2) \quad (4.6)$$

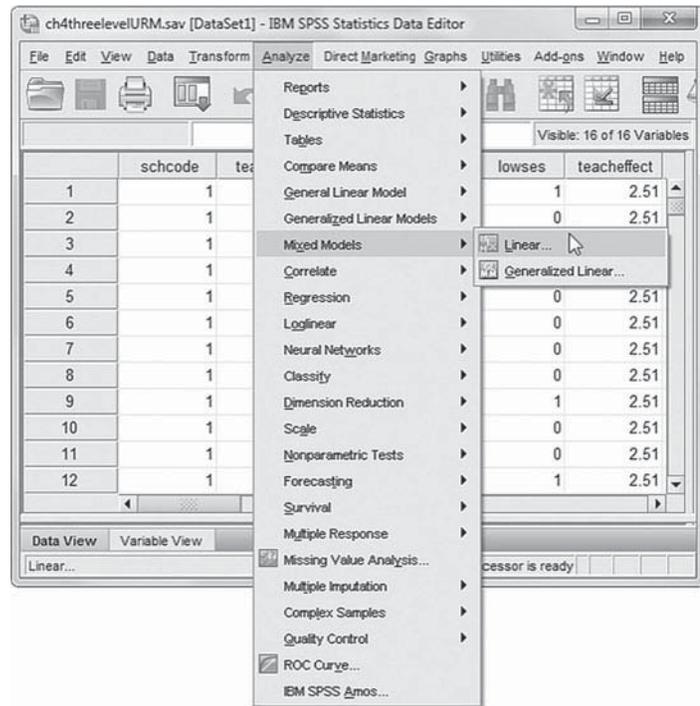
### Defining Model 1 (Null) with IBM SPSS Menu Commands

The appropriate baseline model from which to compare subsequent models is the intercept-only, or no-predictors, model.

Launch the IBM SPSS program application, and select the *ch4threelevelURM.sav* data file.

1. Go to the toolbar and select **ANALYZE, MIXED MODELS, LINEAR**.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

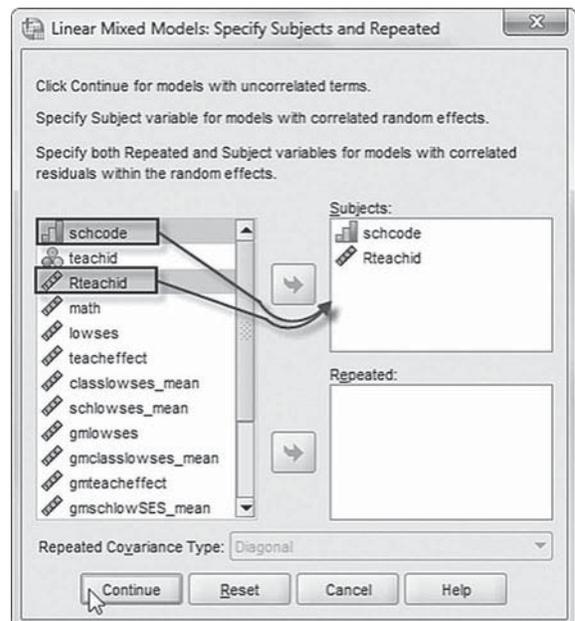


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.

A subject is an observational unit that may be independent of other subjects. For this model, we will designate two subject identifiers (*schcode*, *Rteachid*) for the model.

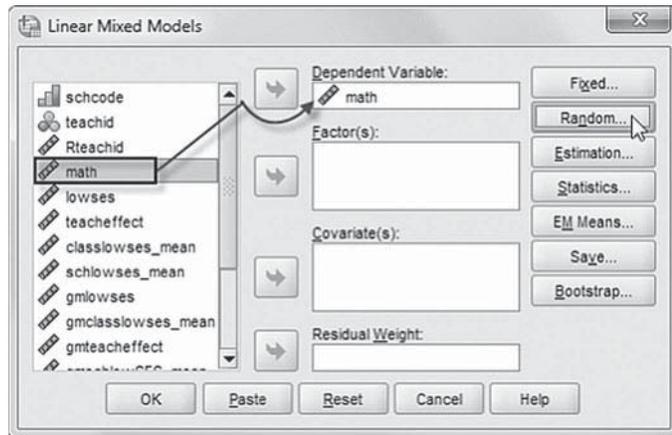
Click to select *schcode* and *Rteachid* from the *Variables* column, and then click the right-arrow button to move the variables into the *Subjects* box.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, covariates, and access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

For this model, we will use *math* as the dependent variable. Click to select *math* from the left column listing. Then click the right-arrow button to transfer *math* into the *Dependent Variable* box.

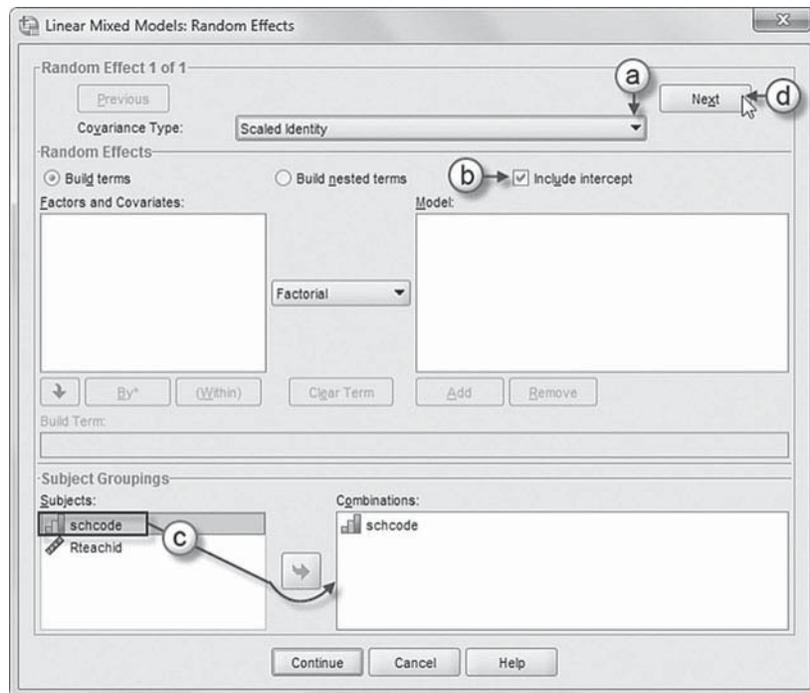


Since this first model (null) excludes predictor variables, we will skip over the **FIXED** button, which enables adding fixed effects to a model.

We will add random effects to this model. Click the **RANDOM** button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen. This is the default screen when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.

- a. Begin by specifying the covariance structure from the default variance components (VC) to scaled identity. Click the pull-down menu and select *Scaled Identity*.



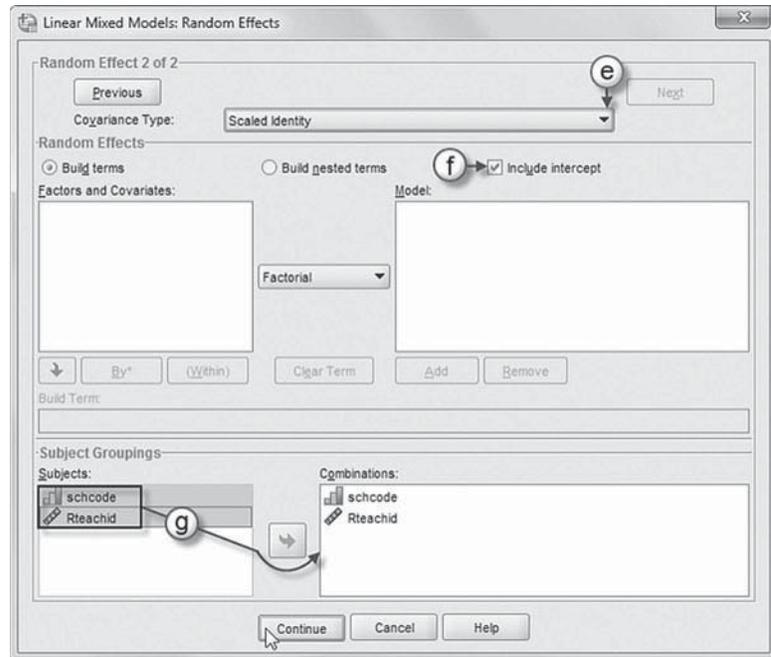
The *Scaled Identity* covariance structure has constant variance and assumes no correlation between any elements (IBM Corporation, 2012).

- b. We want the intercept to be included in the model, so click the *Include intercept* option.
- c. The *Subject Groupings* box displays the *schcode* variable that was specified as a subject variable in the *Specify Subjects and Repeated* dialog box show in step 2. We will specify *schcode* as the subject for the random-effects Level 1 part of this model. Click to select *schcode*, and then click the right-arrow button to move the variable into the *Combinations* box.
- d. After adding *schcode* as a subject in the prior step, the **NEXT** button is activated at the top-right section of the window. Click the **NEXT** button to access the *Random Effect 2 of 2* screen.

**Note:** The NEXT button may not work in earlier or unpatched versions of IBM SPSS when creating multilevel models with random intercepts. An update issued by IBM SPSS for software Version 19 addressed the problem, and Version 20 appears to have resolved the issue. A workaround to activating the NEXT button is to either (a) add or reenter a subject variable into the *Combinations* box or (b) add a variable from the *Factors and Covariates* column to the *Model* box and then remove it before proceeding to the *Random Effect 2 of 2* screen.

The *Random Effect 2 of 2* screen display is similar to the first screen and requires the following changes.

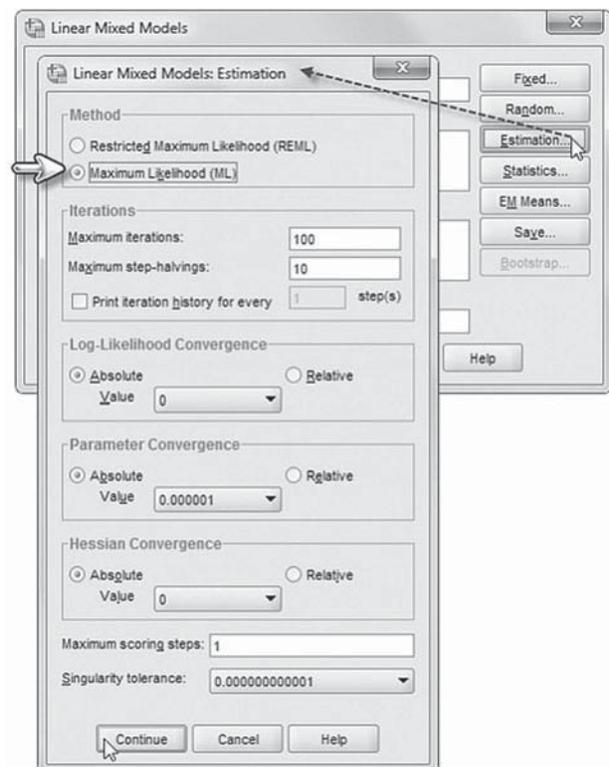
- e. Change the covariance type by clicking on the pull-down menu and selecting *Scaled Identity*.
- f. We want the intercept to be included, so click the *Include intercept* option.
- g. We will specify *schcode* and *Rteachid* as the subjects for the random-effects Level 2 part of this model. Click to select *schcode* and *Rteachid*, and then click the right-arrow button to move the variables into the *Combinations* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. In the *Linear Mixed Models* dialog box, click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: maximum likelihood (ML) or restricted maximum likelihood (REML). In this chapter, we will use maximum likelihood to estimate the models, which facilitates making comparisons between successive models using model fit criteria (Hox, 2002).

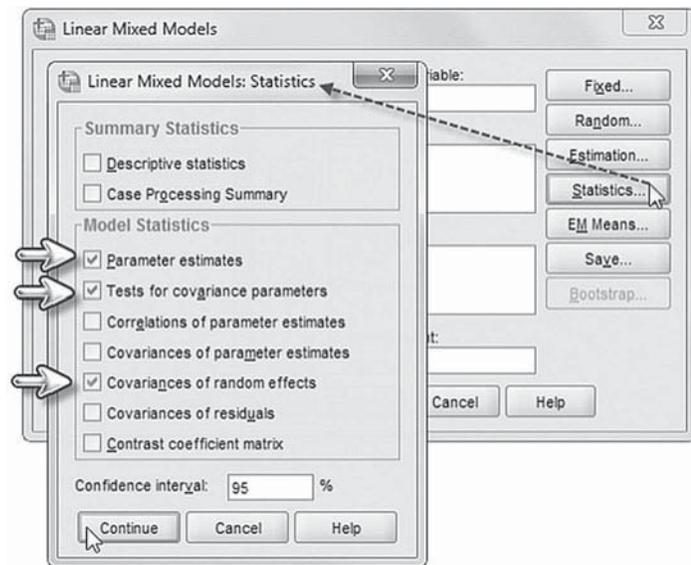
Click to select ML, and then click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



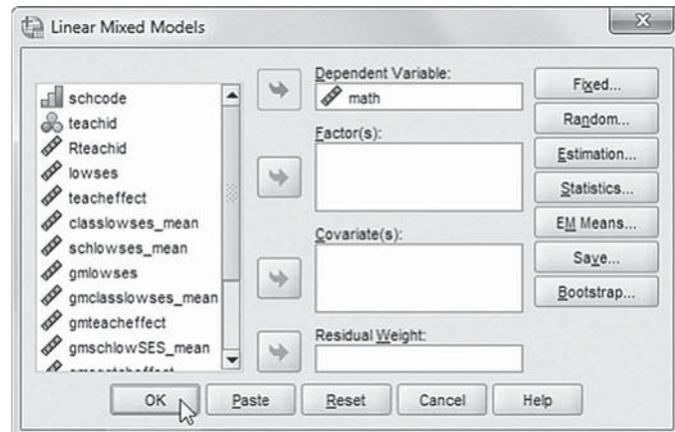
6. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### **Interpreting the Output From Model 1 (Null)**

Table 4.2 presents the variance decomposition for the null (no-predictors) model. The variance component associated with schools is 189.89; with classrooms, it is 131.42; and with individuals, it is 1,333.79. From Equation 4.4, we can calculate the proportion of variance between schools as 0.115  $[189.8867 / (1,333.7850 + 131.4211 + 189.8867)]$  or 11.5%. Following Equation 4.5, the variance is 0.079 (7.9%) between classrooms, and from Equation 4.6, the student-level variance is 0.806 (80.6%). This suggests there is adequate variability at each level to conduct a multilevel analysis.

We reiterate here that in multilevel modeling, explaining variance is more complex than in single-level regression models (Hox, 2010). First, there is the issue of dealing with unexplained variance at several levels. Second, if there are random slopes, the model becomes more complex, and explained variance (at each level) has no unique definition. As we noted in Chapter 3, one approach often used is to examine the change in residual variance that occurs by adding predictors within a sequence of models. The analyst begins with the intercept-only model as we have just presented. This serves as a baseline against which to evaluate subsequent reduction in the variance at each level as other variables are subsequently added to the model. The analyst should keep in mind, however, that when variables are added at Level 1, they can explain (i.e., reduce)

TABLE 4.2 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		1,333.785	20.836	64.012	.000	1,293.565	1,375.255
Intercept [subject = schcode]	Variance	189.887	27.933	6.798	.000	142.324	253.344
Intercept [subject = schcode * Rteachid]	Variance	131.422	15.963	8.233	.000	103.581	166.746

<sup>a</sup> Dependent variable: math.

variance at both Level 1 and at Level 2 (and perhaps even at Level 3). In contrast, variables added at higher levels of the model do not affect the variance present at lower levels. In other situations, a variable added at a lower level may appear to explain negative variance. This may happen because the program may not make a very good “initial” estimate of the variance at each level, depending on the nature of the data (e.g., normality, sample size at each level, unequal sampling distribution of Level 1 variables across groups, etc.). We therefore remind analysts to be cautious about placing too much emphasis on variance reduction approaches in accounting for outcome variance. Adding random slopes can also complicate accounting for variance at each level (Hox, 2010). For these reasons, it can be more challenging to assess the outcome variance explained in multilevel models.

## Model 2: Defining Predictors at Each Level

We will next illustrate the three-level model by building a simplified model with one or two predictors at each level. This results in combining a couple of general steps in the overall modeling strategy that we laid out previously (i.e., no-predictors model, within-group model, group-level model, random slope, and cross-level interactions). We illustrate two primary centering strategies discussed in Chapter 3 in this example.

As we discussed earlier, grand-mean centering (where the grand mean is subtracted from individuals’ values on the variable) results in redefining the sample mean of the predictor to zero (0.0). Grand-mean centering the continuous predictors produces an equivalent model to leaving the predictors in their natural metrics, but it has the advantage of being able to interpret the resulting intercept in the model as the expected value of  $Y$  when they are at their mean values. This has the effect of adjusting the unit intercepts for differences in Level 1 predictors within the units. In contrast, group-mean centering (where each group’s mean is subtracted from individuals’ values on the variable) results in unit intercepts that are unadjusted for differences within the units. More specifically, it produces an intercept equal to the expected value of  $Y$  for an individual when  $X$  is equal to the group’s mean.

We reiterate that group-mean-centered solutions are not the same as grand-mean, standardized, or raw metric solutions. In IBM SPSS, grand-mean-centered or group-mean-centered variables must be created using COMPUTE and saved in the data set. In our examples, we designate variables that we have grand-mean centered by placing “gm” in front of the variable (e.g.,  $SES$  becomes  $gmSES$ ). We designate group-mean-centered variables as “group” (e.g.,  $SES$  becomes  $groupSES$ ). Analysts, of course, can devise their own naming schemes.

Categorical variables, such as dichotomous indicators, also present different options for centering using low SES as an example. We present some of these options in Table 4.3. We have two options if we leave a dummy-coded (coded 0, 1) predictor in its natural metric. If  $lowSES$  is entered as a covariate in MIXED, as is typical in multiple regression analyses, the reference group for the intercept (604.315) will be the group coded 0 (did not participate). For a dummy-coded

variable entered into MIXED as a factor, it is important to note in Table 4.3 that the default reference group is the last category (i.e., participated in free/reduced lunch program) instead of the first category. In this case, the intercept for achievement (587.538) will correspond to the expected scores for students who participated in the free/reduced lunch program (coded 1) in school  $k$ , and the variance associated with the intercept will be interpreted as the variance in outcomes for that group of students across schools (Raudenbush & Bryk, 2002).

If we grand-mean center a dummy-coded variable (i.e., in this case, coded  $-0.41, 0.59$ ) and enter it as a covariate in the model, it results in an intercept that may be defined as the mean outcome in school  $k$ , adjusted for differences among schools in the proportion of students participating in the federal free/reduced lunch program. This is an important distinction, for example, if we are concerned with making comparisons between schools after adjusting the outcomes for differences due to the various backgrounds of students within each school. We can observe in Table 4.3 that centering a dummy-coded variable on the grand mean or leaving it uncentered will result in similar slope coefficients but different estimated intercepts. Leaving the SES variable in its raw metric would result in the intercept representing students in the reference group, after adjusting for the other predictors. We can see that for a dummy-coded variable entered in the model as a factor, the alternative coding schemes (e.g., uncentered and grand-mean centered) will not make a difference in the intercept.

We also point out that some fields prefer effect coding for dichotomous variables. As shown in Table 4.3, when a variable is effect coded ( $-1, +1$ ) and entered as a covariate, the estimate can be interpreted as the change above or below the grand mean for the predictor. Note that the coefficient in the effect-coded solution is half the size of the other estimates (8.389 vs. 16.778), and if we subtract that effect-coded estimate from the intercept of the natural metric solution ( $604.315 - 8.389$ ), we obtain the grand mean of 595.926 in the effect-coded model in Table 4.3 (with slight difference due to rounding).

**TABLE 4.3 Estimates of Fixed Effects<sup>a</sup>**

	Parameter	Estimate	Std. Error	df	t	Sig.
Raw Metric	Intercept	604.315	1.192	183.847	506.943	0.000
	lowses	-16.778	0.849	9,165.850	-19.752	0.000
Grand-Mean	Intercept	597.437	1.138	154.141	524.936	0.000
Centered	gmlowses	-16.778	0.849	9,165.850	-19.752	0.000
Raw Metric (Factor)	Intercept	587.538	1.241	217.152	473.441	0.000
	[lowses = 0]	16.778	0.849	9,165.850	19.752	0.000
	[lowses = 1]	0 <sup>b</sup>	0	.	.	.
Grand-Mean	Intercept	587.538	1.241	217.152	473.441	0.000
Centered (Factor)	[gmlowses = -0.41]	16.778	0.849	9,165.850	19.752	0.000
	[gmlowses = 0.59]	0 <sup>b</sup>	0	.	.	.
Effect Coded	Intercept	595.927	1.140	155.420	522.628	0.000
	lowseseffect	-8.389	0.425	9,165.850	-19.752	0.000
Group-Mean	Intercept	597.278	1.298	156.533	460.172	0.000
Centered	grouplowses	-14.449	0.887	9,068.494	-16.286	0.000

<sup>a</sup> Dependent variable: math.

<sup>b</sup> This parameter is set to 0 because it is redundant.

Finally, as we have noted previously, centering the variable on the group mean produces a different intercept (597.278) and slope coefficient (−14.449) from the other centering solutions presented in Table 4.3. The analyst, therefore, should keep in mind the desired interpretation of the intercept, especially when there are several continuous and dichotomous variables in the analysis. For other categorical variables entered into the model as factors, the default reference group is also the last category.

In our first model, we grand-mean centered the low SES predictor (though we could have just as easily chosen to leave it in its raw metric). At Level 1, we will propose that for individual  $i$  in class  $j$  in school  $k$ , student SES background ( $gmlow\text{ses}$ ) affects math achievement:

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}gmlow\text{ses}_{ijk} + \varepsilon_{ijk}. \quad (4.7)$$

At Level 2 (classrooms), we will add a measure of teacher effectiveness (with higher standardized scores indicating greater effectiveness) in producing student learning in the classroom and an aggregate measure of classroom SES composition:

$$\begin{aligned} \pi_{0jk} &= \beta_{00k} + \beta_{01k}gmteacheffect_{jk} + \beta_{02k}gmclasslow\text{ses\_mean}_{jk} + r_{0jk}, \\ \pi_{1jk} &= \beta_{10k}. \end{aligned} \quad (4.8)$$

We also grand-mean centered these predictors. For this model, we will assume that student SES ( $gm\text{ses}$ ) is fixed at the same value for all Level 2 units. Therefore, we do not include any predictors or a random variance component ( $r_{1jk}$ ) in Equation 4.8.

At Level 3 (schools), we will add aggregated measures of school SES and teacher effectiveness (which we also grand-mean centered) to explain variation in between-school math achievement:

$$\begin{aligned} \beta_{00k} &= \gamma_{100} + \gamma_{001}gmschlow\text{SES\_mean}_k + \gamma_{002}gmaggtcheffect_k + u_{00k}, \\ \beta_{10k} &= \gamma_{100}, \\ \beta_{01k} &= \gamma_{010}, \\ \beta_{02k} &= \gamma_{020}. \end{aligned} \quad (4.9)$$

Equation 4.9 suggests that we will also assume that all Level 2 slopes are fixed, but the adjusted math intercepts vary across schools. The combined equation will then be the following:

$$\begin{aligned} \beta_{00k} &= \gamma_{000} + \gamma_{100}gmlow\text{ses}_{ijk} + \gamma_{010}gmteacheffect_{jk} + \gamma_{020}gmclasslow\text{ses\_mean}_{jk} + \\ &\gamma_{001}gmschlow\text{SES\_mean}_k + \gamma_{002}gmaggtcheffect_k + u_{00k} + r_{0jk} + \varepsilon_{ijk}. \end{aligned} \quad (4.10)$$

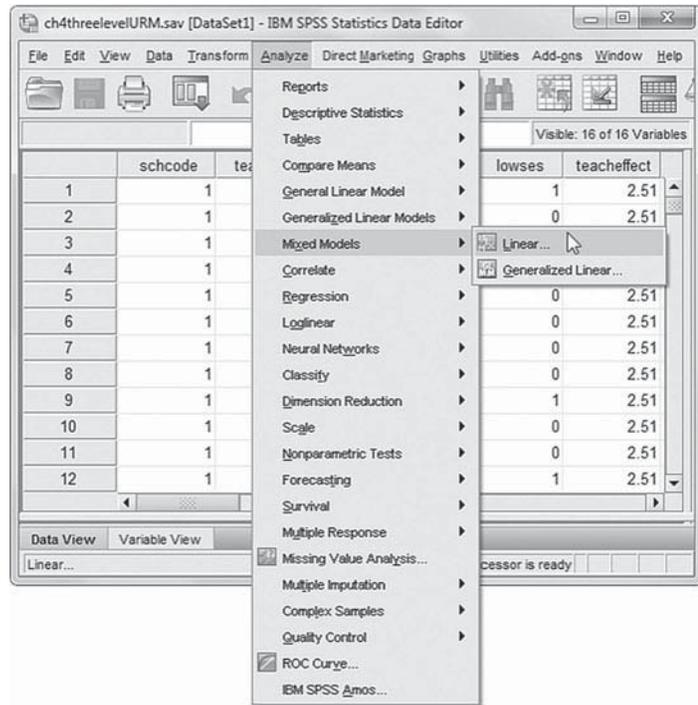
This suggests nine parameters to estimate (six fixed effects, two random effects—the intercept at Levels 2 and 3, and the Level 1 residual). We note that the Level 3 (school) intercepts in this model have been adjusted for individual SES at Level 1 and for student SES composition and teacher effectiveness at Level 2.

### Defining Model 2 with IBM SPSS Menu Commands

**Note:** IBM SPSS settings will default to those used in Model 1.

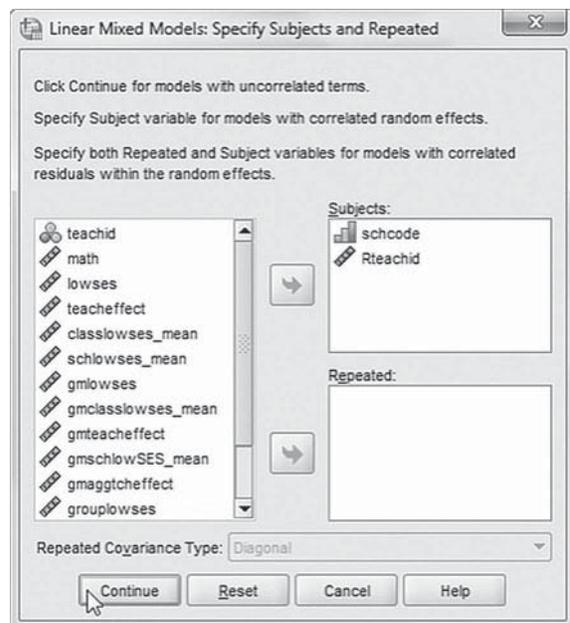
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



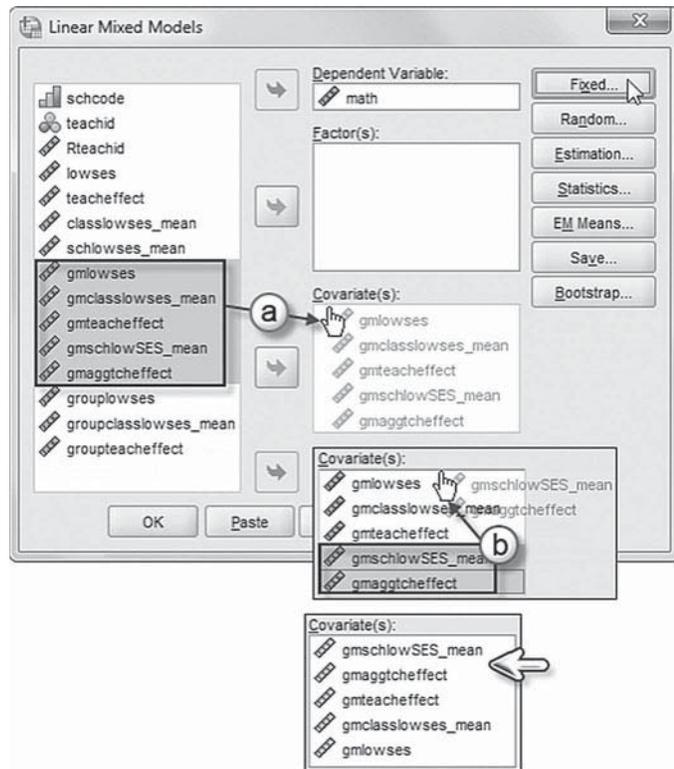
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

- a. We will introduce five additional variables to be used in the model (*gmloswses*, *gmclassloswses\_mean*, *gmteacheffect*, *gmschlowSES\_mean*, *gmaggtcheffect*). First, click to select all five variables, and then “drag” them to the *Covariate(s)* box.
- b. To facilitate reading of the output tables, we will change the sequence of the variables by dragging variables to rearrange them. First, click to select *gmschlowSES\_mean* and *gmaggtcheffect*, and then drag them to the top of the *Covariate(s)* box. Now the variables are listed in first and second place. Continue rearranging the variables to achieve the final sequence order (see insert): *gmschlowSES\_mean*, *gmaggtcheffect*, *gmteacheffect*, *gmclassloswses\_mean*, and *gmloswses*.

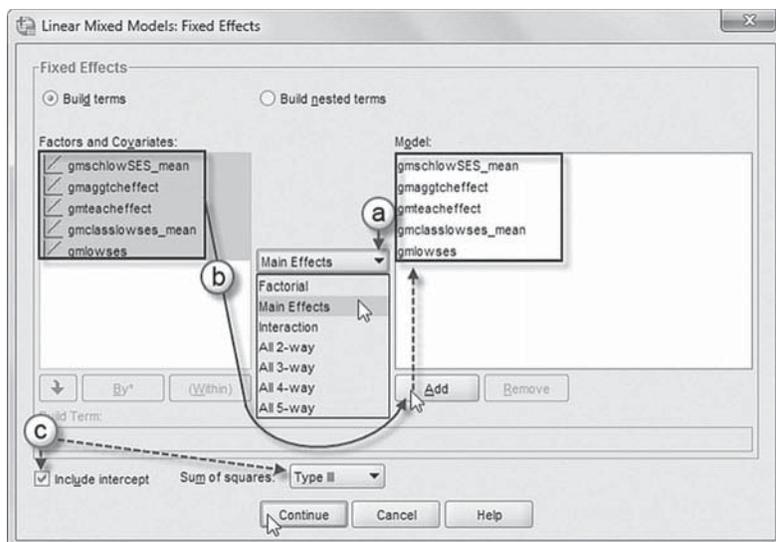


**Note:** An alternate method for arranging the variables in the desired sequence order is to select variables individually and then use the right-arrow button to move each variable into the *Covariate(s)* box.

We may now proceed to define fixed effects for the variables.

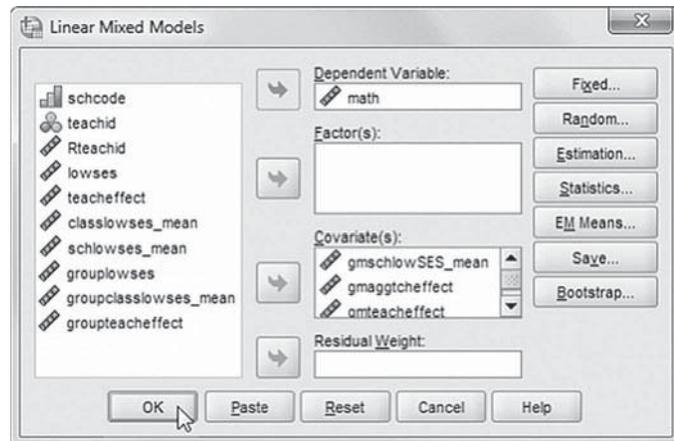
Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
- b. Click to select the five variables from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 2

The fixed-effects output for Model 2 is presented in Table 4.4. The table suggests a composition effect associated with student SES; that is, there is an effect at the individual level ( $\gamma_{100} = -14.534$ ,  $p < .001$ ), which compounds significantly at the classroom level ( $\gamma_{020} = -11.073$ ,  $p < .001$ ) and the school level ( $\gamma_{011} = -31.138$ ,  $p < .001$ ).

As we noted previously, compositional effects refer to the extent to which the size of the organizational-level effect differs from the size of the individual-level effect. With grand-mean centering, the compositional effect for schools is estimated directly as the between-group fixed effect for SES ( $\gamma_{001} = -31.138$ ). At the school level, the effect for SES can be interpreted as the expected difference in math outcomes between two students who have the same individual SES background (and are in classes with the same SES composition) but attend schools differing by 1 unit (i.e., in this case, 1 *SD* in mean SES). Similarly, the composition effect at the classroom level ( $\gamma_{020} = -11.073$ ) is the difference in size between the individual SES and class-level SES effects. At the classroom level, the compositional effect for SES can be interpreted as the expected difference in math outcomes between two students who have the same individual SES but who are in classes differing by 1 unit (1 *SD*) in mean classroom SES.

The output also provides support for our initial contention that there is an academic advantage for students that compounds due to teacher effectiveness. First, the table suggests that the classroom-level effect is significant ( $\gamma_{010} = 7.571$ ,  $p < .001$ ). This suggests that students in a classroom with a teacher whose effectiveness is 1 *SD* above the grand mean would score about 7.6 points higher than their peers in classrooms with teachers at the grand mean of effectiveness.

TABLE 4.4 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	596.742	0.741	148.020	804.861	.000	595.277	598.208
gmschlowSES_mean	-31.138	4.565	436.869	-6.821	.000	-40.110	-22.166
gmaggtcheffect	7.448	1.963	208.719	3.794	.000	3.578	11.319
gmteacheffect	7.571	0.619	656.308	12.223	.000	6.355	8.787
gmclasslowses_mean	-11.073	3.472	1,759.121	-3.189	.001	-17.882	-4.264
gmlowses	-14.534	0.862	8,841.429	-16.868	.000	-16.222	-12.845

Moreover, the effect of teachers on outcomes compounds at the school level ( $\gamma_{002} = 7.448, p < .001$ ). The interpretation of this school-level coefficient is straightforward: For two students having teachers of average effectiveness, attending a school differing by 1 *SD* in collective teacher effectiveness is associated with a 7.45-point increase on the standardized math test. This output, therefore, provides evidence to answer our first research question regarding potential academic advantages associated with teacher effectiveness.

### Model 3: Group-Mean Centering

We contrast these results in Model 1 with a second model where we group-mean centered the within-school variables. It is important to keep in mind that cross-level interactions can sometimes be a source of instability in a model's estimates. If this may be a potential problem, group-mean centering is a good centering choice since this approach removes correlations between variables across levels (Kreft & de Leeuw, 1998). Group-mean centering results in an intercept for each unit that is unadjusted for Level 1 or Level 2 predictors. In group-mean-centered solutions, the effects of between-group predictors will also be unadjusted for within-group predictors.

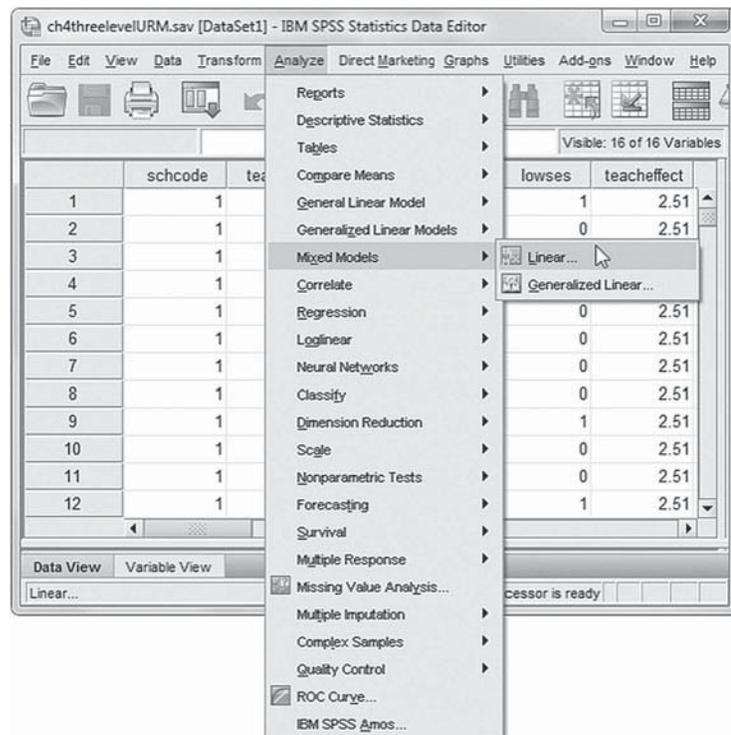
Because grand-mean centering produces group means that are adjusted for the predictors in the model, in some modeling situations the adjustments for some units may not be very reliable. This can occur where there are small within-group sample sizes and considerable variability in slopes for particular covariates. For units with small sample sizes, the random slope may be estimated with little precision, which weakens the likelihood of detecting relationships between groups (Raudenbush & Bryk, 2002). In this situation, it can be difficult to disentangle parameter variance and error variance. In cases where the slope varies considerably across units for a predictor, and there is also variability across units in the levels of the predictors, resulting grand-mean-centered estimates may be less credible than group-mean-centered estimates (Raudenbush & Bryk, 2002). It is also important to note that compositional effects may change the size and significance of other parameters in the model. It is, therefore, important to give attention to decisions about model specification, especially when the focus is on the cross-level interactions.

#### Defining Model 3 with IBM SPSS Menu Commands

**Note:** IBM SPSS settings will default to those used in Model 2.

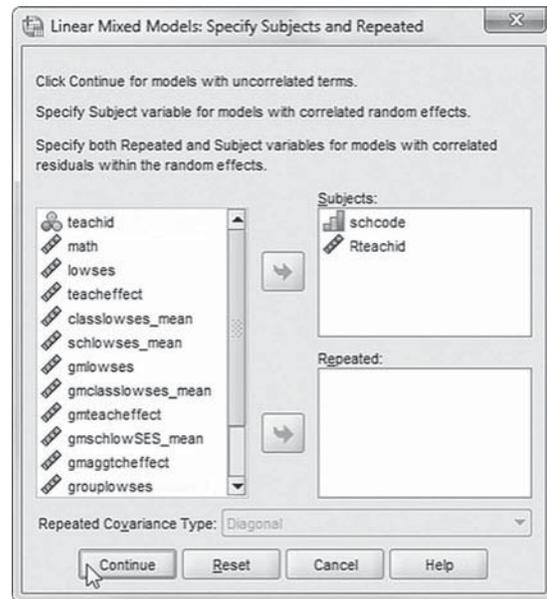
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

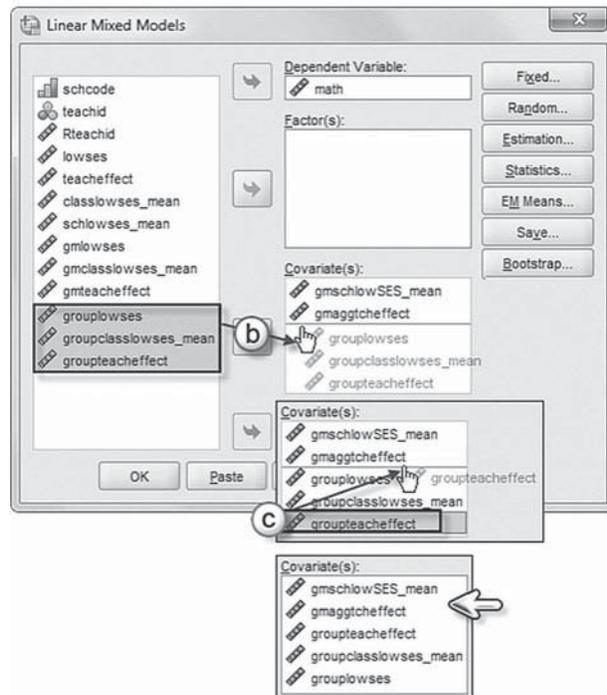
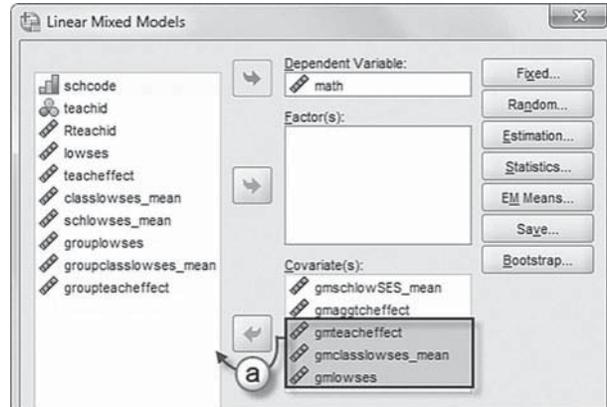


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.
- We will change the model by exchanging the three grand-mean-centered variables for group-mean-centered variables. Remove the grand-mean-centered variables by clicking to select *gmteachef*, *gmclasslowses\_mean*, and *gmllowses* and then clicking left-arrow button (or “drag” the variables to the left).
  - Now click to select three group-mean variables (*grouplowses*, *groupclasslowses\_mean*, and *groupteacheffect*) and drag them to the *Covariate(s)* box.
  - To facilitate reading of the output tables, we will change the sequence of the variables by dragging variables to rearrange them. First, click to select *groupteacheffect*, and then drag the variable below *gmaggtcheffect*. Continue rearranging the variables to achieve the final sequence order (see insert): *gmschlowSES\_mean*, *gmaggtcheffect*, *groupteacheffect*, *groupclasslowses\_mean*, and *grouplowses*.



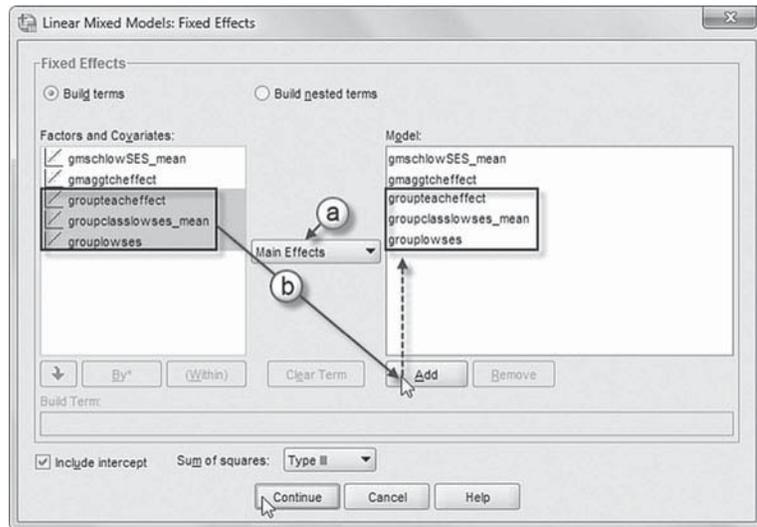
**Note:** An alternate method for arranging the variables in the desired sequence order is to select variables individually and then use the right-arrow button to move each variable into the *Covariate(s)* box.

We may now proceed to define fixed effects for the variables.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

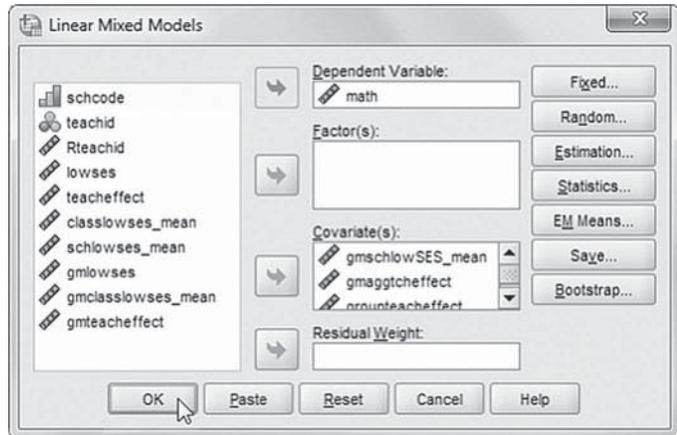
4a. The *Linear Mixed Models: Fixed Effects* default setting from the prior model is *Main Effects*.

b. Click to select the three group-mean-centered variables (*groupteacheffect*, *groupclasslowses\_mean*, and *grouplowses*) from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 3

Given the previous grand-mean-centered estimates in Model 1, if we instead used group-mean centering, we should expect a classroom-level SES group-centered effect of approximately  $-25.6$  [i.e.,  $-14.5 + (-11.1)$  in Table 4.3] and a school-level effect of approximately  $-56.7$  [i.e.,  $-25.6 + (-31.1)$  in Table 4.3]. For teacher effectiveness, we would expect an aggregated Level 3 teacher effectiveness coefficient of about  $15.02$  (i.e.,  $7.57 + 7.45$  in the previous table).

The group-mean-centered estimates are presented in Table 4.5 and are consistent with our expected output translated from the grand-mean estimates. Note that the Level 3 estimates are still grand-mean centered. This is because the estimates of the highest level in the data set cannot be centered on a group—that is, their group is the school sample. Comparison of Tables 4.4 and 4.5 suggests that the only difference between the models is related to the coefficients for the Level 3 composition ( $\gamma_{001} = -56.744$ ) and aggregated teacher effectiveness ( $\gamma_{001} = 15.019$ ) variables and

TABLE 4.5 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	596.759	0.741	148.049	804.847	.000	595.294	598.224
gmschlowSES_mean	-56.744	3.698	192.134	-15.346	.000	-64.037	-49.450
gmaggtcheffect	15.019	1.882	176.732	7.978	.000	11.304	18.734
groupteacheffect	7.571	0.619	656.308	12.223	.000	6.355	8.787
groupclasslowses_mean	-25.606	3.411	1,628.983	-7.507	.000	-32.297	-18.916
groupplowses	-14.534	0.862	8,841.429	-16.868	.000	-16.222	-12.845

<sup>a</sup> Dependent variable: math.

the coefficient for the Level 2 group-centered class composition effect ( $\gamma_{020} = -25.606$ ). It would be easy to misinterpret the meaning of these coefficients without a clear understanding of the difference between the two centering strategies.

### Covariance Estimates

The covariance estimates in Table 4.6 (which are the same for the two models) suggest that the addition of predictors at each level reduces the proportion of variance associated with each level of the data hierarchy (see Table 4.2 on page 139) substantially (i.e., from 1,333.785 to 1,288.363 at the student level, from 131.422 to 73.310 at the classroom level, and from 189.887 to 43.369 at the school level). These reductions in variance can be used to calculate an estimate of  $R^2$  at each level. For example, at the school level (Level 3) the reduction in variance would be calculated as  $(189.887 - 43.369)/189.887$  and results in an  $R^2$  coefficient of 0.772.

It is important to reiterate that predictors entered into the model can affect variance accounted for at the level at which they are entered (e.g., Level 1 or Level 2), and they may also affect the variance accounted for at higher levels. For example, if grand-mean centering is used, adjustments for Level 1 predictors may change the level and variability in the intercept across higher groupings. If group-mean centering is used, however, the Level 1 predictors will not affect the level of the intercept or its variance across groups. In situations where accounting for variance is an important aim, the analyst may want to consider developing a Level 1 model first and then adding Level 2 and Level 3 predictors as separate sets of variables, so that the variance accounted for at each subsequent level is only affected by predictors added at that level.

TABLE 4.6 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		1,288.363	19.974	64.501	.000	1,249.803	1,328.112
Intercept [subject = schcode]	Variance	43.369	9.759	4.444	.000	27.903	67.408
Intercept [subject = schcode * Rteachid]	Variance	73.310	11.640	6.298	.000	53.705	100.072

<sup>a</sup> Dependent variable: math.

Another point to keep in mind is that where there are multiple random effects (e.g., a random intercept and random slope), accounting for variance at successive levels may become more complicated if the slope and intercept are correlated. In this case, predictors entered into the slope equation, for example, may also affect variance estimates in the intercept equation (Raudenbush & Bryk, 2002). Centering on the group means tends to stabilize the model because it removes correlations (Paccagnella, 2006), so the resulting estimates may be more accurate. Group-mean centering may also be advantageous where the slope varies considerably for a particular predictor, and there is also considerable variability in the levels of the predictors across the units (Raudenbush & Bryk, 2002).

#### Model 4: Does the Slope Vary Randomly Across Schools?

We next investigate whether the random teacher effect varies across schools. This will help us answer our second research question. The appropriate baseline slope model is the model with random slopes but no cross-level interactions (Hox, 2002). To indicate a random slope for the teacher effect, we make a change in Equation 4.9 as follows:

$$\beta_{01k} = \gamma_{010} + u_{01k}. \quad (4.11)$$

In this model (Model 4), we will also need to change the covariance matrix from identity (ID) to unstructured (UN) at Level 3 in order to accommodate the two additional random parameters in the model (i.e., the randomly varying slope and the covariance between the intercept and slope). This change will be the following:

$$\begin{bmatrix} \sigma_I^2 & \sigma_{IS} \\ \sigma_{IS} & \sigma_S^2 \end{bmatrix} \quad (4.12)$$

We will leave the Level 2 covariance matrix as an identity matrix, since there is only one random effect (the intercept) at that level. As Equation 4.10 indicated, there were nine parameters to estimate in the previous model. Through substitution, we arrive at the new combined equation, which adds the random slope parameter at Level 3:

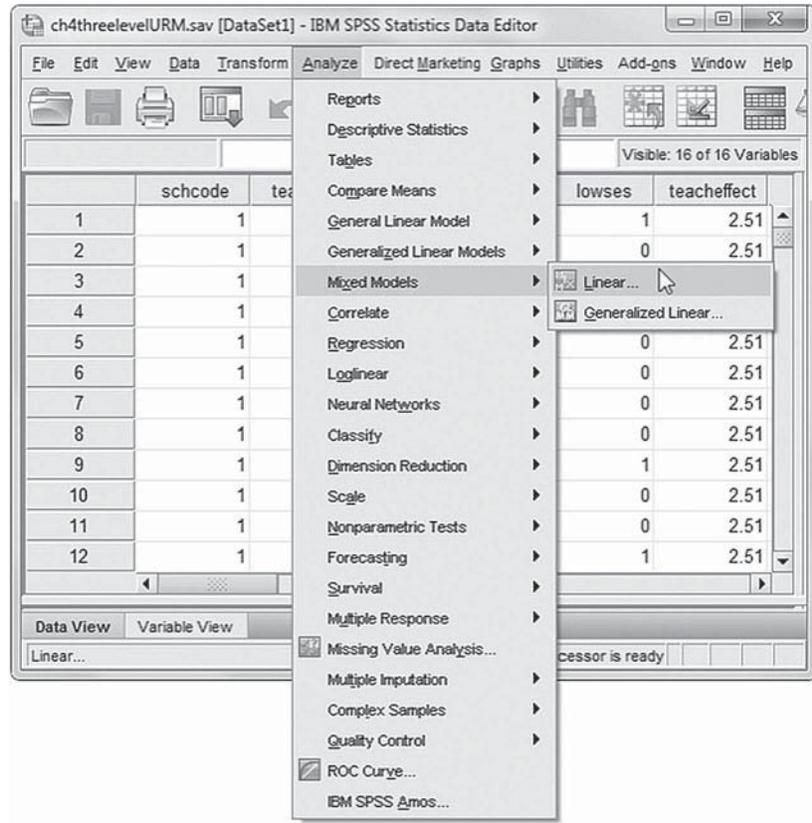
$$\beta_{00k} = \gamma_{000} + \gamma_{100}gmlo\omega ses_{ijk} + \gamma_{010}gmteacheffect_{jk} + \gamma_{020}gmclasslo\omega ses\_mean_{jk} + \gamma_{001}gmschlo\omega SES\_mean_k + \gamma_{002}gmagg\omega tch\omega effect_k + u_{01k}gmteacheffect_k + u_{00k} + r_{0jk} + \varepsilon_{ijk}. \quad (4.13)$$

### Defining Model 4 with IBM SPSS Menu Commands

**Note:** IBM SPSS settings will default to those used in Model 3.

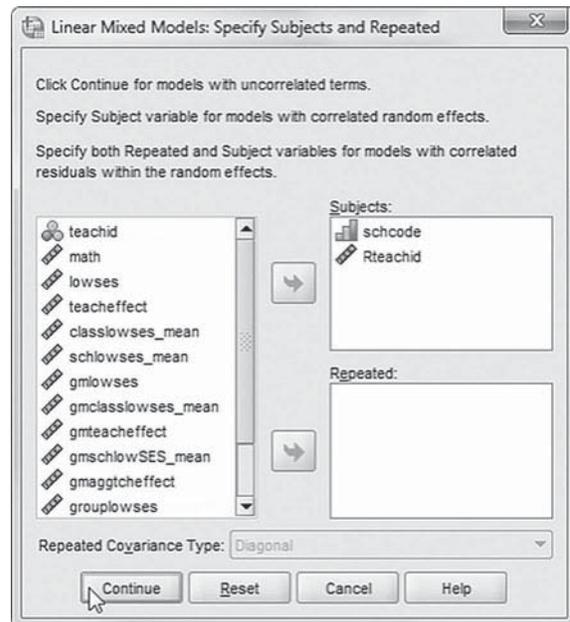
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



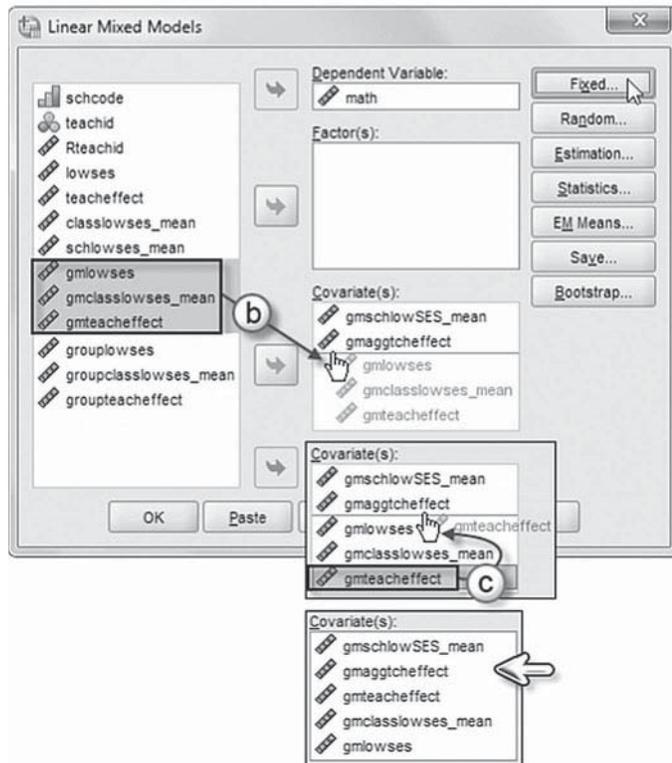
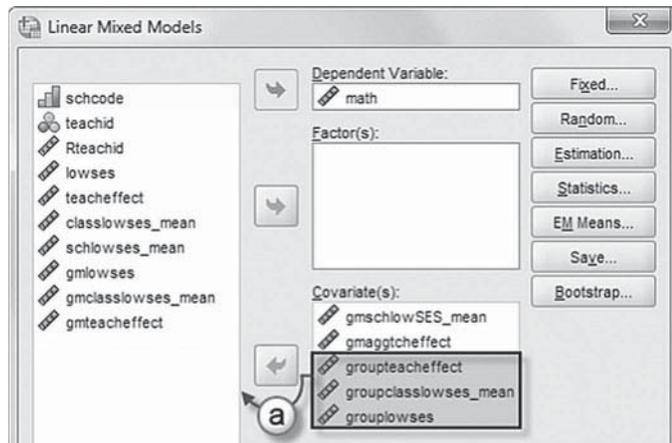
3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

- We will change the model by exchanging the group-mean-centered variables for the grand-mean-centered variables used in Model 2 (*gmlowses*, *gmclasslowses\_mean*, and *gmteacheffect*). Remove the group-mean-centered variables by clicking to select *groupteacheffect*, *groupclasslowses\_mean*, and *grouplowses* and then clicking left-arrow button (or “dragging” the variables).
- Now click to select three grand mean variables (*gmlowses*, *gmclasslowses\_mean*, and *gmteacheffect*), and drag them to the *Covariate(s)* box.
- To facilitate reading of the output tables, we will change the sequence of the variables by dragging variables to rearrange them. First, click to select *gmteacheffect*, and then drag the variable below *gmaggtcheffect*. Continue rearranging the variables to achieve the final sequence order (see insert): *gmschlowSES\_mean*, *gmaggtcheffect*, *gmteacheffect*, *gmclasslowses\_mean*, and *gmlowses*.

**Note:** An alternate method for arranging the variables in the desired sequence order is to select variables individually and then use the right-arrow button (or drag the variable) to move each variable into the *Covariate(s)* box.

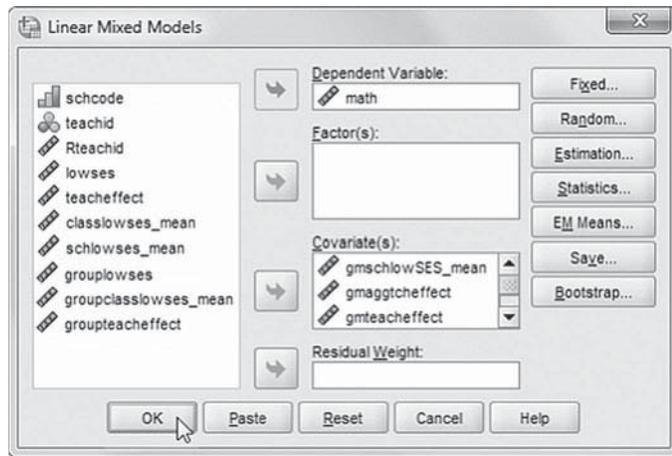
We may now proceed to define fixed effects for the variables.

Click the **FIXED** button to access the *Linear Mixed Models: Fixed Effects* dialog box.





6. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



#### ***Interpreting the Output from Model 4***

The addition of the random slope and the covariance between the slope and intercept at Level 3 in Model 4 makes a total of 11 parameters to estimate. We can verify this from the model dimension output for Model 3 in Table 4.7. To look at this table in a little more detail, there are now two random effects at Level 3 (intercept and teacher effectiveness slope) and one random effect at Level 2 (intercept). In addition, there are two other random parameters (i.e., Level 1 residual, Level 3 slope-intercept covariance). The estimates for these five parameters are summarized in Table 4.8.

**TABLE 4.7 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	gmschlowSES_mean	1		1	
	gmaggtcheffect	1		1	
	gmteacheffect	1		1	
	gmclasslowses_mean	1		1	
	gmlowses	1		1	
Random Effects	Intercept + gmteacheffect	2	Unstructured	3	schcode
	Intercept	1	Identity	1	schcode * Rteachid
Residual				1	
Total		9		11	

<sup>a</sup> Dependent variable: math.

**TABLE 4.8** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.
Residual		1,288.878	19.989	64.478	.000
	UN (1,1)	42.153	9.657	4.365	.000
Intercept + gmteacheffect [subject = schcode]	UN (2,1)	2.826	4.887	.578	.563
	UN (2,2)	5.759	6.568	.877	.381
Intercept [subject = schcode * Rteachid]	Variance	68.701	12.359	5.559	.000

<sup>a</sup> Dependent variable: math.

Table 4.8 provides the new variance components for Model 4. As the table shows, however, the variance in teacher effectiveness slopes (UN 2, 2) is not significant between schools ( $\sigma^2_{2,2} = 5.759$ , Wald  $Z = 0.877$ , one-tailed  $p = .1905$ ). Therefore, in this sample data it would not be necessary to try to build a Level 3 model with cross-level interactions to explain variation in the size of teacher classroom effects across schools. We therefore can answer our second research question by noting that the size of individual teacher effects do not seem to vary between schools. This may suggest that differences in individual teacher effects are more apparent between classrooms but within schools, rather than between schools.

### Developing an Interaction Term

Because classroom teacher effectiveness did not vary across schools, we turn our attention to answering our third research question. Interactions suggest that the relationship between a predictor ( $A$ ) and the outcome ( $Y$ ) is contingent on levels of another predictor ( $B$ ). Because interactions are product terms ( $A*B$ ), they depend on the levels of the predictors from which they are produced. This concerns whether the impact of teacher effectiveness on student learning is contingent on various types of classroom composition features. In this case, we will investigate a covariate\*covariate interaction (class SES\*teacher effectiveness). We initially propose that teacher effectiveness is a stronger predictor of achievement in classrooms with more challenging student composition. If this is true, then we would expect a positive interaction effect between teacher effectiveness and classroom composition.

We will next define an interaction term between teacher effectiveness and classroom SES composition (defined as the percentage of low SES students) and add it to the Level 2 model:

$$\pi_{0jk} = \beta_{00k} + \beta_{01k} (gmteacheffect)_{jk} + \beta_{02k} (gmclasslowses\_mean)_{jk} + \beta_{03k} (gmclasslowses\_mean * gmteacheffect)_{jk} + r_{0jk}. \quad (4.14)$$

We call attention to the importance of thinking about the meaning of the coding of each predictor comprising an interaction. When continuous variables that are part of an interaction are left in natural metrics, it may make it more difficult to interpret the meaning of the interaction (e.g., a motivation score of 0 or a family income score of 0). Because no one in the data set might have a score of 0, it is often useful to recenter the scores in a way that they can be more meaningfully interpreted. Grand-mean centering continuous  $A$  and  $B$  covariates facilitates this type

of meaningful interpretation since if we grand-mean center each, then the interpretation of the interaction slope is that it is the expected value of the slope when the other variable has the value of 0 (i.e., the average effect). For a factor\*covariate interaction (e.g., ethnicity\*family income), the interaction can be interpreted as a test of parallel lines (for each racial/ethnic group) regarding the relationship between the continuous variable (family income) across levels of the factor and  $Y$ . Here, grand-mean centering family income can facilitate interpreting the difference in its “average effect” across levels of the factor. For a factor\*factor interaction (e.g., female\*lowSES), with each variable coded 0 and 1, there are four possible cells (0, 0; 0, 1; 1, 0; and 1, 1). Multiplication in the first three cells will equal 0, leaving only the last cell (1, 1) as defining the added advantage or disadvantage for outcomes due to the interaction effect. The interpretation of the interaction will be the advantage or disadvantage associated with being female and low SES against the other three cells combined. This choice might be the one the analyst desires in interpreting the interaction. Centering the factors ( $A, B$ ) in the interaction will carry a slightly different interpretation of the intercept from leaving them in their natural metrics.

### Preliminary Investigation of the Interaction

Before actually running the full model, we will pause for a moment to consider the proposed interaction in a little more detail. We can create this interaction variable implied in Equation 4.14 using the Menu commands by combining classroom composition and classroom teacher effectiveness (instructions provided in Model A at the end of this section). Interactions can also be computed easily using the syntax statements (or by using COMPUTE and saving the interaction in the data set). We focus on just the two main effects and the interaction to make a point about how centering the interactions differently can affect the interpretation of the coefficients.

In Table 4.9, we summarize the estimates for the natural metric solution. The intercept is 611.325, which can be interpreted as the point where class SES is 0—that is, where there is no student participating in the federal free/reduced lunch program. We might be able to find a better centering solution for this variable, however, since more than 40% of the sample participated in the free/reduced lunch program. The intercept can also be interpreted as the achievement level where teacher effectiveness is 0. Since teacher effectiveness is a factor score (developed from a separate analysis with  $M = 0$ ,  $SD = 1$ ), this can be interpreted roughly as “average” teacher effectiveness, but it is uncentered within the current multilevel model. In the table, both of the main effects (classroom SES and teacher effectiveness) are significant predictors of math scores. The interaction, however, is not significant, so we could consider removing it if desired. When an interaction term ( $A*B$ ) is significant, however, we suggest leaving both the direct effects of  $A$  and  $B$  in the model whether they are significant or not.

**TABLE 4.9** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	611.365	1.507	420.970	405.670	.000	608.403	614.328
teacheffect	5.988	1.424	763.172	4.204	.000	3.192	8.785
classlowses_mean	-35.605	2.913	968.782	-12.225	.000	-41.321	-29.889
teacheffect * classlowses_mean	4.515	3.121	818.673	1.447	.148	-1.611	10.641

<sup>a</sup> Dependent variable: math.

TABLE 4.10 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	596.647	0.896	135.799	666.267	.000	594.876	598.418
gmteacheffect	7.840	0.605	680.886	12.956	.000	6.652	9.028
gmclasslowses_mean	-35.674	2.911	968.896	-12.255	.000	-41.387	-29.962
gmteacheffect * gmclasslowses_mean	4.515	3.121	818.673	1.447	.148	-1.611	10.641

<sup>a</sup> Dependent variable: math.

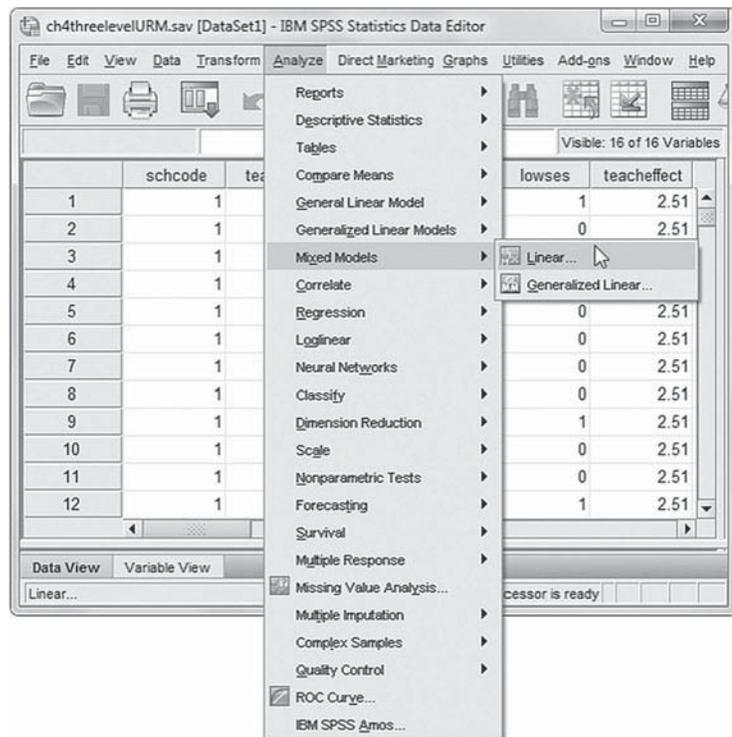
Next, in Table 4.10 we provide the grand-mean-centered solution (instructions provided in Model B at the end of this section). Notice that the intercept is a bit lower (596.647), which results from *classlowses\_mean* and *teacher effectiveness* being centered around their grand means. The intercept can now be interpreted as the point where the proportion of students participating in free/reduced lunch is the average for the sample (0). This is a more meaningful centering because of the high proportion of students in the sample who participate in this program. We reiterate the point that grand-mean centering the interaction slope facilitates interpretation since it implies what the expected value of the slope is when the other variable has the value of 0 (i.e., the average effect).

#### Defining Models A and B (Preliminary Testing of Interactions) with IBM SPSS Menu Commands

**Note:** IBM SPSS settings will default to those used in Model 4.

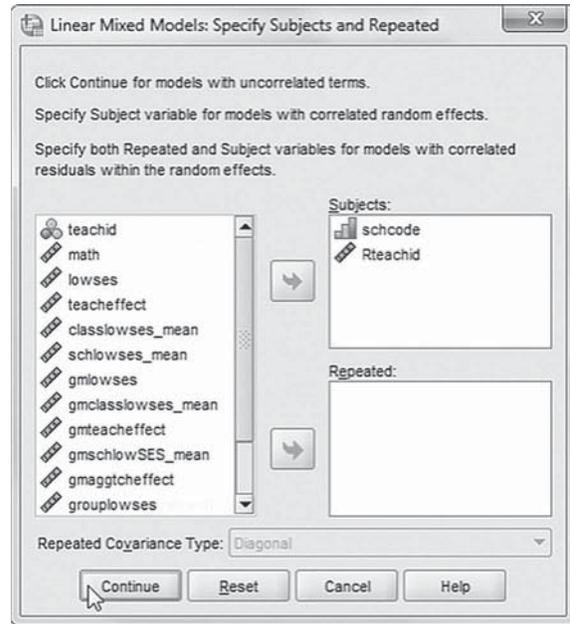
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



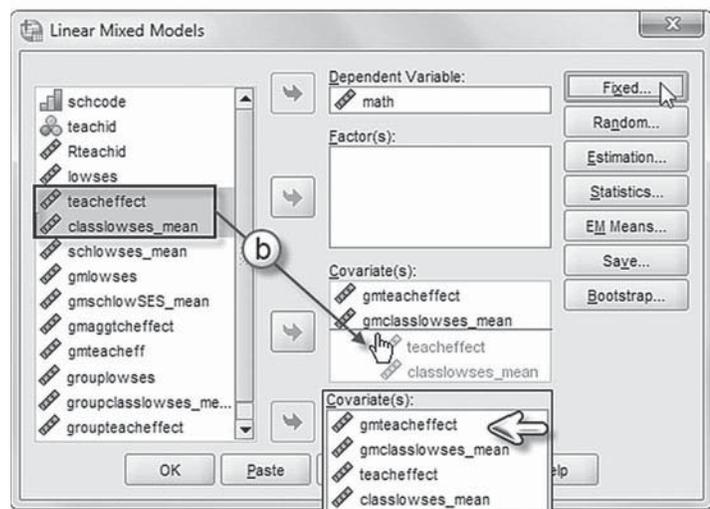
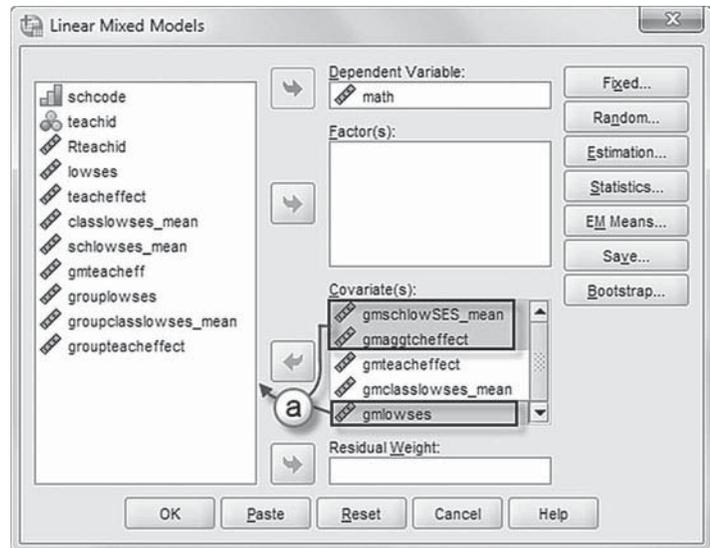
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box settings default to those used in Model 4.

- We will remove variables that aren't needed for the interactions. Click to select *gmschlowSES\_mean*, *gmagggtcheffect*, and *gmlowses*, and then click the left-arrow button (or drag the variables left) to remove them from the *Covariate(s)* box.
- We will now add two variables (*teacheffect* and *classlowses\_mean*) that will be used to test the second interaction. Click to select *teacheffect* and *classlowses\_mean*, and then drag the variables to the *Covariate(s)* box below *gmclasslowses\_mean*.

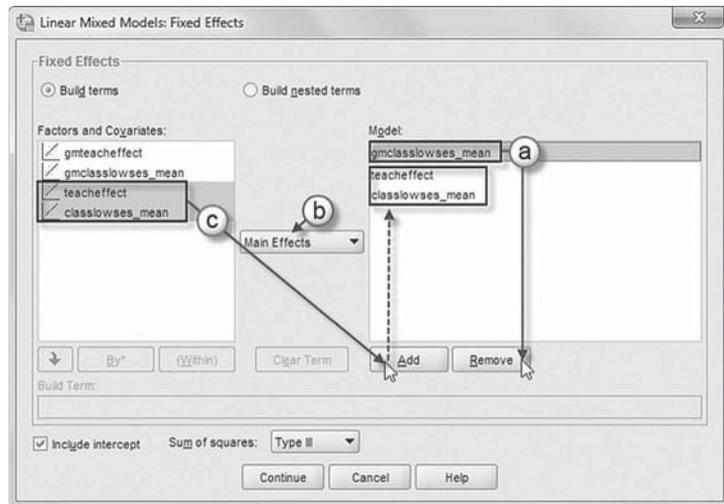


The four variables to be used for interaction testing are shown in the insert: *gmteacheffect*, *gmclasslowses\_mean*, *teacheffect*, and *classlowses\_mean*.

We will introduce a fixed effect to the model, so click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

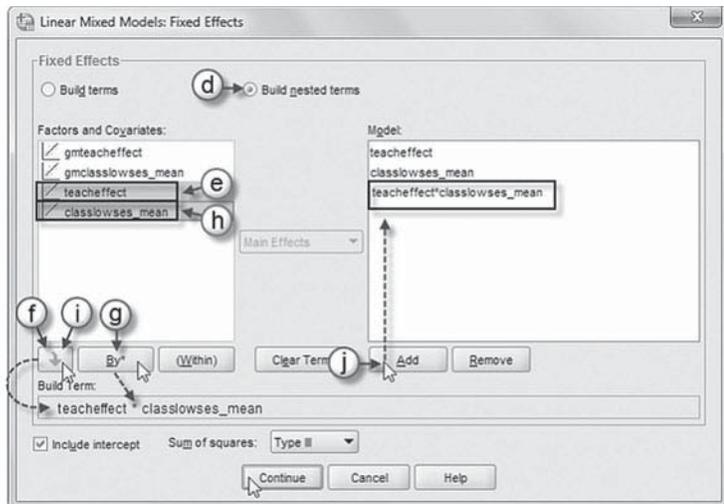
Model A Test Interaction: *teacheffect\*classlowses\_mean*

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from Model 4. We will clear the *Model* box by clicking to select *gmclasslowses\_mean* and then clicking the REMOVE button.
- b. Note that *Main Effects* is the default setting.
- c. Click to select *teacheffect* and *classlowses\_mean*, and then click the ADD button to move the variables into the *Model* box.



We will now add one cross-level interaction (or nested term) to the model: *teacheffect\*classlowses\_mean*. The interaction will tell us if teacher effectiveness (*teacheffect*) predicts achievement in classrooms with more challenging student composition (*classlowses\_mean*).

- d. Note that *Build nested terms* is the default setting from the prior model.
- e. Click to select the variable *teacheffect* from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves *teacheffect* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button, which will insert the computation command symbol: *teacheffect\**.
- h. Click to select *classlowses\_mean* from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move *classlowses\_mean* into the *Build Term* box and complete the interaction term: *teacheffect\* classlowses\_mean*.
- j. Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

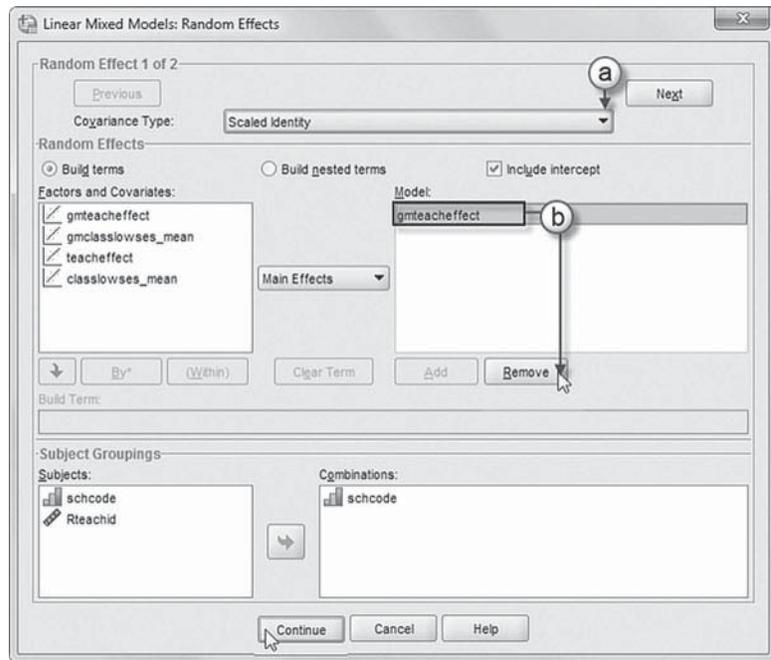
We will modify the model's random effects, so click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Random Effect 1 of 2* screen is displayed first, as it was the last dialog box used in Model 4.

- a. Change the covariance type by clicking the pull-down menu and selecting *Scaled Identity*.

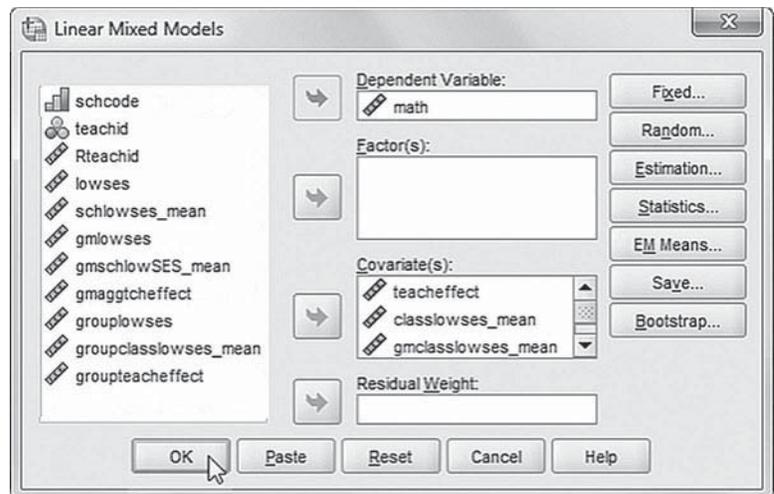
The *Scaled Identity* covariance structure has constant variance and assumes no correlation between any elements (IBM Corporation, 2012).

- b. We will remove *gmteacheffect* from the model. Click to select the variable, and then click the REMOVE button.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

6. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



Model B Test Interaction:  $gmteacheffect * gmclasslowses\_mean$

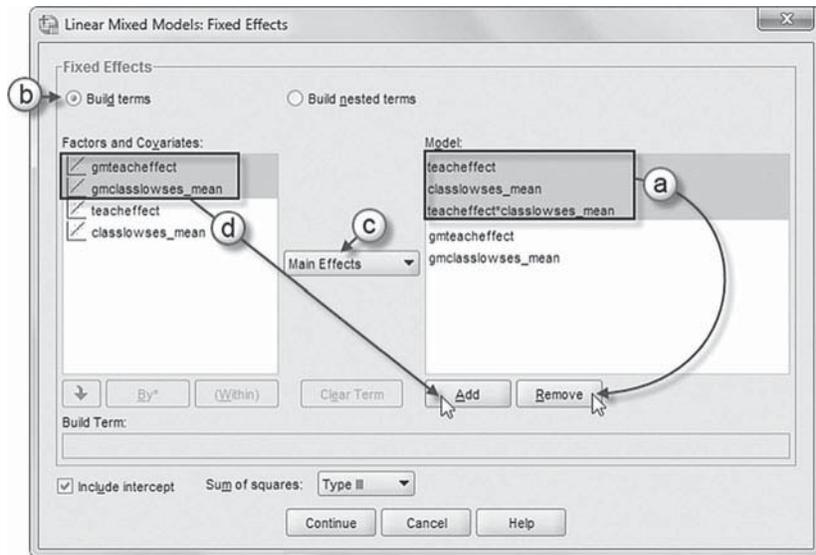
Repeat Model A steps 1 and 2, and then click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

3a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from Model A. We will clear the *Model* box by clicking to select the fixed effects from Model A (*teacheffect*, *classlowses\_mean*, and *teacheffect\*classlowses\_mean*) and then clicking the REMOVE button.

b. Click to select the *Build terms* option.

c. Note that *Main Effects* is the default setting.

d. We will now create a test model using grand-mean-centered variables. Click to select *gmteacheffect* and *gmclasslowses\_mean*, and then click the ADD button to move the variables into the *Model* box.



We will now add one cross-level interaction (or nested term) to the model using grand-mean-centered variables: *gmteacheffect\*gmclasslowses\_mean*. The interaction will tell us if teacher effectiveness (*gmteacheffect*) predicts achievement in classrooms with more challenging student composition (*gmclasslowses\_mean*).

e. Note that *Build nested terms* is the default setting from the prior model.

f. Click to select the variable *gmteacheffect* from the *Factors and Covariates* box.

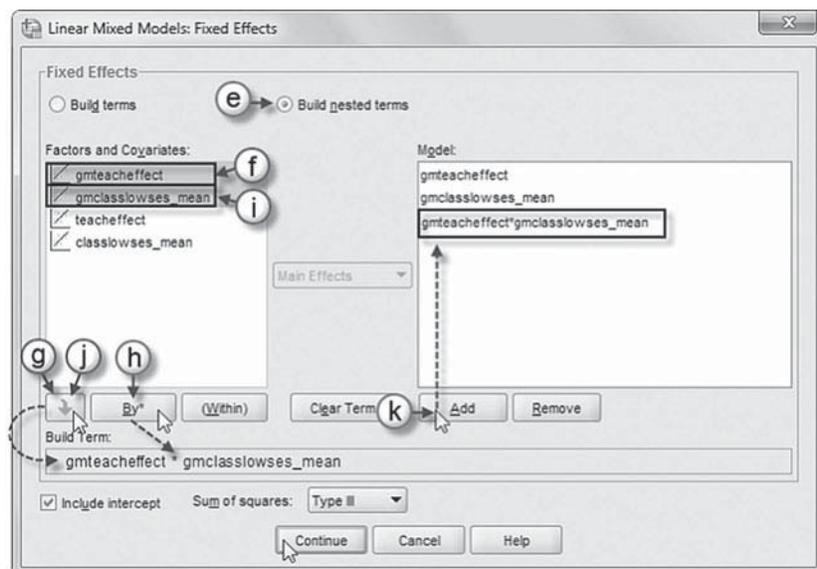
g. Then click the arrow button below the *Factors and Covariates* box. This moves *gmteacheffect* into the *Build Term* box to create a cross-level interaction by linking variables and terms.

h. Next, click the BY\* button, which will insert the computation command symbol: *gmteacheffect\**.

i. Click to select *gmclasslowses\_mean* from the *Factors and Covariates* box.

j. Click the arrow button below the *Factors and Covariates* box to move *gmclasslowses\_mean* into the *Build Term* box and complete the interaction term: *gmteacheffect\*classlowses\_mean*.

k. Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

4. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

### Model 5: Examining a Level 2 Interaction

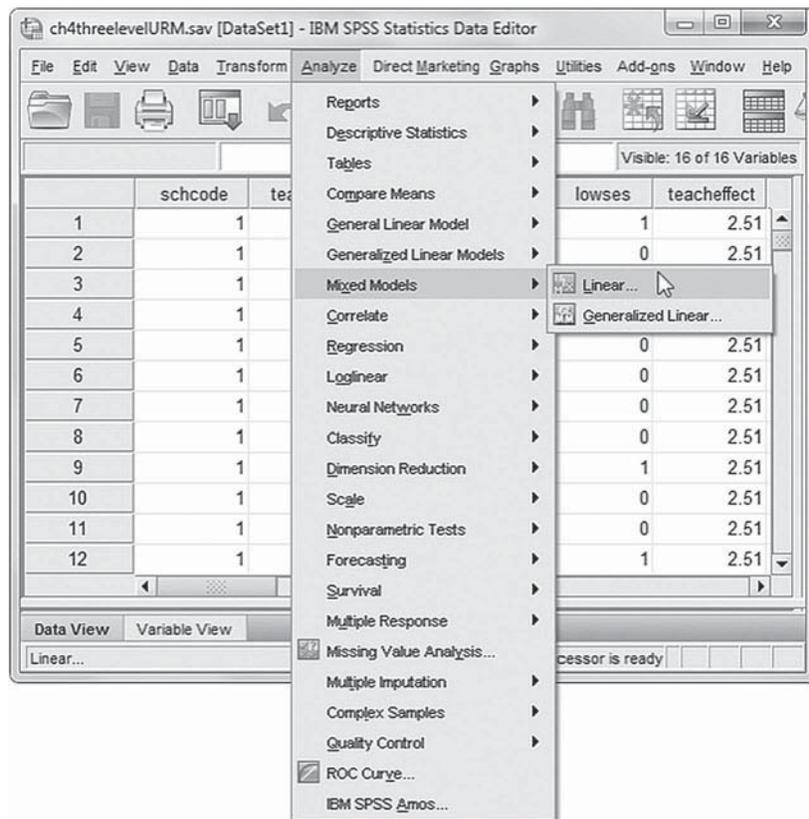
This investigation of adding a Level 2 interaction term to the model also allows us to demonstrate how an analyst can compare successive models to see whether an added variable (or set of variables) makes a difference in the overall fit of the proposed model to the data. We will again use ML estimation to facilitate the comparison of the previous model without the Level 2 interaction against this current model with the interaction added (which will increase the number of estimated parameters from 11 to 12). Using REML to compare successive models would only be optimal when the focus is on a difference in random parameters between the two models. The rest of the model remains as Model 4.

#### Defining Model 5 with IBM SPSS Menu Commands

**Note:** If continuing onward from the last section, “Preliminary Investigation Interaction,” reset the default settings to replicate Model 4 before proceeding.

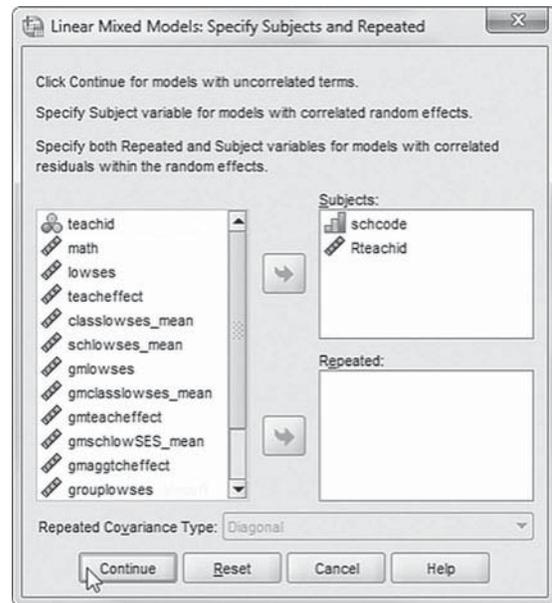
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



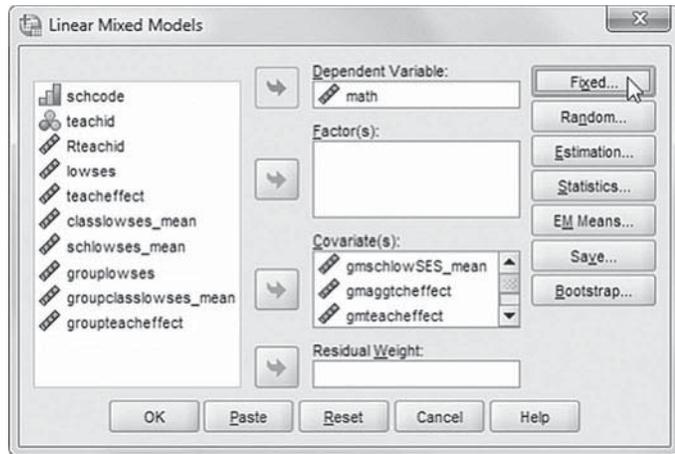
- The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



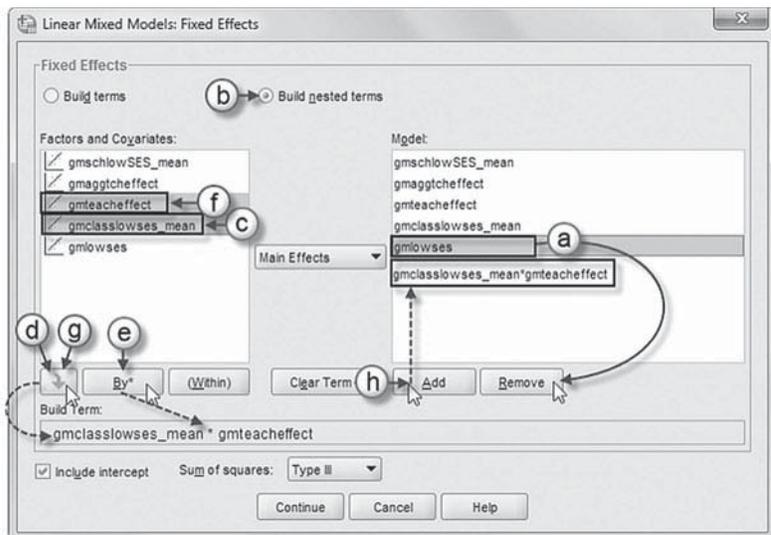
- The *Linear Mixed Models* dialog box settings default to those used in Model 4.

We will introduce a fixed effect to the model, so click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



One cross-level interaction (or nested term) will be created and added to the model:  $gmclasslowses\_mean * gmteacheffect$ . The interaction will tell us if there's a relationship between teacher effectiveness ( $gmteacheffect$ ) and low socioeconomic class composition ( $gmclasslowses\_mean$ ).

- The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from Model 4a. To facilitate reading of the output tables, we will first remove  $gm-lowses$  and then add it back into the model. First, click to select  $gm-lowses$ , and then click the REMOVE button.



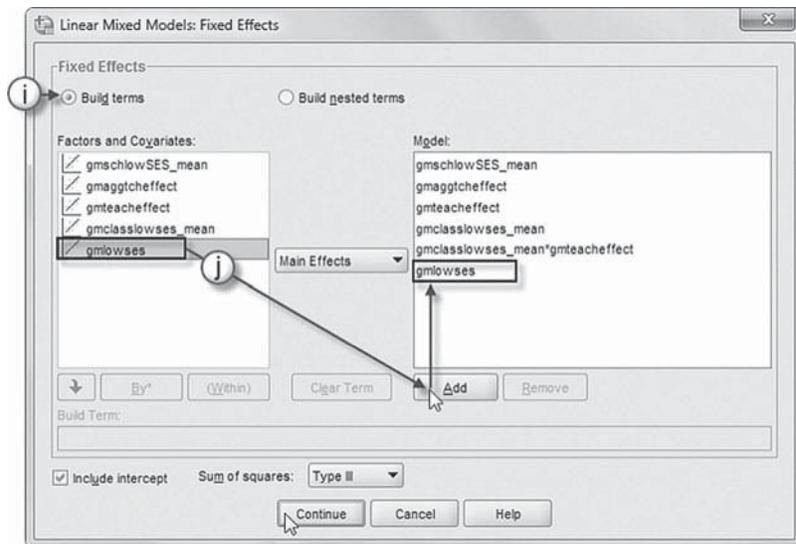
Add Interaction to Model 5:  $gmclasslowses\_mean * gmteacheffect$ 

- b. Change the build terms option by clicking to select *Build nested terms*.
- c. Click to select the variable  $gmclasslowses\_means$  from the *Factors and Covariates* box.
- d. Then click the arrow button below the *Factors and Covariates* box. This moves  $gmclasslowses\_means$  into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- e. Next, click the BY\* button, which will insert the computation command symbol:  $gmclasslowses\_means^*$ .
- f. Click to select  $gmteacheffect$  from the *Factors and Covariates* box.
- g. Click the arrow button below the *Factors and Covariates* box to move  $gmteacheffect$  into the *Build Term* box and complete the interaction term:  $gmclasslowses\_means * gmteacheffect$ .
- h. Click the ADD button to transfer the interaction into the *Model* box.

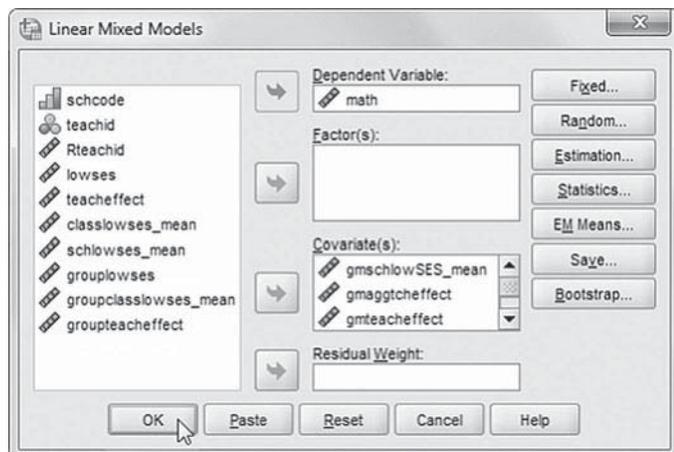
We will now add  $gmloswses$  back into the model.

- i. Click to select the *Build terms* option.
- j. Click to select  $gmloswses$ , and then click the ADD button to transfer the variable into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 5

We present the results for Model 5 in Table 4.11. The first two variables below the intercept are the between-school predictors of math outcomes. Both school SES composition and aggregate teacher effectiveness are significant predictors of achievement. Next are the classroom variables (i.e., classroom composition, teacher effectiveness, and the interaction term). The results suggest

TABLE 4.11 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	596.859	0.733	147.494	814.737	.000	595.411	598.306
gmschlowSES_mean	-31.181	4.517	429.003	-6.903	.000	-40.059	-22.303
gmaggtcheffect	7.676	1.950	208.991	3.937	.000	3.832	11.520
gmteacheffect	7.432	0.677	74.918	10.970	.000	6.082	8.782
gmclasslowses_mean	-10.933	3.469	1,748.372	-3.152	.002	-17.736	-4.130
gmteacheffect *	5.385	3.138	373.263	1.716	.087	-0.785	11.555
gmclasslowses_mean							
gmmlowses	-14.543	0.862	8,837.068	-16.880	.000	-16.232	-12.854

<sup>a</sup> Dependent variable: math.

TABLE 4.12 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		1,288.768	19.985	64.485	.000	1,250.186	1,328.540
Intercept + gmteacheffect	UN (1, 1)	41.227	9.544	4.320	.000	26.190	64.898
[subject = schcode]	UN (2, 1)	3.041	4.932	0.617	.537	6.625	12.708
	UN (2, 2)	6.201	6.536	0.949	.343	0.786	48.932
Intercept [subject = schcode * Rteachid]	Variance	68.321	12.279	5.564	.000	48.036	97.171

<sup>a</sup> Dependent variable: math.

that the interaction between teacher effectiveness and classroom SES composition ( $\beta_{03}$ ) is significant at  $p < .10$  ( $\gamma_{030} = 5.385$ ). Because of coding, this result suggests that in a classroom 1 *SD* above the grand mean (0) in low SES composition, the combined teacher effect would be considerably larger ( $7.432 + 5.385 = 12.817$ ) than the teacher effect (7.432) in a class at the grand mean (0) in SES composition. This provides supportive evidence for answering our third research question, which focused on whether teacher effectiveness might be contingent on classroom composition factors.

Table 4.12 suggests that the variance at Level 2 was little affected by adding the interaction effect to the model.

### Comparing the Fit of Successive Models

We can compare model-fitting evidence from the previous model with no interaction at Level 2 (Model 4) and the current model (Model 5) to determine whether the addition of the interaction term enhanced the fit of the model. Tests of nested models should be conducted with ML when regression coefficients are being compared for their fit to the data. Model 4 without the interaction term included (11 estimated parameters) yields the model fit criteria shown in Table 4.13.

Model 5 (estimated similarly with ID and UN matrices and the Level 2 interaction) yields the model fit criteria (for 12 estimated parameters) shown in Table 4.14.

With ML estimation, the probability of obtaining the observed results given the parameter estimates is referred to as the likelihood function. Since the likelihood is less than 1.0, it is

common to use  $-2$  times the log of the likelihood ( $-2LL$ ) as a measure of model fit to the data. Good models result in a high likelihood of obtaining the observed results (which corresponds to a small value for  $-2LL$ ). A perfect model would have a likelihood of 1, and the log of likelihood would be 0 (which when multiplied by  $-2$  is also 0). The analyst can test the difference in  $-2LL$  (also referred to as model deviance) between two models if the models are nested, which means a specific model can be derived from a more general model by removing parameters from the more general model. The difference in  $-2LL$  for two such models being compared can be conceptualized as a likelihood ratio test, which follows a chi-square distribution, with degrees of freedom ( $df$ ) equal to the difference in the number of parameters estimated between the two models (Azen & Walker, 2011).

In this case, we will assume that the model with the interaction is more general since it has 12 estimated parameters. We then compare it against the more restricted model, which has the interaction removed (i.e., 11 parameters are estimated). The difference in  $-2LL$  between the two models is 2.928. There is 1  $df$  in the model test since one regression parameter is being removed. The required chi-square at  $p = .05$  for 1  $df$  is 3.84. Therefore, we can conclude that the model with 11 estimated parameters (i.e., without the interaction term) fits the data the same as the model with the interaction term added. Another way of looking at this would be that adding the interaction does not “significantly improve” the model’s fit to the data. When the difference between models is small (i.e., not statistically significant), we will usually accept the restricted model since it provides the same fit to the data with fewer parameters estimated (as summarized in Table 4.8). In this case, the likelihood ratio test is consistent with the  $t$  test in Table 4.11 ( $t = 1.716$ ,  $p = .087$ ), which suggests that the interaction is not significant at  $p < .05$ . If the difference in model deviances was statistically significant, we would accept that the more general model (with more parameters) improves the model’s fit to the data. For models that are not nested, we can use the Schwarz’s Bayesian criterion (BIC) or Akaike’s information criterion (AIC) indices in Tables 4.13 and 4.14 to compare models. We prefer smaller values regardless of

**TABLE 4.13 Information Criteria<sup>a</sup>**

$-2$ Log Likelihood	92,325.317
Akaike’s Information Criterion (AIC)	92,347.317
Hurvich and Tsai’s Criterion (AICC)	92,347.346
Bozdogan’s Criterion (CAIC)	92,436.693
Schwarz’s Bayesian Criterion (BIC)	92,425.693

Note: The information criteria are displayed in smaller-is-better forms.

<sup>a</sup> Dependent variable: math.

**TABLE 4.14 Information Criteria<sup>a</sup>**

$-2$ Log Likelihood	92,322.389
Akaike’s Information Criterion (AIC)	92,346.389
Hurvich and Tsai’s Criterion (AICC)	92,346.423
Bozdogan’s Criterion (CAIC)	92,443.891
Schwarz’s Bayesian Criterion (BIC)	92,431.891

Note: The information criteria are displayed in smaller-is-better forms.

<sup>a</sup> Dependent variable: math.

the number of parameters estimated in the models. Other criteria presented as part of the output represent similar ways of comparing models (users can consult SPSS MIXED Help for further information).

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## Summary

The previous discussion suggests that researchers should proceed with caution when building multilevel models. It is often the case that different sets of variables are hypothesized to explain variation in random intercepts and slopes. Raudenbush and Bryk (2002) suggest using a common set of variables when building the models between groups. We decided to add the intercept predictors first and then to consider variation in slopes.

## CHAPTER 5

# Examining Individual Change with Repeated Measures Data

In our previous chapters, the outcome and explanatory variables were only measured on one occasion. One of the limitations of such cross-sectional analyses is that they are not well suited to studying processes that are assumed to be developmental. Because data are collected only at one point in time, they are insufficient to examine possible temporal relationships in a theoretical model. Time is a key factor in understanding how developmental processes unfold. When an outcome is measured several times for an individual, we have a repeated measures design (RMD). A simple example is the pretest and posttest design. For studying developmental processes, however, adding measurement occasions between the pretest and posttest can provide a more thorough examination and often can increase the power of the statistical test used to determine whether a change has taken place (Hox, 2010; Willett, 1989).

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### Ways to Examine Repeated Observations on Individuals

In the past, analyses of repeated observations on individuals were typically conducted using univariate analysis of variance (ANOVA) or multivariate analysis of variance (MANOVA), depending on the goals of the research and the specific features of the data. There are, however, several limitations of both approaches that arise from assumptions underlying their appropriate use in longitudinal research (e.g., see Hox, 2010; Raudenbush & Bryk, 2002). One primary shortcoming is that both approaches are limited to a subset of research situations involving change within and between individuals, where the timing of the repeated measurements is equidistant, subjects are independently and randomly sampled, and the change is considered as a fixed, rather than randomly varying, parameter between individuals. Most importantly for our purposes, analyses using either approach cannot be extended to include higher level groups such as classrooms or schools since this violates the assumption of sampling independence.

A second limitation concerns the normality of the data. For the univariate ANOVA approach, because differences in means of an outcome are tested over levels of a within-subjects factor (*time*), we must restrict the nature of the residual variances between the repeated measures as well as possible covariances between them (referred to as *sphericity*). The sphericity assumption implies that the within-subject model consists of independent (orthogonal) components. One type of sphericity is *compound symmetry*, which requires the repeated measures variances to be equal and any covariances between them to be the same. Where the assumption is not met, researchers can either use a degrees of freedom (*df*) correction to the *F* tests for hypothesis testing (to guard against Type I errors), or they often use the multivariate (MANOVA) approach since it does not require the sphericity assumption. Repeated measures ANOVA (RM-ANOVA) also requires homogeneity of variance for different levels of between-subjects factors such as treatment

and control groups, while MANOVA requires the related assumption of homogeneity of the variance-covariance matrices for dependent variables across between-subjects factors.

A third limitation is the inability to include individuals with partial data in the analysis in either approach. Any individual with missing data on any occasion is eliminated from the analysis through listwise deletion. This can result in a tremendous loss of information about individuals within a longitudinal analysis. Therefore, we suggest having little or no missing data if either approach is to be used. Provided their basic assumptions are met, however, ANOVA and MANOVA remain viable approaches for examining repeated measures data.

Increasingly, however, both the concepts and methods are becoming available that can provide a more rigorous and thorough examination of repeated measures data. One well-known approach, which draws on structural-equation-modeling (SEM) methods, is latent change (or curve) analysis (LCA). In the LCA approach to examining growth, the repeated measures of  $Y$  are defined in a covariance matrix, which accommodates a single-level confirmatory factor analysis (CFA); that is, the repeated measures are treated as observed variables that define latent intercept and growth factors. In this way, a time dimension is incorporated into the specification of the latent variables. The SEM specification of individual latent change amounts to a multivariate specification at a single level. Change involving individuals within groups can then be accommodated as a two-level CFA (Muthén & Muthén, 1998–2006).

Repeated measures data with within-subjects and between-subjects factors can also be specified as another type of linear mixed (or random coefficients) model (Laird & Ware, 1982). In this latter approach, change involving individuals can be conceptualized as a two-level analysis, with the repeated measures of  $Y$  and the change in their levels over time specified at Level 1 and random variation in the individual intercepts and growth rates examined at Level 2. Differences between individuals (e.g., background or an experimental treatment) can be proposed to explain random variation in individuals' growth parameters at Level 2. Moreover, in the mixed model, the repeated measures can be observed at fixed or varying occasions, and the approach can incorporate missing data on *some* occasions. This can be beneficial if there are subjects who drop out during a longitudinal study. The mixed model can also accommodate time-varying covariates at Level 1. Using a multilevel framework, univariate analyses of individual growth can easily be extended to include more than one growth process or differences in growth due to successive groupings such as classrooms (Level 3) and schools (Level 4). We note, however, that the complexity of adding more analytic levels can begin to challenge available computer memory needed to estimate a proposed model's parameters.

Models with time-ordered relationships definitely offer increased possibilities for studying various types of individual change processes using either the multivariate SEM or univariate two-level mixed model approaches. They encourage researchers to ask a number of different questions of the data: Is there a change in the level of the means over time? If so, what is the shape of the change trajectory? Is the change the same for different groups of individuals?

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## Considerations in Specifying a Linear Mixed Model

There are a number of considerations to keep in mind in specifying a repeated measures analysis using a linear mixed-modeling approach. First, we should decide whether we are examining one or more growth processes. For our presentation in this chapter, we will use a univariate approach to examine our proposed research questions; that is, we assume there is only one outcome being examined over time. In Chapter 7, we consider a multivariate (e.g., parallel) growth model—one that facilitates the examination of individual changes in reading and math achievement simultaneously. Our primary hypothesis in a univariate RMD concerns whether or not there is a difference in the levels of the means of the dependent variable over time (Raykov & Marcoulides, 2008). If we are able to reject the null hypothesis that the means are the same, then we can assume that some change has taken place.

For investigating individual development in a single outcome, we can test this assumption of equal means using RM-ANOVA; however, as we suggested earlier, we must assume *sphericity*. The sphericity assumption refers to the structure of the repeated measures covariance matrix and stems from the assumption that the repeated observations should be independent and therefore have constant variance and ideally be uncorrelated with each other. This is an important assumption underlying the RM-ANOVA approach (which can be tested using Mauchly's sphericity test) since it affects the calculation of significance levels of the related  $F$  tests for proposed hypotheses. Mauchly's test provides a test of whether the repeated measures used to define within-subjects growth are represented by a spherical covariance matrix. The test, however, is highly sensitive to even mild departures from the required covariance structure. If sphericity is not met (which is generally the case with real data), there are other options including adjusting the  $F$  tests in RM-ANOVA for purposes of hypothesis testing or using the multivariate approach. Another alternative is the mixed model approach. A clear advantage of this latter approach is that it provides greater flexibility in identifying a suitable Level 1 covariance structure that captures the nature of the observed relationships between the repeated measures of  $Y$  in varied longitudinal studies, regardless of time-trend patterns within individuals and among the groups being compared.

Second, after determining an approach to use, we need to consider the expected within-subjects (Level 1) effect for *time*, which concerns individuals' possible change over the *temporal* period of the study. The time effect describes whether individuals are indeed changing over some relevant *interval* of time and by *how much*. This part of the within-subjects model represents the change we would anticipate that each individual would experience over the course of the study (Singer & Willett, 2003). For example, the relevant time interval may be days, weeks, months, or years. In a mixed model, the potential difference in means across measurement occasions can be summarized as a time-related slope. The time slope is generally the most important parameter in the model because it provides a test of whether the outcome means are equal across occasions, as well as information about the *rate* at which individual  $i$  changes over a particular time interval  $t$ .

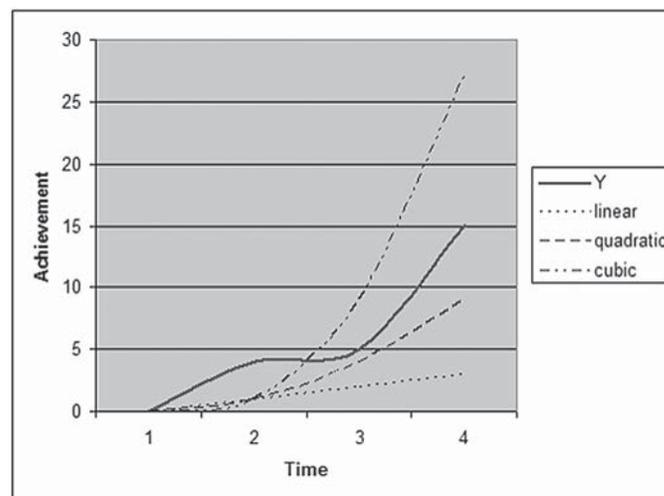
If we find that the means of  $Y$  are not the same across time, we next would want to investigate further how individuals are changing over time. The analysis of repeated measures facilitates the representation of several different growth trajectories. These can concern, for example, a naturally occurring developmental trend (e.g., students' acquisition of vocabulary between 12 and 36 months of age) or perhaps individuals' reading skills before and after a classroom treatment is introduced. When a categorical variable represents an increasing level of a treatment dosage, age, or a series of successive measurements, the growth trend is often summarized using polynomial curves within individuals (Level 1) because they are very adaptable to a variety of different growth trajectories and can be estimated using standard linear-modeling procedures (Hox, 2010). Most common is a linear growth trend, which assumes that the rate of individual change is the same across each interval of time ( $a$ ). However, it is not always the case that individuals are changing at the same rate over time. In some cases, adding higher order polynomials to the model for defining the time-related trend can improve prediction.

The polynomial equation is not nonlinear; in fact, mathematically it is linear (Hox, 2010). If there are  $k$  measurements, there is a  $k - 1$  polynomial that can be used to fit the model. A quadratic trend is interpreted as a change in the rate of change (i.e., accelerating or decelerating) over an interval of time ( $a^2$ ). A cubic trend ( $a^3$ ) is S-shaped, which describes changes in the rate of change over time such as accelerating, decelerating, and accelerating again. For example, for three measurement occasions, the highest degree term is  $a^2$  (quadratic), while for four measurement occasions, the highest degree term is cubic, or one less than the number of measurement occasions ( $k - 1$ ). Of course, we might prefer to interpret the results of a linear growth model for ease of interpretation, but sometimes a higher order polynomial may describe the data more accurately. Generally, we only include up to the highest significant polynomial in the model. In terms of actually describing the change over time, it becomes increasingly more difficult to interpret models of higher polynomial degrees. There also may be cases where developmental

processes cannot be well approximated by using a polynomial function. For example, the logistic curve (which cannot be transformed to a linear model) may be better in approximating a process that is slower in the beginning of the trend, speeds up in the middle, and then slows at the end (representing an *S* curve). In other cases, however, a logistic or exponential function may be well approximated by using a cubic polynomial if there are at least four measurements (Hox, 2010).

It is important to note that the parameters of a higher order polynomial model (e.g., cubic) have no direct interpretation in the growth process over any particular interval, such that interpretation must be made by looking at average plots of the growth or some more typical individual growth curves (Hox, 2010). To illustrate this point, in Figure 5.1 we have plotted the actual growth in an academic outcome *Y* (solid line) over four intervals against what the growth might look like if it were strictly linear, quadratic, or cubic (dotted lines). At the bottom of the figure, we can observe that linear growth rises (or declines) constantly over time. In contrast, quadratic growth tends to rise more quickly (or decline) relative to the constant rate of growth represented in the linear trajectory. We can also notice that the cubic growth curve increases much more rapidly over the four time periods compared to linear or quadratic growth curves. The actual growth trajectory plotted in the figure for *Y* appears to have elements of linear, quadratic, and cubic growth. It appears to change linearly between intervals 1 and 2, to slow between intervals 2 and 3 (quadratic trajectory), and then to accelerate more rapidly between intervals 3 and 4 than between intervals 1 and 2. The cubic polynomial actually combines the effects of four coefficients (i.e., linear, quadratic, and cubic slope coefficients, and the intercept). Different values of each coefficient will move the average trajectory up or down with respect to the horizontal axis (i.e., changing the intercept) and alter the steepness of the cubic curve (changing the cubic coefficient), its slope (changing the linear component), or the curvature of the parabolic element (changing the quadratic coefficient). As we reiterate, when a cubic element is present in a trajectory, it suggests a focus on the whole growth trend and not just any particular interval.

Third, after settling on a reasonable growth trajectory to describe individual development, we can consider possible between-subjects variables that might affect individuals' growth trajectories. A third hypothesis often tested in a RMD is a test of parallelism (Raykov & Marcoulides, 2008); that is, are the trajectories the same for different levels of a factor (e.g., subjects in treatment or control groups)? In repeated measures multilevel designs, it is also important to distinguish within-subjects (Level 1) variables, which change over time, from between-subjects (Level 2) variables, which are considered to be static over the temporal period of the study. For



**FIGURE 5.1** Examining several different individual growth curves.

example, we can enter motivation as a time-varying covariate at Level 1 that predicts changes in reading scores. In contrast, we could also enter motivation as a between-subjects covariate (i.e., continuous predictor), in which case it would be considered as static over the temporal period of the study. Covariates, which are continuous variables that might affect the rate of growth, can also be added to the model. When covariates are added, the means for each occasion are adjusted for the presence of the covariates.

## An Example Study

Consider study where we wish to examine students' growth trajectories in math achievement over time (a within-subjects factor with three levels) and to assess whether their socioeconomic status (SES) background (a between-subjects covariate) and perceptions about their math teacher's effectiveness (a between-subjects factor) are related to different achievement growth patterns.

## Research Questions

We may be interested first in whether a change takes place in student math achievement over time. This type of question addresses whether the levels of the means for the outcome are the same or different over the occasions of measurement. The assumption is that if we can reject the null hypothesis of no difference in means across measurement occasions, it implies that a change in individuals has taken place. A second question concerns what the shape of the developmental change might look like for individuals in the study. For example, we might ask whether the rate of individual change per occasion is linear or whether the change might be more complex. Once we have described the shape of individuals' change trajectories over time, we might ask a third question: Are there differences in development between groups of individuals? These differences might be due to an experimental treatment or other types of factors (e.g., background). In this case, we examine whether student growth is related to their individual perceptions about the teaching skill of their teachers. We might also wish to adjust our estimates for the presence of covariates that might affect individuals' development over time. In this example, we use students' SES as a covariate.

## The Data

The data used for this study consist of 8,670 secondary students. We will assume that they have been randomly sampled from a larger population of students. The variables used in the example are summarized in Table 5.1.

**TABLE 5.1** Data Definition of *ch5growthdata-vertical.sav* ( $N = 8,670$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement <sup>1</sup>
<i>id</i>	Individual	Individual student identifier (8,670 students) across three time (test) occasions.	Integer	Ordinal
<i>nschcode</i>	School	School identifier (515 schools).	Integer	Ordinal
<i>Rid</i>	Individual	A within-group level identifier <sup>b</sup> representing a sequential identifier for each student within each school.	1 to 46	Ordinal
<i>Index1</i>	Within Individual	Identifier variable resulting from indexing the math outcomes to create a new identifier with a number sequence (1, 2, 3) corresponding to the three time occasions measuring students' math achievement.	1 = First Time 2 = Second Time 3 = Third Time	Scale

(Continued)

TABLE 5.1 (Continued)

Variable	Level <sup>a</sup>	Description	Values	Measurement <sup>1</sup>
<i>time</i>	Within Individual	Variable representing three linear occasions in time measuring students math achievement.	0 = First Time 1 = Second Time 2 = Third Time	Scale
<i>quadtime</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into a “squared” quadratic sequence (0, 1, 4) to capture any changes (acceleration or deceleration) in the rate of change that might occur over the three measurement occasions.	0 = First Time 1 = Second Time 4 = Third Time	Scale
<i>orthtime</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into an orthogonal linear (-1, 0, 1) sequence.	-1 = First Time 0 = Second Time 1 = Third Time	Scale
<i>orthquad</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into an orthogonal quadratic (1, -2, 1) sequence.	1 = First Time -2 = Second Time 1 = Third Time	Scale
<i>test</i>	Within Individual	The dependent variable representing each students’ individual scores on the repeated math measurements.	24.35 to 99.99	Scale
<i>effective</i>	Individual	Two-category predictor variable representing teachers’ effectiveness in teaching math.	0 = Not Effective 1 = Effective	Nominal
<i>ses</i>	Individual	Predictor interval variable (z score) measuring student socioeconomic status composition within the schools.	-2.41 to 1.87	Scale
<i>timenonlin1</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into a time sequence variation that encompasses the whole 3-year period of time (0.00, 0.50, 1.00) measured from 0 to 1.	0.00 = First Time 0.50 = Second Time 1.00 = Third Time	Scale
<i>timenonlin2</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into a time sequence variation representing the whole 3-year period of time (0.00, 0.60, 1.00) measured from 0 to 1.	0.00 = First Time 0.60 = Second Time 1.00 = Third Time	Scale
<i>timenonlin3</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into a time sequence variation representing the whole 3-year period of time (0.00, 0.70, 1.00) measured from 0 to 1.	0.00 = First Time 0.70 = Second Time 1.00 = Third Time	Scale
<i>timenonlin</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into a time sequence variation representing the whole 3-year period of time (0.00, 0.53, 1.00) measured from 0 to 1.	0.00 = First Time 0.53 = Second Time 1.00 = Third Time	Scale

<sup>a</sup> Individual = Level 1; school = Level 2; Within Individual = repeated measures, Level 1; Individual = Level 2; School = Level 3.

<sup>b</sup> Results from ranking student cases (*id*) with the school group identifier (*nschcode*).

<sup>1</sup> Measurement icon settings displayed in subsequent model screenshots may differ from Table 5.1 but will not affect the output.

Using a random-coefficients approach to investigate individual change provides considerably more flexibility than either the univariate ANOVA approach or the multivariate approach, especially in situations where there may be missing data, varying occasions of measurement, and more complex error structures. It is generally useful to spend some time examining the nature of the data initially. To examine individual change using MIXED, the data must first be organized differently. As we noted in Chapter 2, the time-related variable describing the shape of the growth trajectory (e.g., linear or quadratic) is entered into the data set as a variable, with the successive math measurements structured vertically or stacked (i.e., instead of as a horizontal, multivariate

	id	schcode	time	quadtime	test	effective
1	1	1	0	0	521	1
2	1	1	1	1	543	1
3	1	1	2	4	564	1
4	2	1	0	0	602	0
5	2	1	1	1	623	0
6	2	1	2	4	645	0
7	3	2	0	0	575	0
8	3	2	1	1	589	0
9	3	2	2	4	607	0

**FIGURE 5.2** Vertical data matrix for repeated measures analysis in IBM SPSS.

set of variables as in MANOVA or SEM) for each subject within the data set  $(y_{1i}, y_{2i}, \dots, y_{ti})'$ . As this suggests, the number of lines for each individual is defined by the number of measurement occasions. In this case, therefore, we will have  $8,670 \times 3$ , or 26,010 lines, in the database. We can use DATA and RESTRUCTURE menu commands to restructure the data vertically. Figure 5.2 presents data on three subjects in a hypothetical data set.

Closer inspection of the data suggests that there are three observations per individual on the math “test” outcome, and individual and school identifiers, as well as any predictors (e.g., perceived teacher effectiveness), are repeated in the data set for each time interval. The outcome (*test*) represents each individual’s scores on the repeated math measurements. We can also see that the repeated observations of *test* are nested within individual identification (*id*) numbers, and student IDs are nested within school identifiers (*schcode*). The grouping variables (*id*, *schcode*) are used to identify each predictor as belonging to a particular level of the data hierarchy. As we noted, often a polynomial function will describe individual growth pretty well. As Figure 5.2 also indicates, the time-related variables (linear and quadratic) are also entered as data. Most often, researchers assume that individuals are changing at a constant rate over time, which can be represented as linear growth, especially over a short period of time, but it may also be the case that subjects are experiencing more complex patterns of growth over time. Since there are three occasions, we can also enter a quadratic component to test for the presence of a change in the rate of growth occurring over time.

### Examining the Shape of Students’ Growth Trajectories

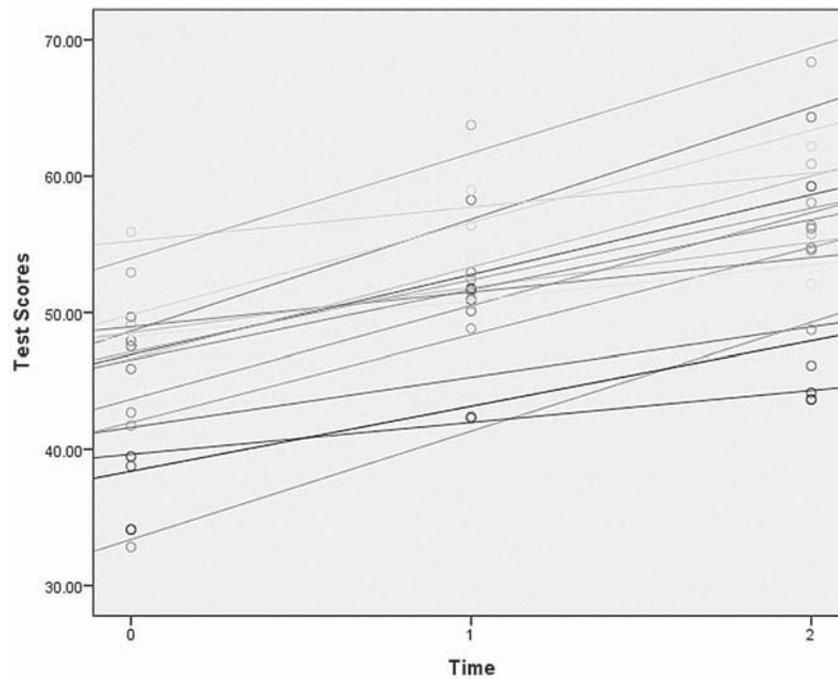
We will begin by examining descriptive statistics showing the level of the outcome means on each occasion, as summarized in Table 5.2. The table suggests that the average math achievement for the first occasion is 48.632, while for the last occasion it is 57.094, indicating considerable change over time. The table suggests that the grand mean is 52.945, which falls somewhere between the first and second measurement occasions. The grand mean is often not of much interest in examining growth since it just represents the average achievement level across the three measurement occasions. Examining the means more closely, one can see that the change between the first two test means is about 4.5 points, while between the second two means it is about 4.0. This

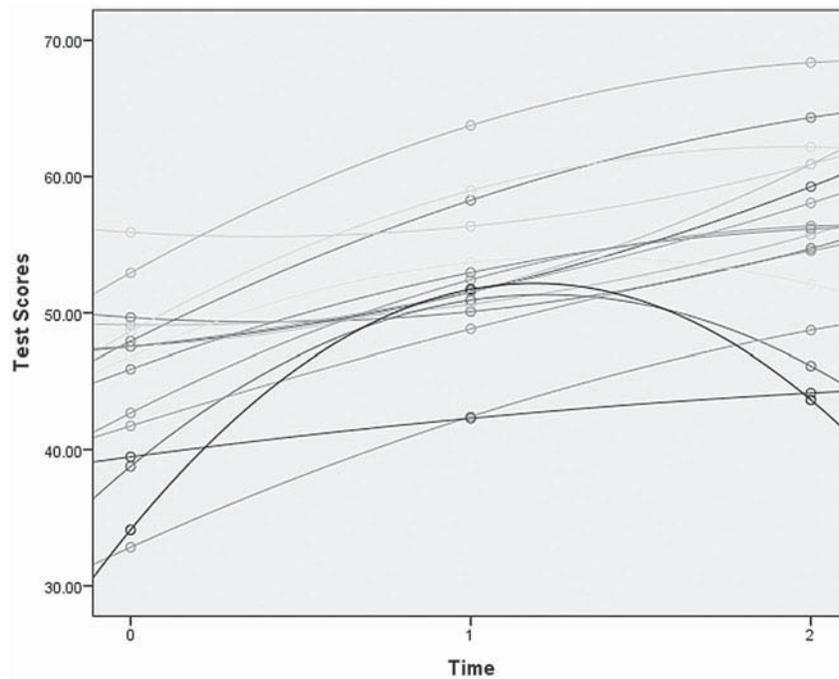
**TABLE 5.2 Means for Each Measurement Occasion**

Test	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean	
				Lower Bound	Upper Bound
0	48.632	9.713	0.104	48.428	48.837
1	53.107	9.888	0.106	52.899	53.316
2	57.094	9.894	0.106	56.886	57.303
Total	52.945	10.421	0.065	52.818	53.071

suggests slightly less growth during the latter part of the trend compared with the initial part. The differences in observed means summarized in Table 5.2 suggest that they probably are not the same over time (hypothesis 1).

Visual inspection of the data can provide important preliminary clues about the shape of change in math that is taking place among individuals over time. Figure 5.3 provides a plot of the linear growth trajectories of the first 17 subjects in the data set. The plot of these individuals' scores over time suggests that individuals are increasing in their knowledge. Readers will note that the intercepts (i.e., individuals' status at Time 0) appear to vary considerably (i.e., from about 32 to 55) and the steepness of the growth over time also seems to vary within this subset of individuals. Many times, with a few waves of data, a linear model will be adequate to describe individuals' growth. Notice, however, in the graph of these individuals' growth, the linear model does not seem to capture the change over time of all individuals equally well; that is, not all of the individuals' observed scores fall on their predicted growth lines.

**FIGURE 5.3** Individual linear math growth trajectories.



**FIGURE 5.4** Individual nonlinear math growth trajectories.

For purposes of contrast, Figure 5.4 is a graph of the same 17 subjects, this time using a quadratic trajectory. With three time points, one can observe that the fit of the curved lines to the data points will be perfect. For some individuals, the plot of their trajectories in the figure suggests that a linear shape might be adequate to describe the growth. For others, however, it appears that their growth might be better described by a curvilinear trajectory. These plots show visually our preliminary interest in determining whether a linear shape, or both linear and quadratic components, would be required to describe the shape of individuals' growth trajectories accurately.

#### ***Graphing the Linear and Nonlinear Growth Trajectories with IBM SPSS Menu Commands***

We can use the IBM SPSS menu commands to display the information for the subset of individuals in the study, as shown in Figures 5.2, 5.3, and 5.4 in the following series of instructions.

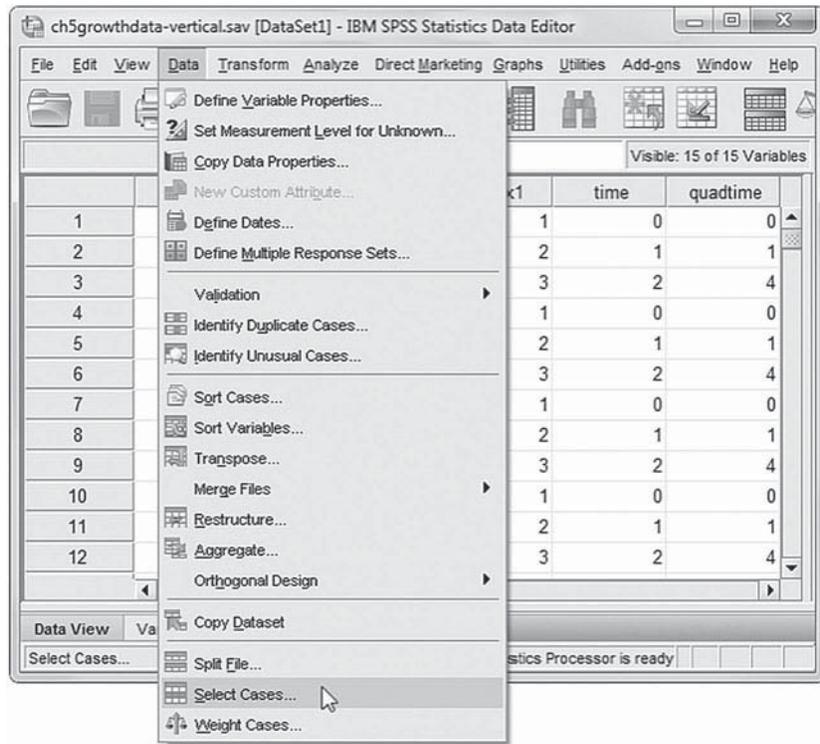
## Select Subset of Individuals

Launch the IBM SPSS application and select the *ch5growthdata-vertical.sav* data file.

We will begin by selecting the subset of individuals.

1. Go to the toolbar and select DATA, SELECT CASES.

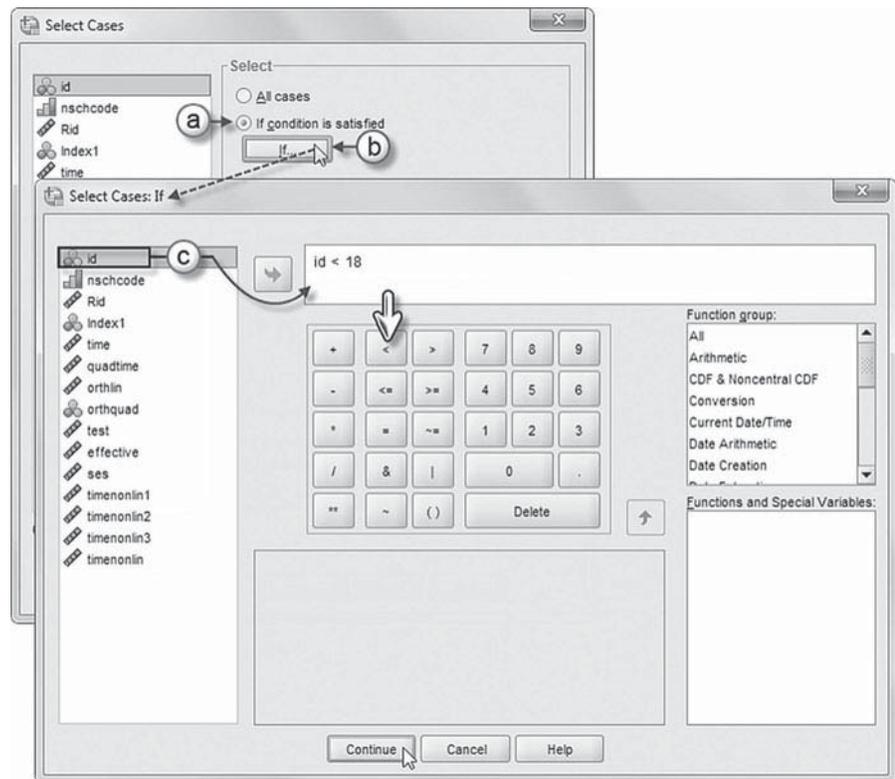
This command will open the *Select Cases* dialog box.



- 2a. Within the *Select Cases* dialog box, click to select *If condition is satisfied*.

- b. Then click the IF button, which will activate the *Select Cases: If* box.

- c. Click the variable *id* from the left column listing, and then click the right-arrow button to move *id* into the box.

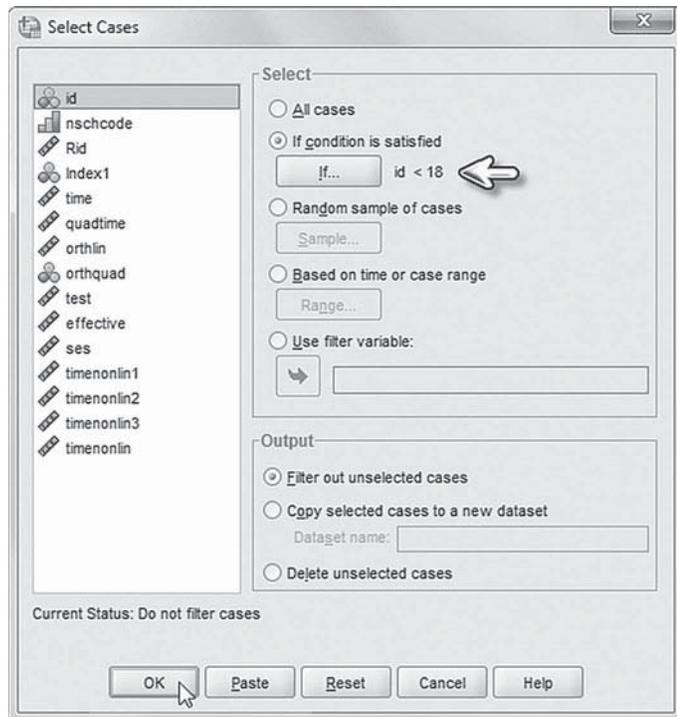


Use the keypad to enter the less than sign (<) followed by the number 18. The resulting command (*id < 18*) instructs IBM SPSS to select only the first 17 cases of the data set.

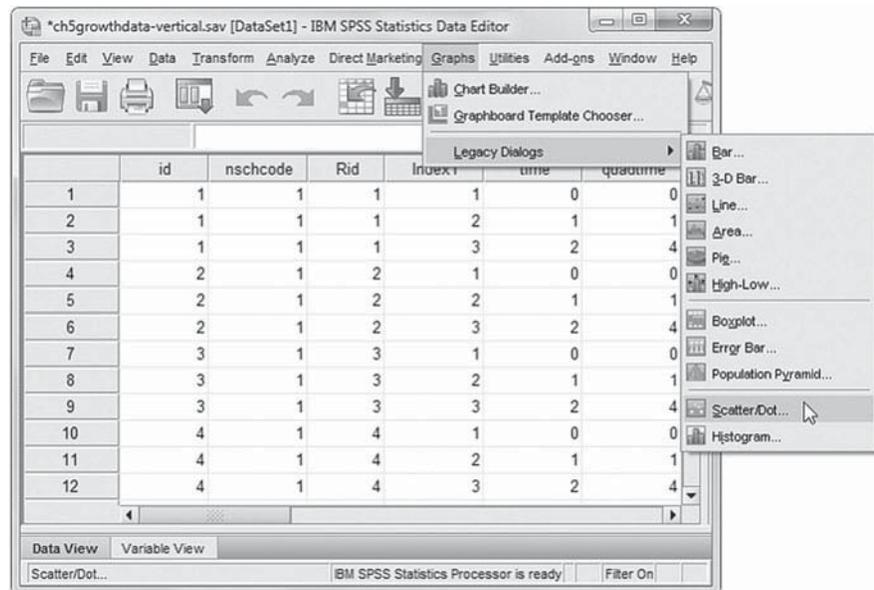
Click the CONTINUE button to return to the *Select Cases* dialog box.

3. Notice that the IF condition statement  $id < 18$  is listed.

Click the OK button to return to the main menu.

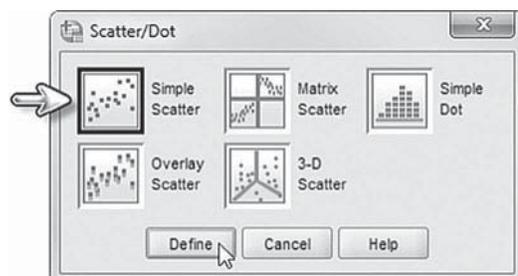


4. To graph the 17 cases, go to the toolbar and select GRAPHS, LEGACY DIALOGS, SCATTER/DOT.



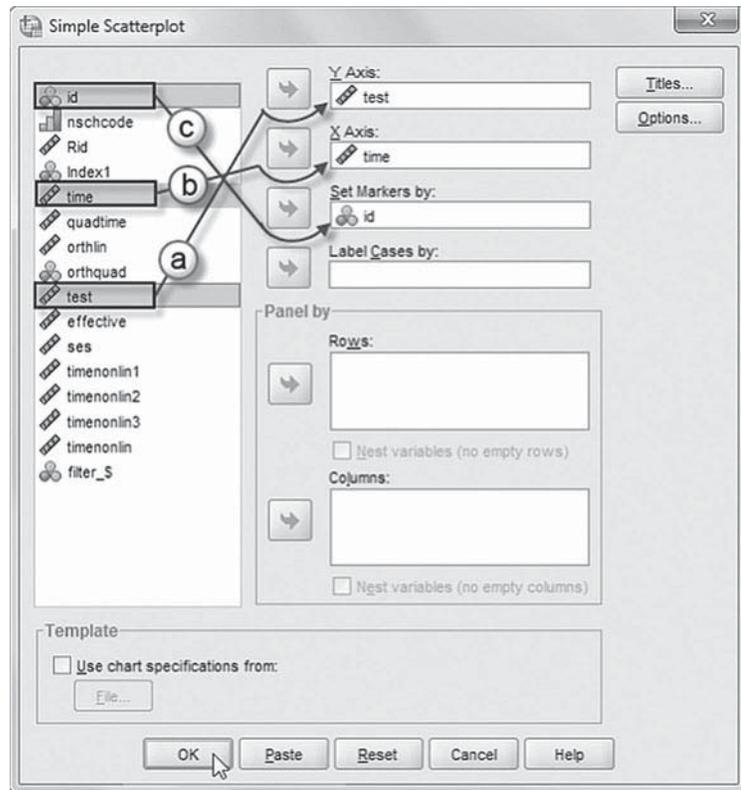
5. Click on the SIMPLE SCATTER icon to select this option among those shown.

Click DEFINE button to open the *Simple Scatterplot* dialog box.



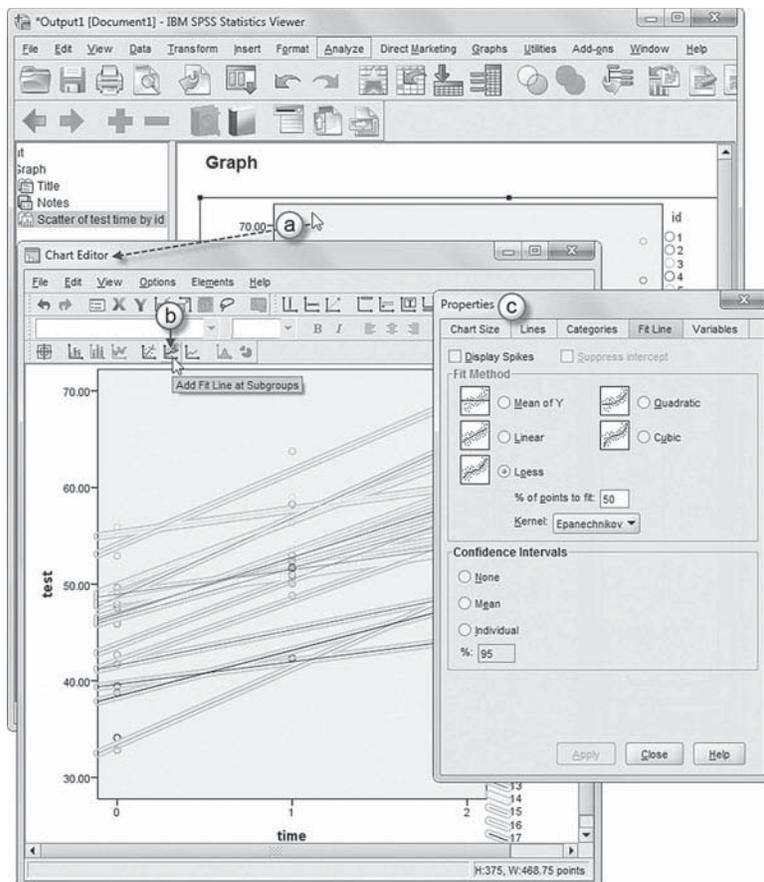
- 6a. Within the *Simple Scatterplot* dialog box click to select the variable *test* from the left column listing. Then click the right-arrow to move the variable into the *Y Axis* box.
- b. Click to select *time* from the left column listing, and then click the right-arrow button to move the variable into the *X Axis* box.
- c. Click to select *id* from the left column listing, and then click the right-arrow button to move the variable into the *Set Markers by* box.

Then click the OK button to generate the scatterplot.

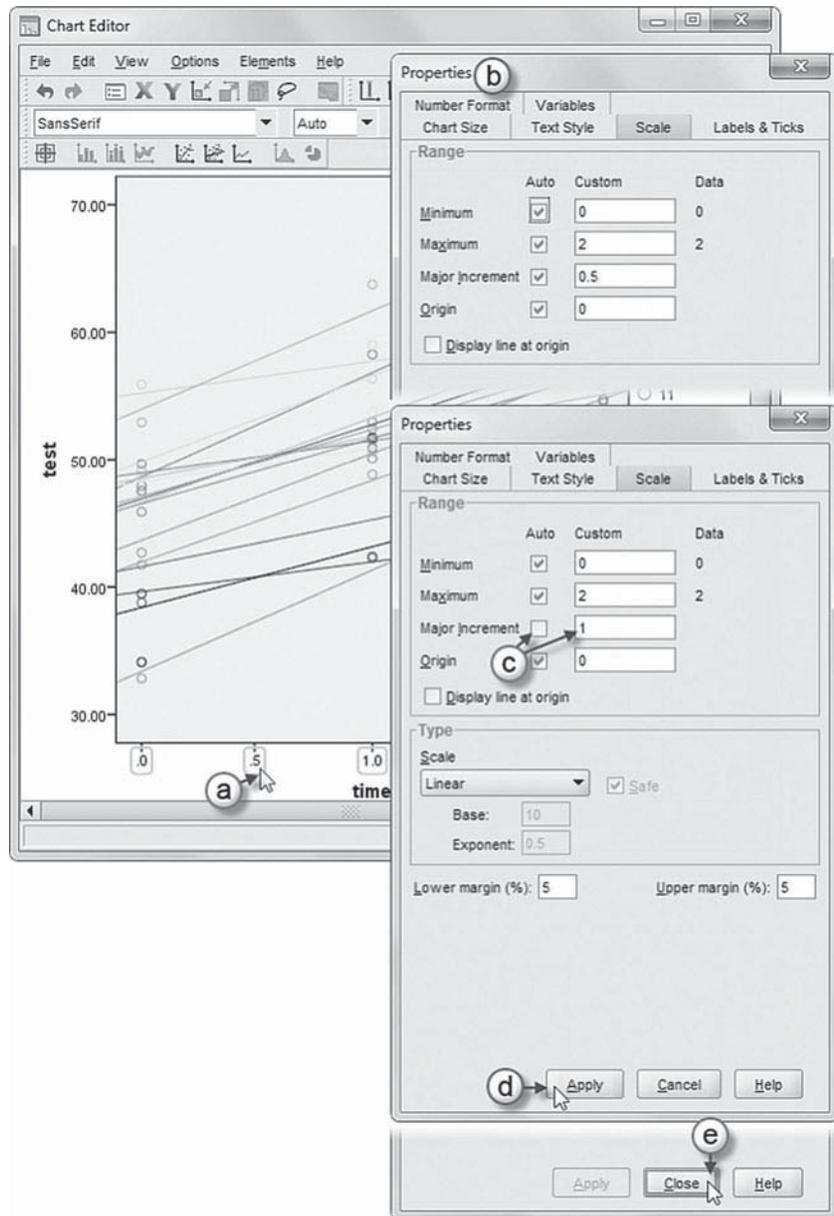


Generate Figure 5.3 (Linear Trajectory)

- 7a. Double-click on the graph in the output to select it and activate the *IBM SPSS Chart Editor*.
- b. In the *Chart Editor*, click on the icon **ADD FIT LINE OF SUBGROUPS**, which will insert lines on the graph.
- c. Clicking the **Add Fit Line of Subgroups** icon also activates the *Properties* box, which provides assorted options.

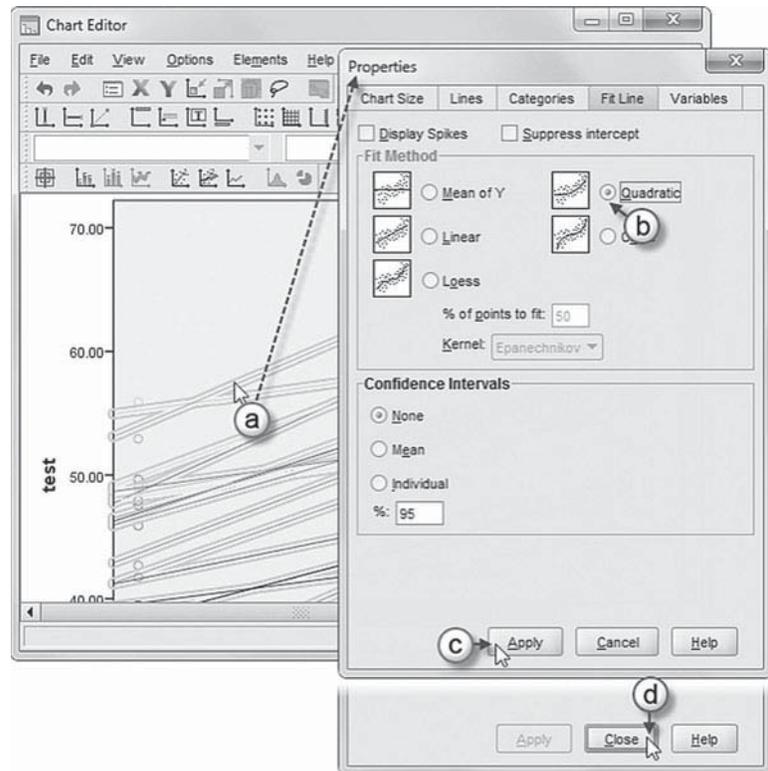


- 8a. IBM SPSS default settings insert 0.5 increments to the  $X$  axis representing time. To change the increment, double-click the number to access the *Properties* dialog box.
- b. In *Properties* box, the *Scale* tab displays the *Range* options for changing the increments.
- c. Change the *Major Increment* by unchecking the box and then replacing 0.5 with 1.
- d. Click the APPLY button to make the change and activate the Close button.
- e. Click the CLOSE button to exit from the *Properties* box.



### Generate Figure 5.4 (Nonlinear Quadratic Trajectory)

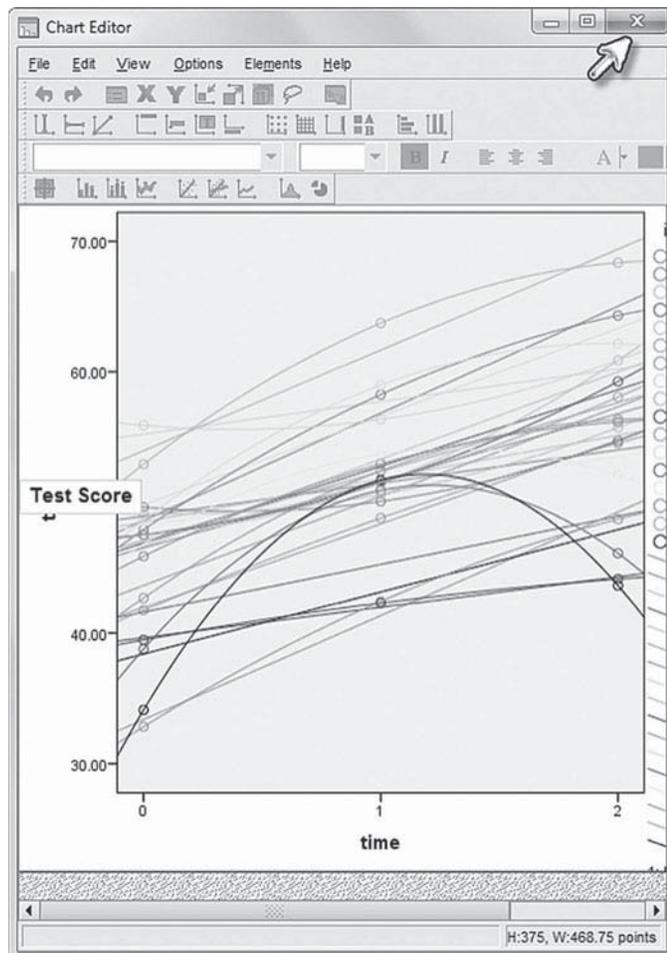
- 9a. To display the 17 subjects using a quadratic trajectory as shown in Figure 5.4, double-click the graph to select the graph's fit lines, which will also open the *Properties* box.
- b. In the *Properties* box, click to select *Quadratic*.
- c. Click the APPLY button to make the change and activate the Close button.
- d. Click the CLOSE button to exit from the *Properties* box.



10. The *Chart Editor* displays the graph's quadratic trajectory.

Graph labels for the *Y* and *X* axes may be changed by clicking and typing the preferred name.

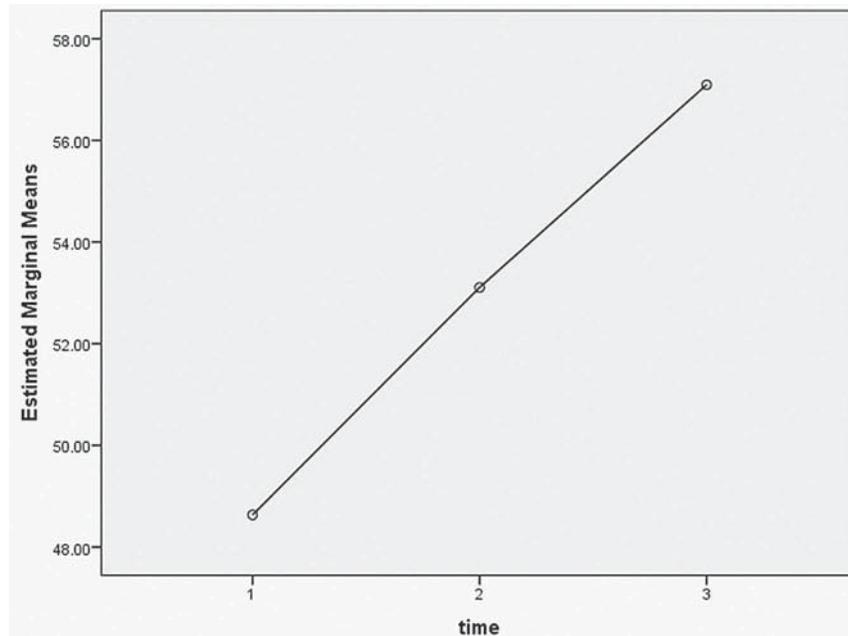
Close the *Chart Editor* box, and return to the IBM SPSS output document by either clicking the "x" located in the upper-right-hand corner or by selecting FILE, CLOSE or CTRL+F4.



### Coding the Time-Related Variables

There are various ways that the time-related within-subjects factor may be coded in defining possible individual changes over time. Careful thought should be given to coding the time variable, as it can affect the interpretation of the model's parameters (Hox, 2010). We first illustrate the polynomial approach for describing individual growth in math. Where there are three repeated measures, the linear time variable (*time*) is most often coded 0 for year 1, 1 for year 2, and 2 for year 3 (0, 1, 2). This coding pattern is useful because it identifies the intercept as students' *initial* (year 1) math achievement level. This approach is often preferred since the intercept can be interpreted as the mean when the predictors in a model are all zero (0). For linear growth, the slope would then be defined as the change occurring between each interval (i.e., between 0 and 1 and between 1 and 2). We can also define a quadratic component (*quadtime*) to capture any changes (acceleration or deceleration) in the rate of change that might occur over the three measurement occasions. We simply "square" the *time* intervals; therefore, *quadtime* is correspondingly coded 0, 1, and 4. If instead we wished to define the intercept as students' ending math achievement status (year 3), we could instead code the linear variable -2, -1, 0, and the quadratic variable would then be -4, -1, 0. In some situations, we might wish to code the time-related variables such that the second measurement (i.e., year 2) represents the intercept.

We can see in Figure 5.5 that the shape of the "average" growth trend is not quite linear. The average trend suggests a slight slowing of the growth rate between the second and third intervals. This suggests that a trajectory with both linear and quadratic components may be necessary to define student growth optimally (hypothesis 2).



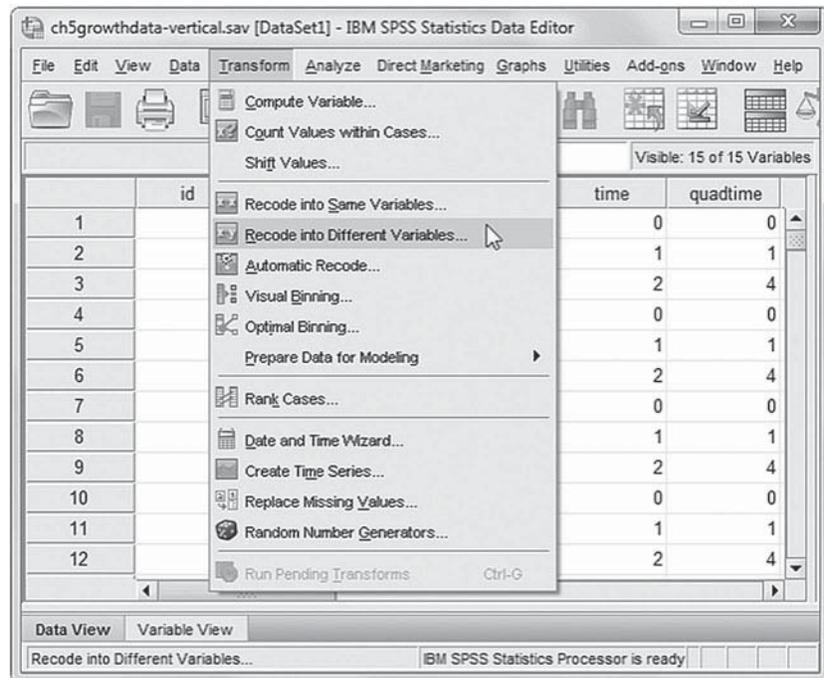
**FIGURE 5.5** Curvilinear average math growth trend.

### Coding Time Interval Variables (time to quadtime) with IBM SPSS Menu Commands

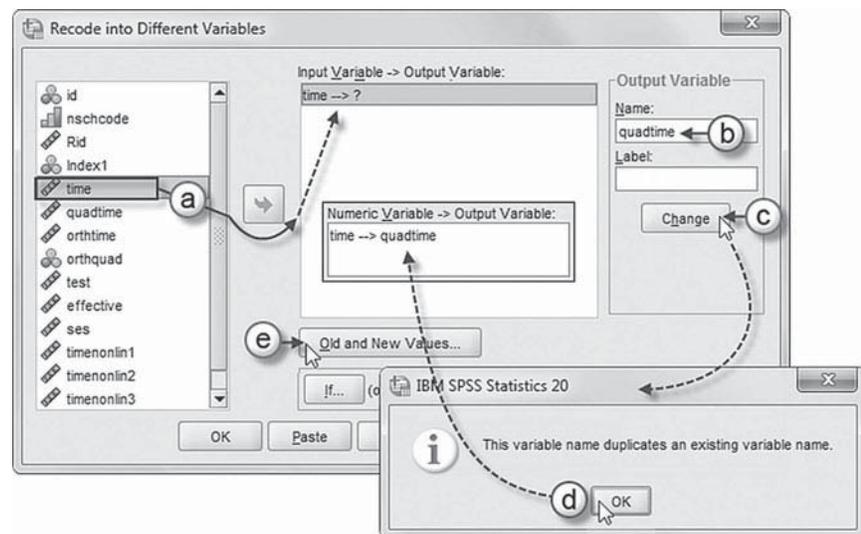
**Note:** Continue using *ch5growthdata-vertical.sav*. If continuing from the prior set of graphing instructions (Figures 5.2, 5.3, and 5.4), remove the *Select Cases* conditional setting before proceeding, as the following models will use all cases in the data set. To clear the filter, go to the toolbar, select DATA, SELECT CASES, RESET, OK.

1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.



- 2a. The *Recode into Different Variables* enables creating a new variable using a variable from the current data set. First, click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.



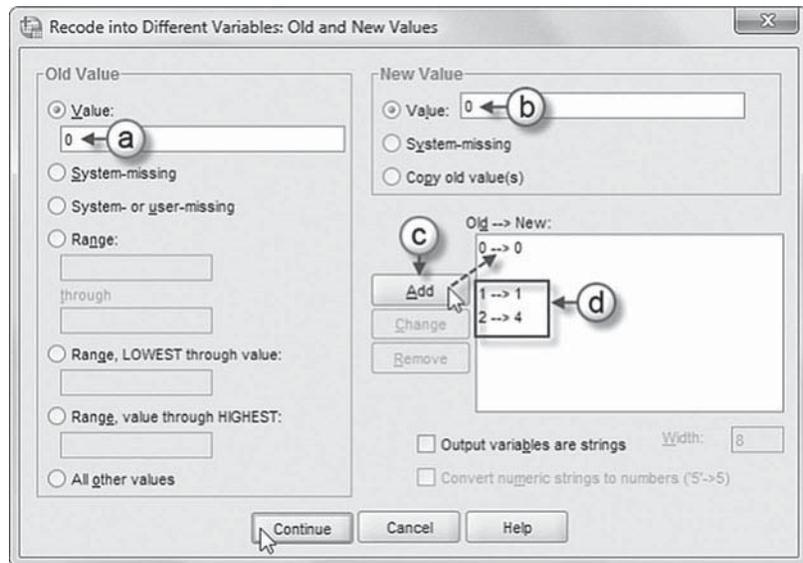
- b. Now enter the new variable name by typing *quadtime* into the *Output Variable Name* box.
- c. Then click the CHANGE button, which will add *quadtime* and complete the RECODE command for *time → quadtime*.

**Note:** A warning message appears as *quadtime* is an existing variable in the data set.

- d. Click OK to continue, which will overwrite the preexisting *quadtime* variable. If you prefer not to overwrite the original variable, rename the output variable (e.g., *quadtime1*).
- e. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

3. Within the *Recode into Different Variables: Old and New Values*, we will begin changing the *time* values (0, 1, 2) to reflect *quadtime* (0, 1, 4).

- Begin by entering the first value for *time* (0) in the *Value* (old) box.
- Next, enter the new value (0) for *quadtime* in the *Value* (new) box.
- Click the button to place the first command  $0 \rightarrow 0$  into the *Old  $\rightarrow$  New* box.
- Repeat steps 3a to 3c to complete the remaining coding changes for *quadtime* values:
  - $1 \rightarrow 1$
  - $2 \rightarrow 4$



Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

Click the OK button to generate the recoded variable *quadtime* and corresponding time values (0, 1, 4).

One disadvantage of using polynomial functions in defining growth trajectories, however, is that there are strong correlations between the components comprising the function (e.g., linear, quadratic, and cubic). This occurs because the components of the polynomial function must be defined as data for each individual within the data set. As we previously described, with three repeated measures, the linear component would be defined as 0, 1, 2 and the quadratic component would be defined as 0, 1, 4. One can see these two components required for defining a curvilinear trajectory will be highly correlated for the individuals in the study. In fact, for the linear and quadratic contrasts the correlation is 0.958. Readers familiar with multiple regression will recognize that this type of strong correlation can potentially cause problems in estimating the model parameters optimally.

If the occasions are equally spaced, as in our example, and there is little or no missing data, one approach for dealing with multicollinearity is to transform the polynomials to be orthogonal, or uncorrelated (Hox, 2010). Polynomials can be transformed to be orthogonal by recoding using available tables for defining orthogonal contrasts. Recoding helps simplify calculations and interpretations involved in polynomial regression. Because of the way the orthogonal polynomials are defined, the intercept for the transformed model (coded 0) will be centered near the middle of the growth sequence (i.e., representing the grand mean instead of initial status). Readers can consult a table of orthogonal polynomials for other numbers of repeated measures (e.g., Guilford & Frunchter, 1978). Although not required, we can then standardize the orthogonal estimates so that they will be on the same scale of measurement (Hox, 2010). As Hox suggests, even in data situations where the repeated measurements may not be exactly spaced, using orthogonal polynomials will tend to reduce any potential multicollinearity problem. Of course, in situations where the higher order polynomial components are not needed, it would not be necessary to use orthogonal transformation.

**TABLE 5.3 Orthogonal Coding for Three and Four Measurement Occasions for Equally Spaced Intervals**

Model Description	k-1 = 2	k-1 = 3
Linear	-1, 0, 1	-3, -1, 1, 3
Quadratic	1, -2, 1	1, -1, -1, 1
Cubic		-1, 3, -3, 1

We summarize orthogonal coding for three and four repeated measures in Table 5.3. For three repeated measures, we can simply use RECODE to transform the linear trend (0, 1, 2) to the orthogonal coding (-1, 0, 1), as illustrated in Table 5.3. This simple transformation creates a grand-mean centering. Centering at the midpoint will minimize the correlation between the time-related variables, which tends to stabilize the model estimation procedure (Raudenbush & Bryk, 2002). The quadratic component is then recoded from 0, 1, 4 to 1, -2, 1. As Raudenbush and Bryk indicate, for higher order polynomials, the highest order coefficient will have an invariant interpretation, while the lower order coefficients will have meanings that depend on the centering strategy employed. We reiterate that in defining the growth trajectory using higher order polynomials, we typically only include up to the highest significant component.

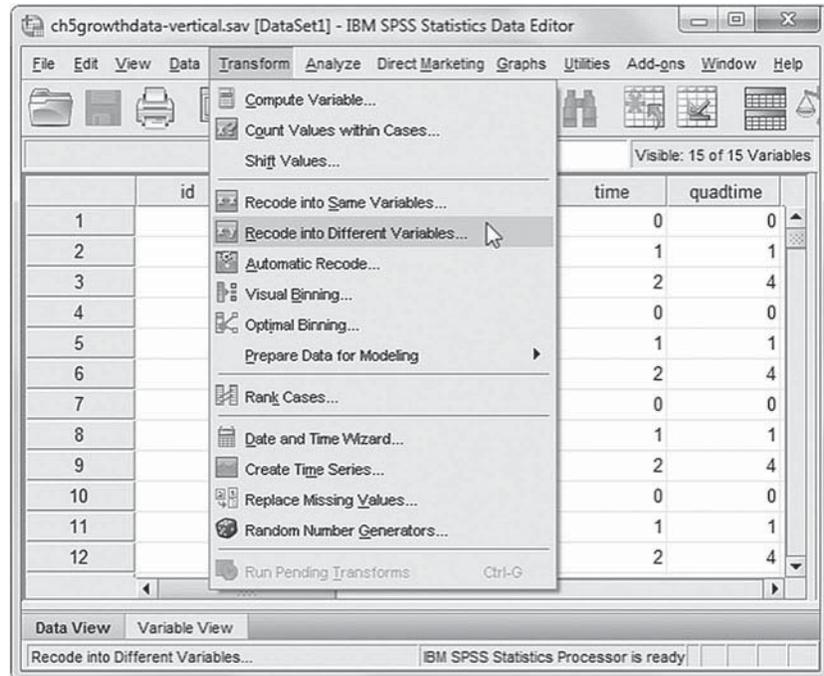
### **Coding Time Interval Variables (*time* to *orthtime*, *orthquad*) with IBM SPSS Menu Commands**

Continue using the *ch5growthdata-vertical.sav* data.

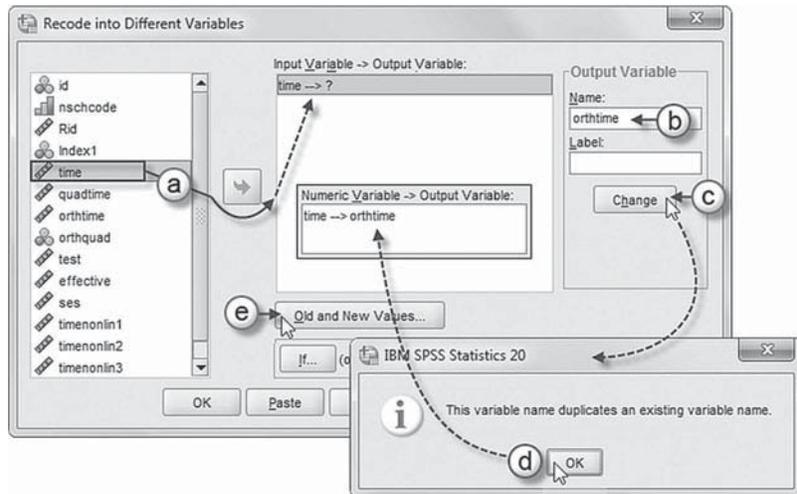
1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.

**Note:** If continuing from performing the prior coding example (*time* to *quadtime*), click the RESET button before proceeding to clear the default settings.



- 2a. The *Recode into Different Variables* screen enables creating a new variable using a variable from the current data set. First, click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.



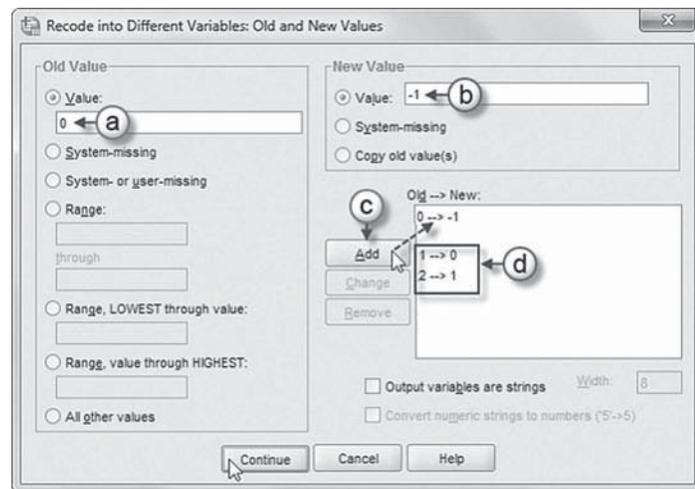
- b. Now enter the new variable name by typing *orthtime* into the *Output Variable, Name* box.
- c. Then click the **CHANGE** button, which will add *orthtime* and complete the **RECODE** command for *time* → *orthtime*.

**Note:** A warning message appears as *orthtime* is an existing variable in the data set.

- d. Click **OK** to continue, which will overwrite the preexisting *orthtime* variable. If you prefer not to overwrite the original variable, rename the output variable (e.g., *orthtime1*).
- e. Click the **OLD AND NEW VALUES** button, which will then display the *Recode into Different Variables: Old and New Values* screen.

3. Within the *Recode into Different Variables: Old and New Values*, we will begin changing the *time* values (0, 1, 2) to reflect *orthtime* (-1, 0, 1).
- a. Begin by entering the first value for *time* (0) in the *Value (old)* box.
- b. Next, enter the new value (-1) for *orthtime* in the *Value (new)* box.
- c. Click the button to place the first command  $0 \rightarrow -1$  into the *Old → New* box.
- d. Repeat steps 3a to 3c to complete the remaining coding changes for *orthtime* values:

$1 \rightarrow 0$   
 $2 \rightarrow 1$



Click the **CONTINUE** button to return to the *Recode into Different Variables* main dialog box.

Click the **OK** button to generate the recoded variable *orthtime* and corresponding time values (-1, 0, 1).

**Note:** To generate *orthquad* (coded 1, -2, 1), repeat all steps but rename the output variable (*orthquad*) and code the *Old* → *New* values as follows:

$0 \rightarrow 1$   
 $1 \rightarrow -2$   
 $2 \rightarrow 1$

A second approach, which we illustrate later in the chapter, is to code the time-related variable in a way that represents the growth taking place over the whole trend, rather than over a specific interval. This is referred to as a level-and-shape model in the SEM literature on LCA (e.g., McArdle & Anderson, 1990; Raykov & Marcoulides, 2006). In the LCA approach, it is possible to estimate more general types of change, where maximum likelihood (ML) estimation is used to provide estimates of some of the factor loadings, which define the shape (or growth) factor (McArdle, 1988; Meredith & Tisak, 1990). One of the advantages of the latent variable approach to modeling change is that it permits development in the repeated measures variable to occur in any fashion (e.g., growth followed by decline followed by growth). Importantly, this makes it generally easier to fit empirical data than many other models where growth is assumed to follow a particular function (Raykov & Marcoulides, 2006). Although it is not possible to provide all of these possibilities using a mixed model approach, since the repeated measures must be specified as series of data points occurring within the specific temporal period of the study, there is one often used SEM specification that can be adapted. More specifically, since the slope is interpreted as the change in  $Y$  for a unit change in  $X$ , if we code the first measurement occasion as 0 and the last measurement occasion as 1, we can then describe the slope as the change in  $Y$  occurring over the entire trend for a unit change in  $X$ . We then use the middle measures to estimate the general *shape* of the trend in between these two endpoints.

If we obtain a graph of the average growth trajectory for the sample or several representative individual trajectories, we can often experiment a little to find an appropriate coefficient for the middle measurement occasion (or occasions). For example, we might start by coding the middle measurement in our example (year 2) as 0.5 if the trend is assumed to be linear, or some other value (e.g., 0.7) if a nonlinear trend is hypothesized. This strategy can of course be adjusted if there are several repeated measures to be specified between the first and last measurement. We can save predicted values from the estimated model using various coding schemes and generate a new graph to see which coding scheme seems to best capture the actual shape of the observed data. We can also examine various model fit indices to determine which coding approach (or implied hypothesis about the general shape of the growth) may capture the individual growth trajectories most accurately. As Raykov and Marcoulides (2008) caution, the downside of this approach is that in some instances one may not be able to obtain a specific quantitative description of the development occurring over the timeframe under consideration.

Various curved or *S*-shaped polynomial growth trajectories create additional challenges in terms of interpretation and model building. It is also possible to treat the time variable as categorical and model each occasion separately either by using a reference occasion or by eliminating the intercept in the model so that each occasion can be modeled separately (see Hox, 2010, for further discussion). We emphasize that this type of examination of the time-related variable is typically conducted preliminarily before settling on a final set of coefficients that describe individual growth and then investigating a subsequent set of predictors that might explain variability in individuals' growth trajectories. The important point is that various ways of defining individuals' growth trajectories (e.g., alternative ways of coding the time-related variable) may result in somewhat different estimates of individual growth, owing to the underlying assumptions of each type of statistical model.

### Specifying the Two-Level Model of Individual Change

After setting up the data set appropriately and considering possible ways to code the time-related variables, we are ready to build a series of models. At Level 1, each person's successive measurements over time are defined by an individual growth trajectory and random error. At Level 2, differences in trajectories between groups of individuals can be examined. Following Raudenbush and Bryk's (2002) notation, we will use two subscripts to describe individuals ( $i$ ) and occasions of measurement ( $t$ ). We assume the observed status,  $Y_{it}$ , at time  $t$  for individual  $i$  is

a function of a systematic growth trajectory plus random error. At Level 1, the systematic growth for each individual in reading can be represented as a polynomial of degree  $P$  for  $k - 1$  repeated measures. In this example, with three measurements, the highest polynomial will then be 2 ( $3 - 1 = 2$ ), or quadratic, with the Level 1 model at time  $t$  for individual  $i$  written as

$$Y_{it} = \pi_{0i} + \pi_{1i}a_{it} + \pi_{2i}a_{it}^2 + \varepsilon_{it} \quad (5.1)$$

where  $a_{it}$  and  $a_{it}^2$  are time-varying variables of interest (e.g., which are coded to indicate the linear and quadratic components hypothesized to describe the shape of the trajectories);  $\pi_{0i}$  is an intercept;  $\pi_{1i}$  and  $\pi_{2i}$  describe the linear and quadratic growth rates, respectively; and  $\varepsilon_{it}$  represents variation in estimating growth within individuals. We note that in specifying a growth model, we often use the Greek letter pi to represent the Level 1 coefficients, so that we can maintain the typical beta coefficients to describe between-individual relationships and gamma coefficients to describe between-group relationships. Because we have coded the first repeated measure as 0, the intercept parameter (i.e., the point where the trajectory crosses the  $Y$  axis) is interpreted as the child's true score at initial status (or the beginning) of the study. As Singer and Willet (2003) note, the Level 1 model assumes that all the individual change trajectories have the same algebraic form in Equation 5.1, but not every individual has exactly the same trajectory.

The slope parameters ( $\pi_{1i}$  and  $\pi_{2i}$ ) represent the predicted change in individuals over a specified time interval. The linear component describes the rate of change per unit of time. A quadratic component can be interpreted as a "change" in the rate of change (e.g., accelerating or decelerating). As shown in Figure 5.5, the linear component of the time-related variable ( $a$ ) is coded 0, 1, 2 in the data set (referred to as "time"), which ensures that the intercept is interpreted as students' true initial status (i.e., their corrected achievement level at Time 1). We can also add a quadratic component to the model to test for a change in the rate of growth over time. This component is also coded as a variable (*quadtime*). The interval values for the variable can be generated using COMPUTE and multiplying the time variable by itself (*time\*time*).

If this coding scheme is used, the linear component ( $\pi_{1i}$ ) represents the yearly growth rate for each child in the study. The quadratic component ( $\pi_{2i}$ ) represents any increase or decrease in the rate of change for each time interval. Alternatively, the time-related variable might also be conceptualized as students' age in months at the time of each measurement. The intercept and slope coefficients represent the model's *structural*, or fixed, effects. As mentioned previously, we have also created orthogonal polynomials in our example data in order to remove the sizable correlation between the linear and quadratic components of individuals' growth trajectories. We have added these orthogonal linear and quadratic components into the data set as *orthtime* and *orthquad*, respectively.

We can also specify one or more *time-varying* covariates ( $X_t$ ) at Level 1. Time-varying covariates (i.e., predictors that also change over repeated measurement occasions) provide a way of accounting for temporal variation that may increase (or decrease) the value of an outcome predicted by the individual's growth trajectory (Raudenbush & Bryk, 2002). For example, we could consider a situation where a variable like motivation, which changes over time, might also affect students' learning in math or reading. Alternatively, we could also consider motivation as a static variable (i.e., individuals' average motivation level), which requires only one value for the covariate. In this latter case, we would then enter the covariate at the between-subjects level (i.e., Level 2). One of the advantages of the time-varying formulation at Level 1, however, is that the effect of motivation level on student achievement can then be modeled as a random parameter at Level 2.

## Level 1 Covariance Structure

The other part of the Level 1 model is the *stochastic* part, or the part that describes the variation in measuring each individual  $i$  on occasion  $t$ . This part of the model implies that there is some error ( $\varepsilon_{it}$ ) associated with measuring each individual's true growth trajectory. The errors are *unobserved*, which means that we must make some assumptions about their distribution at Level 1 (Singer & Willett, 2003). Often, a simple residual structure is assumed from occasion to occasion and person to person, with each error independently and normally distributed, a mean of 0, and constant variance:

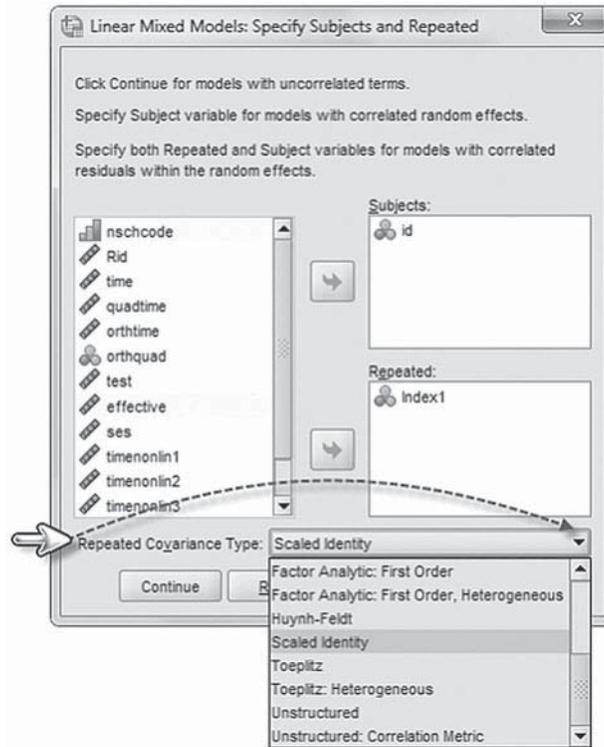
$$\varepsilon_{it} \sim N(0, \sigma_{\varepsilon}^2) \quad (5.2)$$

where  $\sim$  means “distributed as,”  $N$  refers to normal distribution, 0 refers to the mean, and  $\sigma_{\varepsilon}^2$  refers to the variance. One way to represent this simplified error structure is as a scaled identity matrix, which provides a single residual variance associated with measurement occasions. With longitudinal data, however, this type of simple Level 1 error structure may have less credibility (Singer & Willett, 2003).

Restrictions about the within-individual residuals over time can be relaxed. This is often necessary because the error covariance matrix for the repeated measures typically will exhibit some correlation between occasions. For example, often the repeated measures will correlate more strongly when they are taken closer together and less strongly as the time interval increases. It is often useful to examine different error structures (e.g., compound symmetry, autocorrelated, or unstructured) in preliminary analyses, depending on the nature of the repeated measurements per subject. The fit of particular covariance structures to the data can then be compared to see which covariance structure provides the best overall choice. IBM SPSS provides a considerable number of choices for the Level 1 residual covariance matrix in a repeated measures model (a complete list of covariance structures can be obtained from the MIXED Commands in the IBM SPSS Help Menu).

## Repeated Covariance Dialog Box

The Level 1 covariance matrix can be specified by opening the *Repeated Covariance* dialog box in the Commands menu (Figure 5.6) or with a REPEATED syntax command. There are actually a number of different uses for this dialog box, which we will briefly summarize. The first is the typical growth specification we have just described, where this is used to specify the individual variation around the repeated measures of  $Y$ . A second use is when the focus is on defining a multivariate model, for example, when we have several survey items used to define an underlying (or latent) dependent variable such as job satisfaction. The individual items defining the construct can be specified vertically for each individual in the data set using an *index* variable and the *Repeated Covariance* dialog box used to describe variance and covariance relationships between the items, which is similar to representing repeated measures over time. We cover these multivariate situations in more detail in Chapter 7. A third use would be where the researcher has different amounts of data on an outcome for individuals at Level 1. For example, workers might be measured performing a series of work-related tasks. One individual might have one such measure of task performance, while others might have five or six (or even more) measures. This specification amounts to having multiple pieces of data nested within individuals at Level 1, but not incorporating a growth parameter in the sense that individuals are assumed to be changing in performance over time. In this case, the *Repeated Covariance* dialog box is useful in organizing the differing amounts of performance information regarding individuals in the study.



**FIGURE 5.6** The *Repeated Covariance Type* dialog box and available covariance structures.

We note that if the *Repeated Covariance* dialog box (or repeated syntax statement) is not used, the default Level 1 matrix will be a scaled identity covariance matrix. As Equation 5.3 indicates, the scaled identity matrix (abbreviated as ID in MIXED syntax) assumes a constant variance across occasions, where  $\sigma^2$  is the variance, and no covariances between occasions. Therefore, it has only one estimated parameter, as suggested in Equation 5.3. We note that because a covariance matrix is a square matrix, the same elements appear above and below the diagonals. This simplified within-subject error structure may sometimes be sufficient for repeated measures studies of short duration, if the focus is not primarily on defining the relationships between successive measurements:

$$\sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5.3)$$

Specifying an identity covariance matrix for the repeated measures structure amounts to accepting that Mauchly's test for sphericity holds.

A special form of sphericity is *compound symmetry* (abbreviated CS) or *uniformity* (Hox, 2010). Conditions for compound symmetry are met if all the variances are equal in the population being sampled and all the covariances (the off-diagonal elements of the covariance matrix) are equal. This means there is one variance and one constant covariance in the Level 1 covariance matrix (or one more parameter to estimate than in Eq. 5.3). If the observed covariances are roughly

equal and the variances are similar too, we can generally assume that compound symmetry is not violated and, therefore, sphericity should not be a problem (Raykov & Marcoulides, 2008). It turns out that for a two-level model of repeated measures with random intercept only, the residual variance at any occasion can be defined as  $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$  (i.e., the sum of the occasion-level and person-level residual variances, respectively) in the diagonals of the matrix, and the covariance between any two occasions (i.e., the off-diagonal elements) is  $\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{bmatrix}$ . For this simple type of repeated measures model, the matrix of variances and covariances among occasions would then be defined as follows:

$$\begin{bmatrix} \sigma_\varepsilon^2 + \sigma_{u_0}^2 & \sigma_1 & \sigma_1 \\ \sigma_{u_0}^2 & \sigma_\varepsilon^2 + \sigma_{u_0}^2 & \sigma_1 \\ \sigma_{u_0}^2 & \sigma_{u_0}^2 & \sigma_\varepsilon^2 + \sigma_{u_0}^2 \end{bmatrix} \quad (5.4)$$

This suggests that the typical linear trend two-level model with single residual term at the occasion (Level 1) and person (Level 2) levels is the same as assuming compound symmetry in the univariate repeated measures ANOVA model (Hox, 2010; Raudenbush & Bryk, 2002). In fact, for a simple linear two-level model with random intercept only, specifying an ID covariance matrix at Level 1 (as in Eq. 5.3) and Level 2 (since there is one random effect) will produce an identical model to specifying CS at Level 1 and ID at Level 2.

An alternative Level 1 structure assuming different variances across measurement occasions could also be summarized as a diagonal (DIAG) covariance matrix. This type of covariance matrix assumes heterogeneous variances for each measurement occasion in the diagonals of the matrix and 0s for the off-diagonal elements, which indicates no covariances between occasions:

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} \quad (5.5)$$

Relative to the scaled identity or compound symmetry covariance matrices, however, the diagonal covariance matrix will have more parameters to estimate for the assumed heterogeneous variances. The limitation of this type of covariance structure is also that it assumes no relationship between measurement occasions (similar to the scaled identity matrix).

Often we find that the repeated measures structures for longitudinal data may have a complex covariance structure. This autoregressive error covariance matrix (AR1) assumes that the Level 1 variance remains constant across occasions but facilitates specifying an autocorrelation coefficient between occasions. The autocorrelation coefficient rho ( $\rho$ ) represents the correlation between any two adjacent occasions, where  $|\rho| \leq 1$ . It follows, then, that  $\rho^2$  represents the correlation when there is a skip between occasions. This structure is useful when it is likely that the correlations become weaker as there is longer distance in time between them:

$$\sigma_\varepsilon^2 \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix} \quad (5.6)$$

The autoregressive covariance structure differs from compound symmetry, which assumes that the covariance between residuals is the same despite the time lag between them. One advantage of the autoregressive covariance structure is that it can be specified with only two estimated parameters (i.e., a variance parameter and the correlation parameter). This makes it a relatively simplified covariance structure, but one that does not assume that all covariances between occasions are the same. We note that it is also possible to assume heterogeneity among the occasion variances; that is, the diagonal elements can be replaced with separate variance estimates in the autoregressive structure (abbreviated as ARH1).

In contrast to an autoregressive error structure, a completely *unstructured* (UN) covariance matrix provides separate occasion variance estimates in the diagonals and separate covariances estimated for the off-diagonal elements:

$$\begin{bmatrix} \sigma_1^2 & \sigma_{21} & \sigma_{31} \\ \sigma_{21} & \sigma_2^2 & \sigma_{32} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix} \quad (5.7)$$

This type of error structure, however, can become overly complex when the number of measurement occasions increases beyond three or four.

A general goal is to identify a parsimonious covariance structure that will adequately describe the data, both at Level 1 and Level 2. In this example, we will first assume an identity Level 1 covariance matrix within individuals to examine how much variance in the outcome lies within and between individuals. MIXED provides several fit indices (e.g., AIC and BIC) that can also be used to evaluate various combinations of fixed effects and covariance structures. In simulation studies of growth curve modeling, the AIC has been noted to work well in selecting the true model, provided the sample size is not below 100 individuals (Liu, Rovine, & Molenaar, 2012). We discuss these indices later in this chapter.

More complex covariance structures can also make it more difficult to arrive at a solution that converges (i.e., provides reasonable estimates for all proposed parameters). Often, it is necessary to exercise some type of compromise between the complexity of some covariance structures and the parsimony provided by others in arriving at solution that defines model covariance structures adequately. We illustrate some of these differences in Level 1 covariance matrices subsequently. As Hox (2010) notes, if the focus is primarily on the model's fixed effects, one can often assume a more simplified variance and covariance structure across occasions, as some misspecification in the random part of the model does not generally affect the model's fixed effects (see also Verbeke & Lesaffre, 1997). In some circumstances, however, different user choices regarding the error variance structure at Level 1 may affect the outcome of tests for random effects at higher levels.

### Model 1.1: Model with No Predictors

There is some difference of opinion among researchers about what is the proper model to begin with in building a growth model. We can start with a “no-predictors” model if desired. However, when we calculate the intraclass correlation (within individuals = Level 1; between individuals = Level 2) we will only get a rough estimate since the within-individual variance may be different at each measurement occasion. Moreover, this initial estimate of the variance components may not be very accurate. This is because when the variance components are initially estimated, the estimation procedure is assuming *random sampling* at both levels. What this means is that the initial estimate of the variance in the outcome at the between-individual level (Level 2) can ignore possible variation in the between-individual variance component that may be due

to additional within-individual variance that actually exists. If this occurs, the initial between-individual estimate must be corrected by the ML estimation procedure as subsequent variables are added (Hox, 2010).

In this first type of null model, for example, we could test whether the grand-mean intercept for math varies across individuals. From Equation 5.1, this is simply defined at Level 1 without the time-related variables as follows:

$$Y_{ti} = \pi_{0i} + \varepsilon_{ti}, \quad (5.8)$$

where  $\pi_{0i}$  is the average achievement across the three occasions, and  $\varepsilon_{ti}$  represents errors in predicting the average achievement for individuals. Between individuals, we can describe the average growth across occasions as

$$\pi_{0i} = \beta_{00} + u_{0i}, \quad (5.9)$$

where  $\beta_{00}$  is the intercept describing the average initial status mean between individuals, and  $u_{0i}$  is the Level 2 random component associated with describing differences in average achievement between individuals. Substituting Equation 5.9 into Equation 5.8, we would arrive at the combined equation

$$Y_{ti} = \beta_{00} + u_{0i} + \varepsilon_{ti}, \quad (5.10)$$

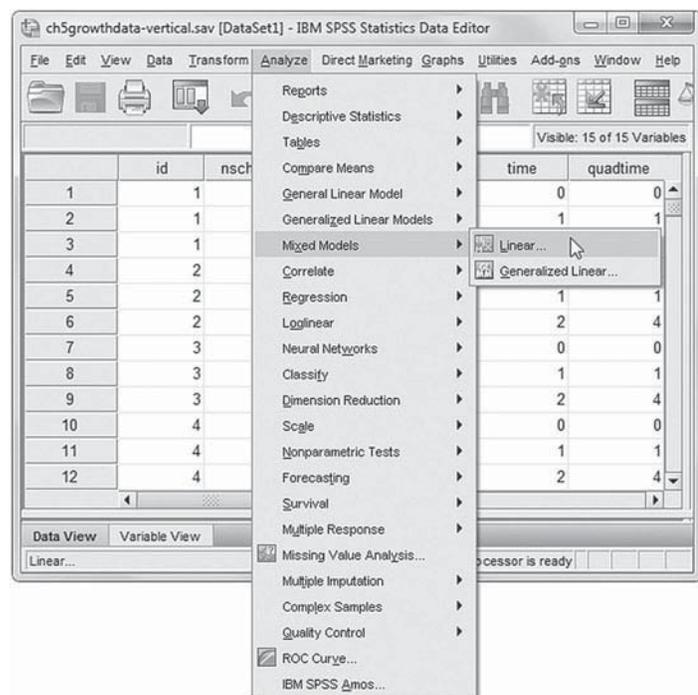
which indicates three parameters to estimate. These include the fixed effect describing average math achievement, the between-individual random variance, and the Level 1 residual variance.

### Defining Model 1.1 (Null) with IBM SPSS Menu Commands

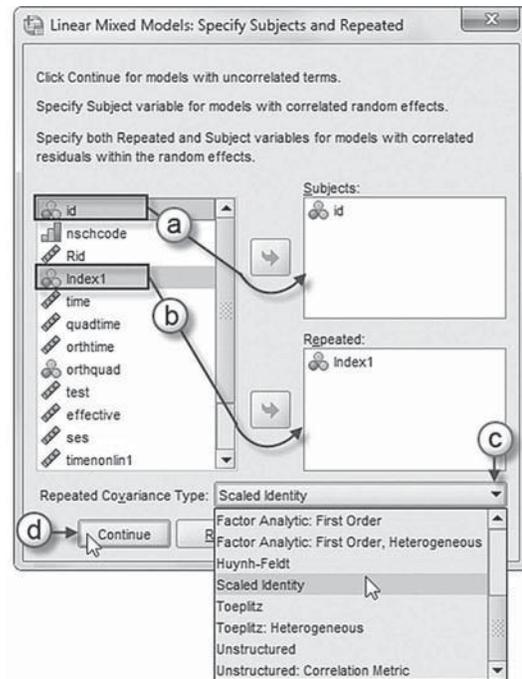
Continue using *ch5growthdata-vertical.sav*.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



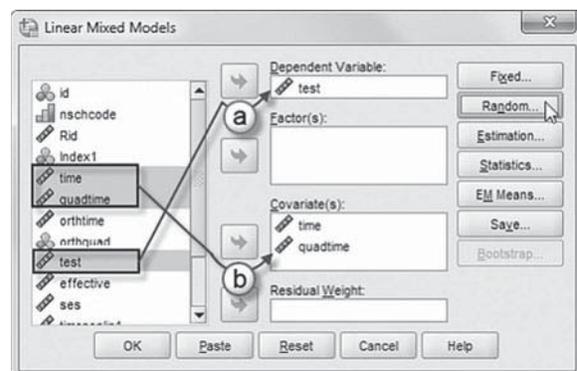
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.
  - a. A subject is an observational unit that may be independent of other subjects. For this model, we will designate *id* (individual identification numbers) as the subject variable. Click to select *id* from the *Variables* column, and then click the right-arrow button to move the variable into the *Subjects* box.
  - b. The *Repeated* box allows specifying variables that identify repeated observations. For this model, *Index1* identifies repeated observations over three time periods. Click to select *Index1*, and then click the right-arrow button to move the variable into the *Repeated* box.



The combination of values for *id* and *Index1* defines a particular student across three time periods.

- c. The *Repeated Covariance Type* specifies a model's covariance structure. For this model, we will use the *Scaled Identity* (ID) covariance type. Click the pull-down menu and select *Scaled Identity*. The ID structure has constant variance and assumes that no correlation occurs between elements.
  - d. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, covariates, and access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.
  - a. For this model, we will use math achievement (*test*) as the dependent variable. Click to select the *test* variable from the left column listing. Then click the right-arrow button to transfer *test* into the *Dependent Variable* box.
  - b. The null model does not have predictors, but since we will be designating a random effect in step 3, we will need to introduce variable(s) at this point as factors or covariates as a workaround. (Omitting this step will prevent specifying a random effect.)



Factors and covariates may be specified in predicting the dependent variable. Factors are categorical predictors that may be numeric or string. Covariates are scale predictors that must be numeric.

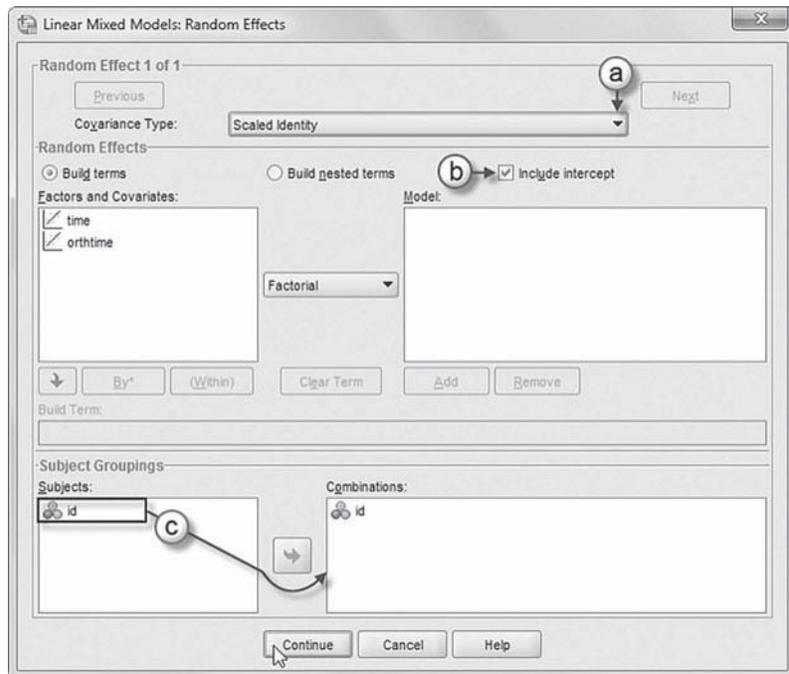
We will designate two predictor variables, which will be used later in the upcoming model (Model 5.1A). Locate and click the variables *time* and *quadtime* from the left column listing, and then click the right-arrow button to move the variables into the *Covariate(s)* box.

Since we are not defining effects for the two predictors, we will skip over the FIXED effects button and go to set up the model's random effects.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* screen allows specifying random effects, interactions, in-intercept terms, and subject groupings.

- Begin by specifying the covariance structure from the default variance components (VC) to scaled identity. Click the pull-down menu and select *Scaled Identity* (ID). The scaled identity structure has constant variance and assumes that no correlation occurs between elements.
- We want the intercept to be included in the model, so click *Include intercept*.
- The *Subject Groupings* box displays the *id* variable that was selected as a subject variable in the *Select Subjects and Repeated* dialog box show in step 2a. We will specify *id* as the subject for the random-effects part of this model. Click to select *id*, and then click the right-arrow button to move the variable into the *Combinations* box.

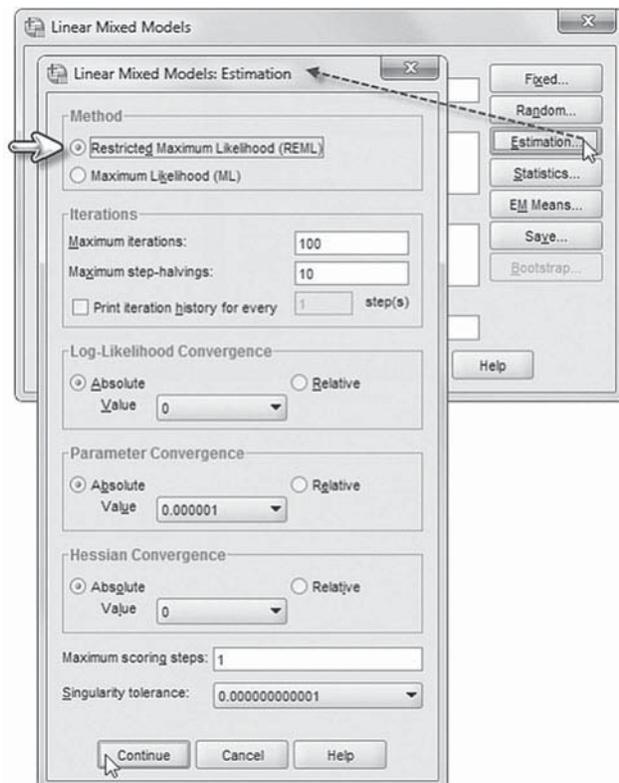


Click the CONTINUE button to return to the *Linear Mixed Models* main dialog box.

5. The *Linear Mixed Models: Estimation* dialog box displays two estimation method choices: ML or restricted maximum likelihood (REML).

In this chapter, we will use the default setting of REML to estimate the models.

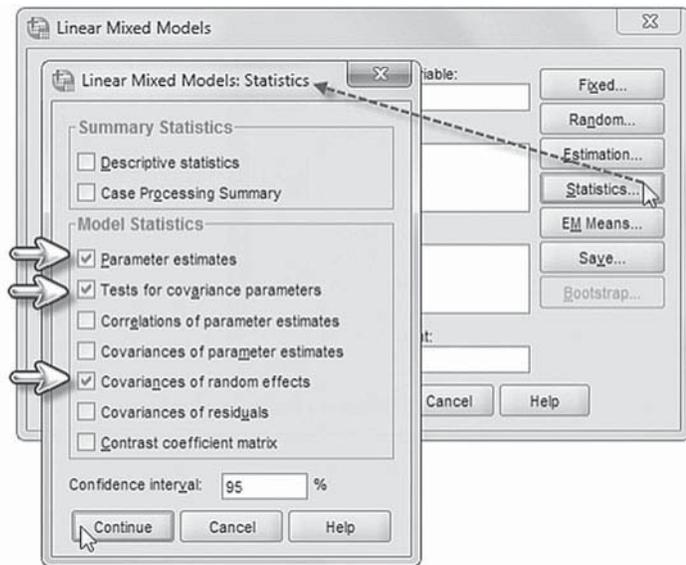
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



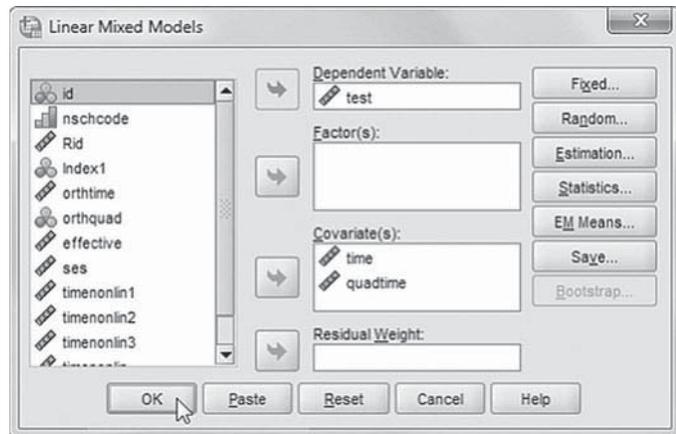
6. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### **Interpreting the Output From Model 1.1 (Null)**

The grand mean for achievement is 52.945 (not tabled), as summarized previously in Table 5.2. The variance component table (Table 5.4) can be used to determine how much variability in math achievement is present at each level. At Level 1, the variance ( $\sigma^2_{\epsilon}$ ) summarizes the population variability in the average individual's achievement estimates around her or his own true growth trajectory (Singer & Willett, 2003). The estimate is 78.101. The Level 2 variance is 30.505 (Wald  $Z = 33.777, p < .001$ ), which suggests there is sufficient variation in intercepts across individuals. The null hypothesis is that the population parameter for the variance is 0 (Singer & Willett, 2003). Keep in mind that the Wald  $Z$  statistic provides a two-tailed test and because the null hypothesis is that the population variance is 0, we should use a one-tailed test for variances. As a rough estimate, we can calculate the proportion of variance in math achievement that is between individuals as about 0.281 [ $30.505/(30.505 + 78.101)$ ], or 28.1%.

TABLE 5.4 Initial Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.
Repeated Measures	Variance	78.101	0.839	93.113	.000
Intercept [subject = id]	Variance	30.505	0.903	33.777	.000

<sup>a</sup> Dependent variable: test.

### Model 1.1A: What Is the Shape of the Trajectory?

Alternatively, we can start with Model 1.1A as a model that includes the time-related variables (*time* and *quadtime*). If one begins with the linear component and quadratic components, and they are not transformed, this model will provide a variance component for the intercept at Time 0 (at initial status). The Level 1 model is specified in Equation 5.1, where we have defined linear and quadratic time-related components describing the shape of individual growth in math achievement over the three time points. In describing the shape of the growth trajectory, we can also estimate  $k - 1$  random effects. Since there are three time points in this example, we can estimate two random effects. This allows us to estimate a Level 2 randomly varying intercept and either a randomly varying linear component or quadratic component.

In this model, we will initially treat both time-related components as fixed (i.e., with no  $u_{2i}$  and  $u_{2i}$  residual variances) between individuals:

$$\pi_{1i} = \beta_{10} \quad (5.11)$$

$$\pi_{2i} = \beta_{20}. \quad (5.12)$$

Substituting the Level 2 equations (Eq. 5.10 for the intercept and Eqs. 5.11 and 5.12 for the time-related slopes) into the Level 1 equation (Eq. 5.1) results in the following combined equation:

$$Y_{ti} = \beta_{00} + \beta_{10}a_{ti} + \beta_{20}a_{ti}^2 + u_{0i} + \varepsilon_{ti} \quad (5.13)$$

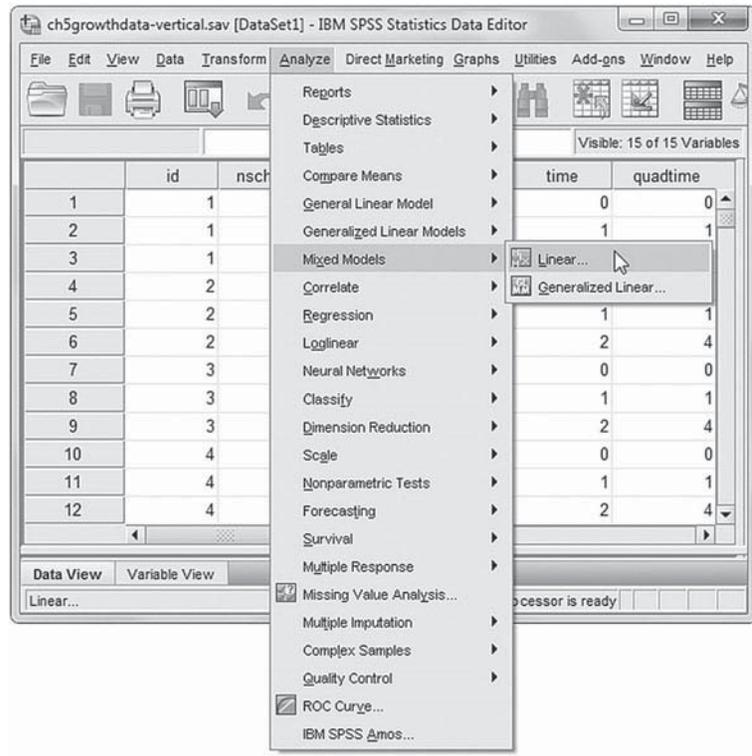
This suggests five parameters to estimate including three fixed effects, one random intercept ( $u_{0i}$ ), and the Level 1 residual ( $\varepsilon_{ti}$ ).

### Defining Model 1.1A with IBM SPSS Menu Commands

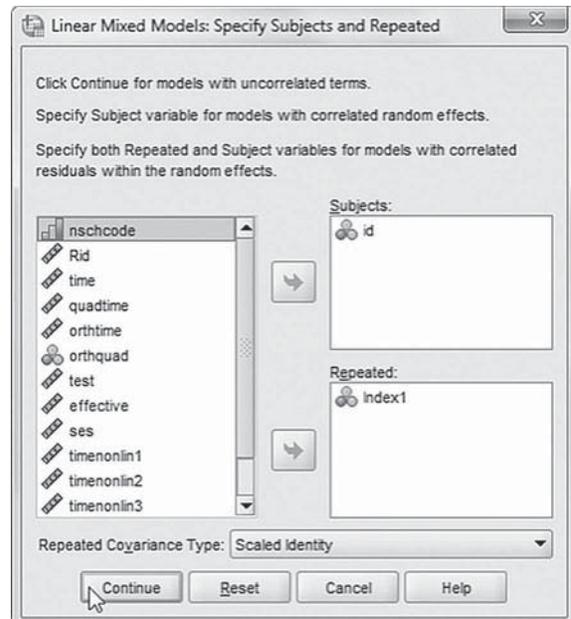
Continue using the *ch5growthdata-vertical.sav* data. Settings will default to those used in Model 1.1 (Null).

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



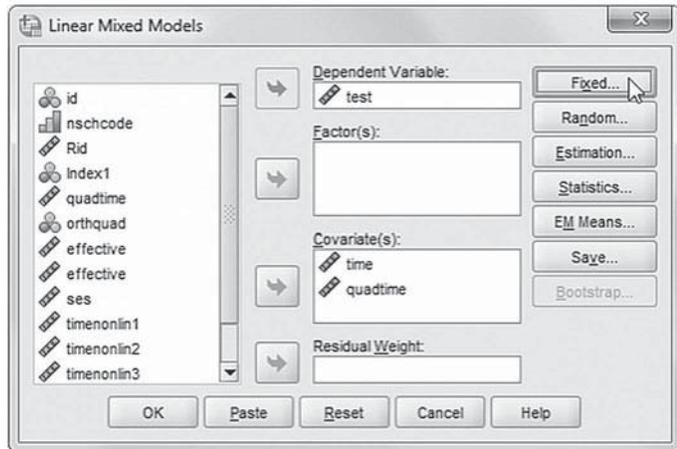
2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main dialog box displays *test* as the dependent variable and the predictor variables *time* and *quadtime* as covariates.

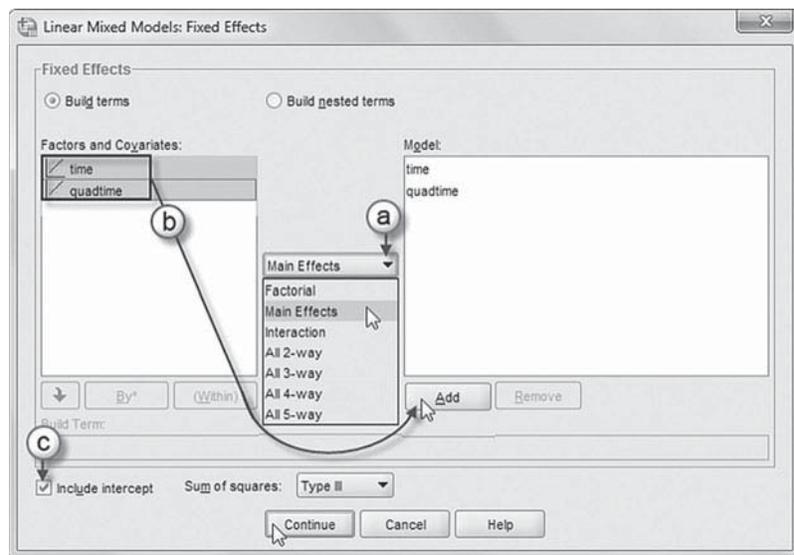
Once the predictor variables have been specified, we may now proceed to define fixed effects for the variables.

Click the **FIXED** button to access the *Linear Mixed Models: Fixed Effects* dialog box.



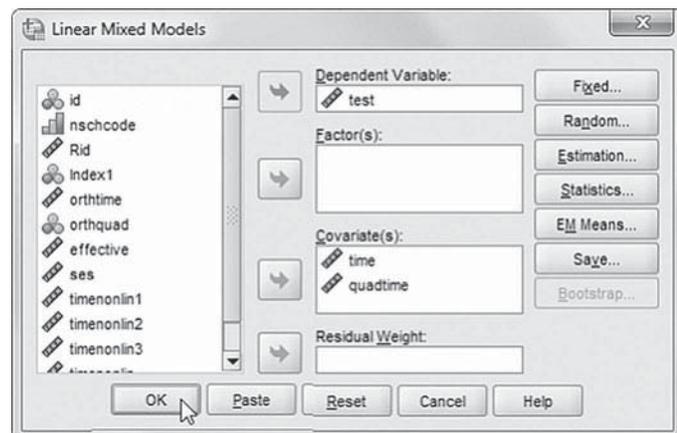
- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.

- b. Click to select *time* and *quadtime* from the *Factors and Covariates* box, and then click the **ADD** button to move the variable into the *Model* box.
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the **OK** button to run the model.



**Interpreting the Output From Model 1.1A**

The fixed effects are presented in Table 5.5. We can observe that both the linear and quadratic polynomials are significant in explaining student growth in math, which suggests that both should be retained in subsequent analyses. The linear component is the portion of the sum of squares (SS) attributable to the linear regression of  $Y$  on  $X$ , and the quadratic component measures the additional improvement in fit due to that component. Notice that the intercept corresponds to students' math achievement level at the beginning of the study. We can use the estimates in Table 5.5 to obtain the observed means for the second and third intervals summarized in Table 5.2.

Next, readers will notice in Table 5.6 that when we include the time-related variables in the initial model, the within-individual variance is reduced from 78.10 in Table 5.4 to 60.19. The estimate of the between-individual variance, however, is actually a little larger (36.48) than the initial estimate in Table 5.4 (30.51) with no time-related parameter in the model. This would seem to be explaining "negative" variance between individuals (at Level 2) since the variance component is reduced by adding the time-related variables. As we have noted previously, this problem regarding negative variance is actually quite common in multilevel analyses of repeated measures. It often occurs because the variability among subjects in the repeated measures portion (Level 1) of the outcome is usually much larger than the sampling model assumes (Hox, 2010). This leads to overestimating occasion variance and underestimating between-individual (Level 2) variance in the intercept-only model. If we calculated the proportion of the variance between individuals based on the variance components in Table 5.6 for Model 1.1A, we would find that the between-individual proportion of the variance in math achievement is now 0.377 (36.477/96.664), or almost 38%, rather than 28% from Model 1.1 with no predictors.

**TABLE 5.5 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	48.632	0.106	20,242.098	460.579	.000	48.425	48.839
time	4.719	0.212	17,338.000	22.216	.000	4.303	5.135
quadtime	-0.244	0.102	17,338.000	-2.391	.017	-0.444	-0.044

<sup>a</sup> Dependent variable: test.

**TABLE 5.6 Estimates of Covariance Parameters Including Time Variables<sup>a</sup>**

Parameter		Estimate	Std. Error	Wald Z	Sig.
Repeated Measures	Variance	60.187	0.646	93.107	.000
Intercept [subject = id]	Variance	36.477	0.885	41.198	.000

<sup>a</sup> Dependent variable: test.

### Does the Time-Related Slope Vary Across Groups?

We emphasize there is no “one way” to proceed in developing an initial growth model. We generally favor this latter approach (Model 1.1A), where the analysis begins with focusing on whether the means of the repeated measures differ across time and defining the shape of the growth trajectory. This initial model could include both the linear and quadratic components since both were found to be significantly related to student growth. Therefore, our first model focuses on defining the shape of students’ growth trajectories and determining whether the intercept and slopes vary across individuals. We can now test whether the time-related slopes are randomly varying across individuals at Level 2:

$$\pi_{1i} = \beta_{10} + u_{1i}, \quad (5.14)$$

$$\pi_{2i} = \beta_{20} + u_{2i}, \quad (5.15)$$

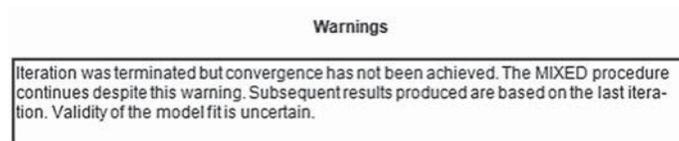
where the  $\beta$  coefficients represent the intercepts, and the  $u$  coefficients represent the variance estimates for the equations. We note that determining which time-related component or components to use in building the Level 2 explanatory models can take a bit of trial and error. It is generally the case that alternative models with various trajectory shapes and covariance structures should be preliminarily investigated in order to arrive at an optimal solution (e.g., see Liu et al., 2012).

As we have noted, when we estimate a polynomial growth model, we need to keep in mind that we are limited in the number of random effects by the number of repeated measures. We can estimate a random effect for the intercept and linear or quadratic term, but not both. If we choose the linear term, this means we will need to assume that the slowing indicated by the quadratic parameter is the same for every person in the sample. Therefore, it is often the choice to consider the quadratic as a fixed effect within individuals in subsequent models to explain differences in students’ growth trajectories. Since it does not vary across individuals, we can fix its variability at 0 by removing the random term ( $u_{2i}$ ), as in Equation 5.12. If we do try to estimate the quadratic effect also, we receive the warning message in Figure 5.7 regarding the model’s failure to converge on a solution.

With the quadratic component fixed, through the substitution of Equations 5.10 (intercept) and 5.14 and 5.12 (time-related slopes) into Equation 5.1, we arrive at the single-equation model for examining the fixed and random components without Level 2 predictors:

$$Y_{it} = \beta_{00} + \beta_{10}time_{it} + \beta_{20}quadtime_{it} + u_{1i}time_{it} + u_{0i} + \varepsilon_{it}. \quad (5.16)$$

Equation 5.16 suggests that the intercept and linear component (time) are randomly varying across individuals. Because the quadratic component is merely defining the shape of growth



**FIGURE 5.7** Warning message.

within individuals, this suggests building the explanatory model for student growth rates on the randomly varying linear component.

## Level 2 Covariance Structure

It is also possible to define different covariance structures at successive levels of the proposed model. For Level 2, the dimensionality of the covariance matrix describing the variances and covariances between random effects depends on the number of random effects in the model. In this case, because we are treating the quadratic component as fixed across individuals due to our preliminary examination, we will assume a  $2 \times 2$  *unstructured* covariance matrix of random effects for the intercept ( $I$ ) and slope ( $S$ ) at Level 2:

$$\begin{bmatrix} \sigma_I^2 & \sigma_{I,S} \\ \sigma_{I,S} & \sigma_S^2 \end{bmatrix} \quad (5.17)$$

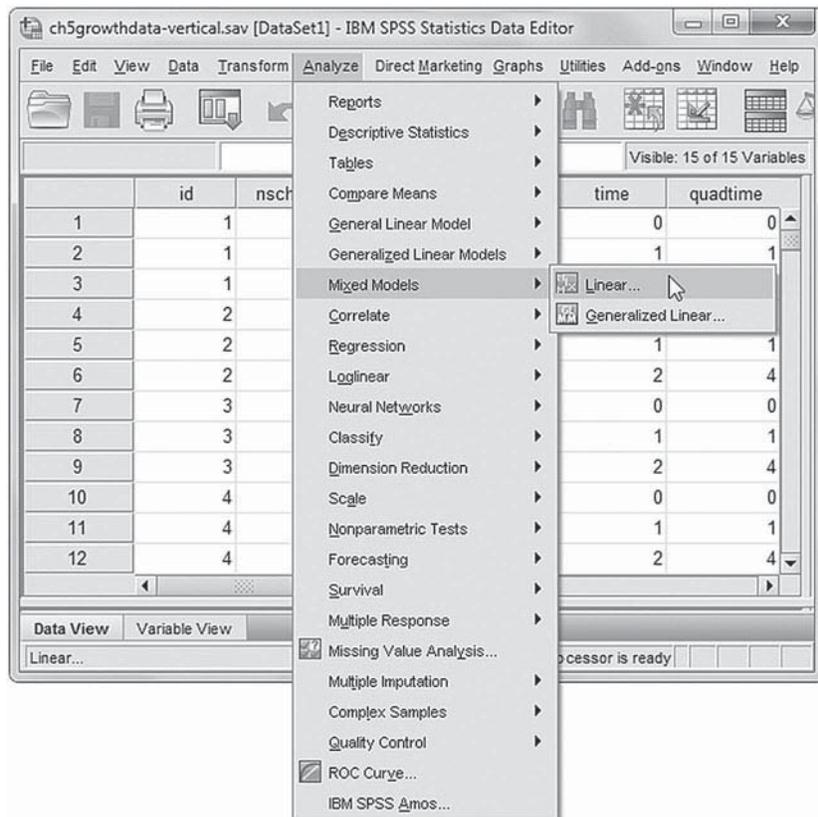
The variances are contained in the diagonals of the matrix, and the covariance is represented by the off-diagonal element. When we combine the covariance parameter estimated in Equation 5.17 with the six parameters defined in Equation 5.16 (i.e., three fixed effects, one random Level 2 intercept, one random Level 2 slope, and the Level 1 residual), we will have a total of seven parameters to estimate in our final version of Model 1.

### Defining Model 1.1B with IBM SPSS Menu Commands

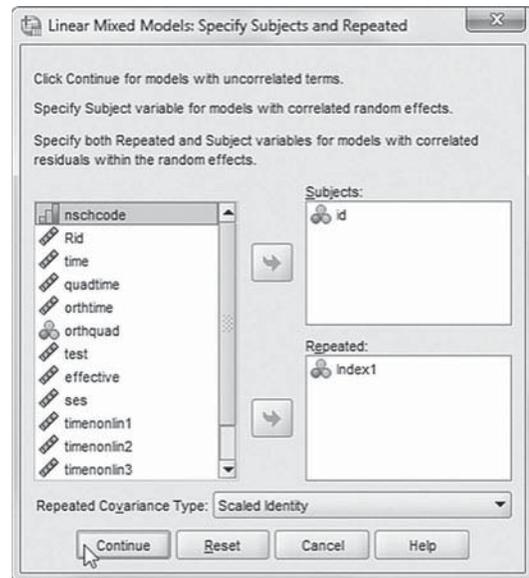
Continue using *ch5growthdata-vertical.sav*. Settings will default to those used in Model 1.1A.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



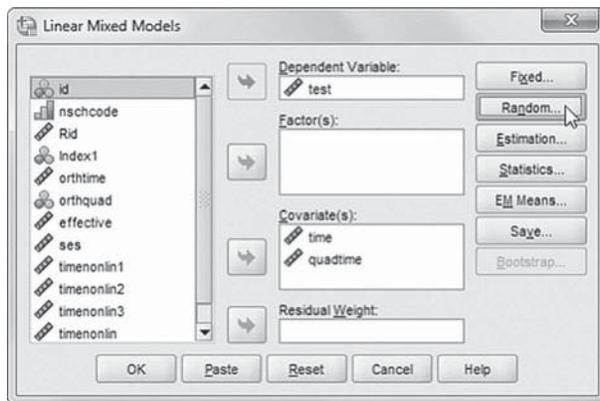
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



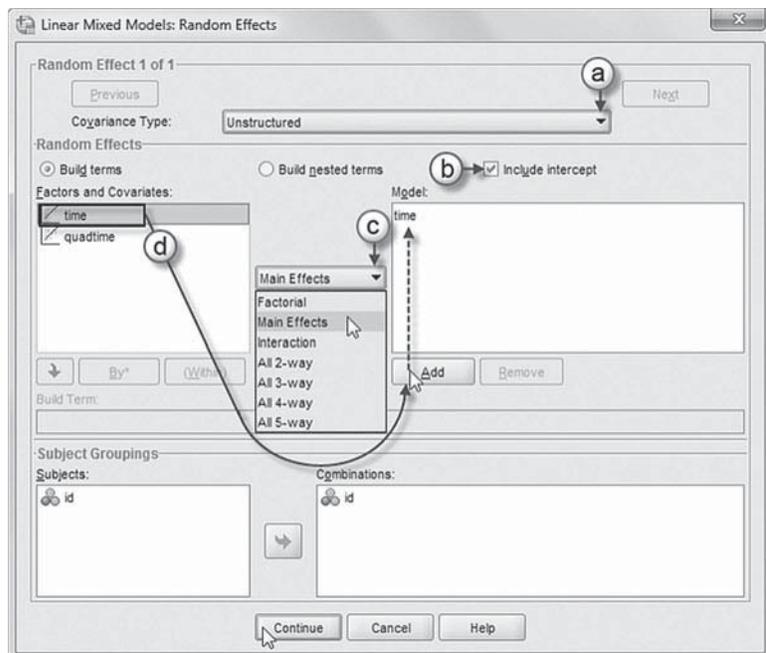
3. The *Linear Mixed Models* main dialog box displays the settings used in the prior model.

We will now add random effects to this model.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



- 4a. Within the *Linear Mixed Models: Random Effects* box, change the covariance type by clicking on the pull-down menu and selecting *Unstructured*.
- b. Click to select *Include intercept*.
- c. Change *Factorial Effects* by clicking on the pull-down menu and selecting *Main Effects*.
- d. Click to select *time* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.



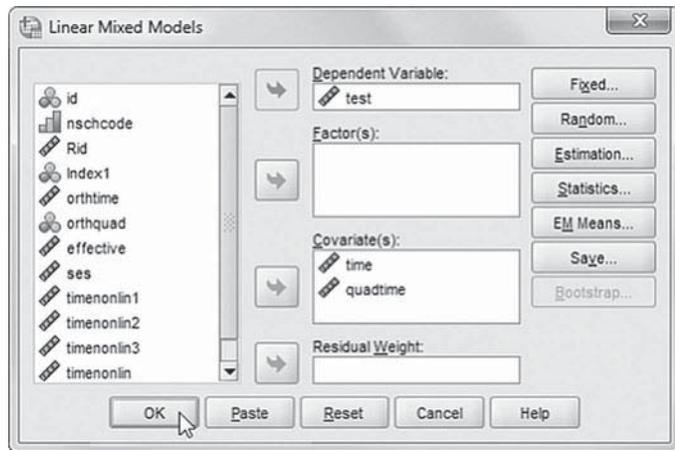
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

### Interpreting the Output From Model 1.1B

Results of this first model test are presented in the following tables. As in previous chapters, it is often useful to examine the total number of parameters being estimated and the number of random and fixed effects to make sure they correspond with what the analyst might have in mind. The first output (Table 5.7) confirms that there are seven total parameters being estimated, which is consistent with Equations 5.16 and 5.17.

The fixed-effect results in Table 5.8 are summarized as  $\beta$  parameters since they are Level 2 parameters (as suggested in Eq. 5.16). For the true intercept ( $\pi_{0i}$ ), the estimate for initial status ( $\beta_{00}$  in Eq. 5.16) is 48.632. This estimate is consistent with estimated mean for Time 0 in Table 5.2. For the linear growth rate ( $\pi_{1i}$ ), the estimate ( $\beta_{10}$  in Eq. 5.16) is 4.72 points per year. For the quadratic growth rate ( $\pi_{2i}$ ), the estimate ( $\beta_{20}$  in Eq. 5.16) is  $-0.244$ . The significance of each fixed effect is tested with a  $t$  test (i.e., defined as the ratio of the unstandardized estimate to its standard error). We note that MIXED applies the Satterthwaite (1946) correction for calculating the degrees of freedom used in testing parameter significance, which explains the presence of decimals in the degrees of freedom column in Table 5.8. The significant  $t$  tests for the growth



**TABLE 5.7 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	time	1		1	
	quadtime	1		1	
Random Effects	Intercept + time	2	Unstructured	3	id
Repeated Effects	time	3	Identity	1	id
Total		8		7	

<sup>a</sup> Dependent variable: test.

**TABLE 5.8 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	48.632	0.103	10,597.930	472.847	.000	48.431	48.834
time	4.719	0.206	10,329.959	22.948	.000	4.316	5.122
quadtime	-0.244	0.098	8,669.000	-2.485	.013	-0.436	-0.052

<sup>a</sup> Dependent Variable: test.

TABLE 5.9 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Variance	55.720	0.846	65.837	.000	54.085	57.403
Intercept + time	UN (1, 1)	35.992	1.437	25.048	.000	33.283	38.922
[subject = id]	UN (2, 1)	-1.247	0.764	-1.631	.103	-2.745	0.251
	UN (2, 2)	4.467	0.648	6.891	.000	3.361	5.936

<sup>a</sup> Dependent variable: test.

terms suggest that both should be retained in the model and that, on average, individuals' growth rates slow slightly over time.

Next, we can examine the covariance parameters in Table 5.9. The Level 1 estimate is 55.72 (Wald  $Z = 65.837$ ,  $p < .001$ ). At Level 2, we specified an unstructured covariance matrix, which means that there are variance estimates for the random intercept (UN 1, 1), the random linear slope (UN 2, 2), and an estimate of the covariance between them (UN 2, 1). The table suggests that there is significant variability in the random intercept to be explained between individuals (Wald  $Z = 25.048$ ,  $p < .001$ ). The linear time slope also varies significantly across individuals (Wald  $Z = 6.891$ ,  $p < .001$ ). The third parameter represents the covariance between the Level 2 initial status and linear growth estimates. We note that the relationship between these two parameters can depend on how the intercept is specified (i.e., with the intercept is defined as initial status, the grand mean, or end status) as well as the presence of other variables in the model (Hox, 2010). In this model, the results suggest that the covariance parameter between the initial status intercept and growth rate is not significantly different from 0 (Wald  $Z = -1.631$ ,  $p > .05$ ).

Because the covariance can be positive or negative, the two-tailed test Wald  $Z$  test implies that we could fix the covariance (UN 2, 1) in subsequent models if we wished to simplify the covariance structure. We also note in passing that researchers have cautioned about the use of these single parameter tests of significance of the model's variance components (Hox, 2010; Raudenbush & Bryk, 2002). The Wald  $Z$  test can also perform poorly under multicollinearity problems and in small sample sizes. For small samples, the likelihood ratio test (which can be estimated from MIXED output) tends to be more reliable than the Wald  $Z$  test.

### Examining Orthogonal Components

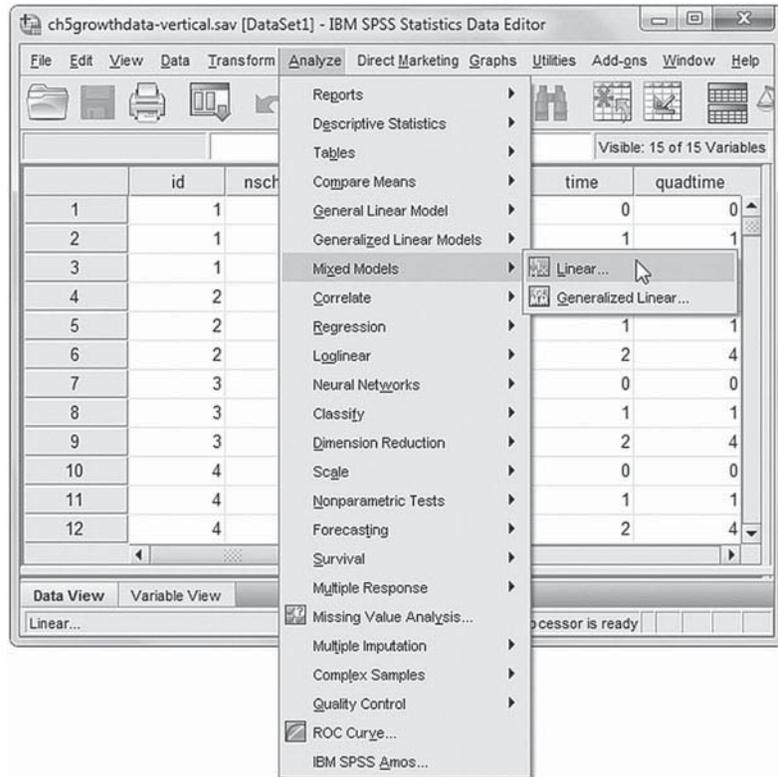
We reiterate that analysts should keep in mind that the polynomial components when untransformed are highly correlated. As we have discussed previously, when repeated measures ANOVA is used, the program automatically transforms the polynomials to be orthogonal. When we use MIXED, the recommendation is that we also transform the coded polynomial components so that they are orthogonal, or we consider other possible ways of coding the time variables. In this next model, we will use the transformed polynomial contrasts for time (*orthtime* and *orthquad*).

### Defining Model 1.2 with IBM SPSS Menu Commands

Continue using the *ch5growthdata-vertical.sav* data. Settings will default to those used in Model 1.1B.

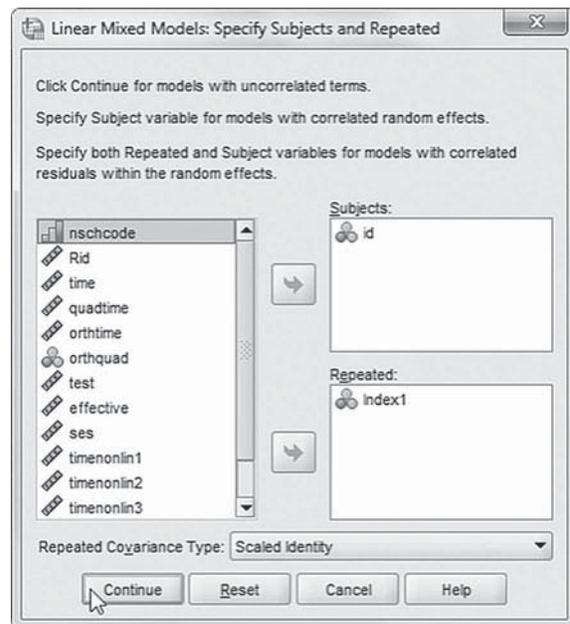
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

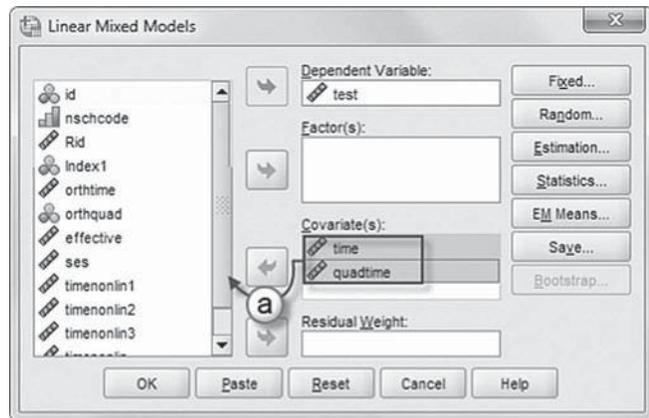


2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from the prior model.

We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.

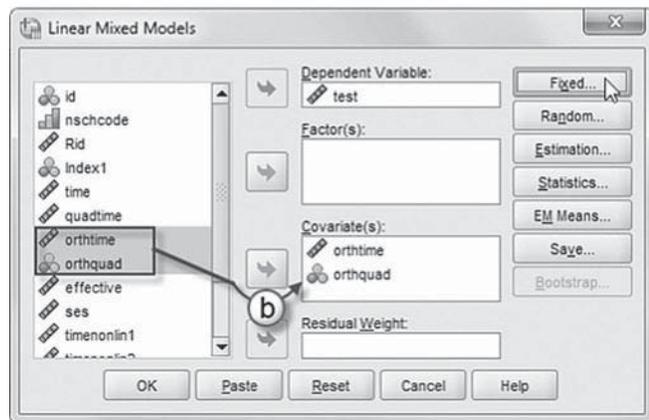


3. We will change the predictors by using the transformed polynomial contrasts for time in the model (*orthtime* and *orthquad*).
  - a. Begin by selecting *time* and *quadtime* and then clicking the left-arrow button to remove the variables from the *Covariate(s)* box.
  - b. Now click to select *orthtime* and *orthquad*. Then click the right-arrow button to add them to the *Covariate(s)* box.

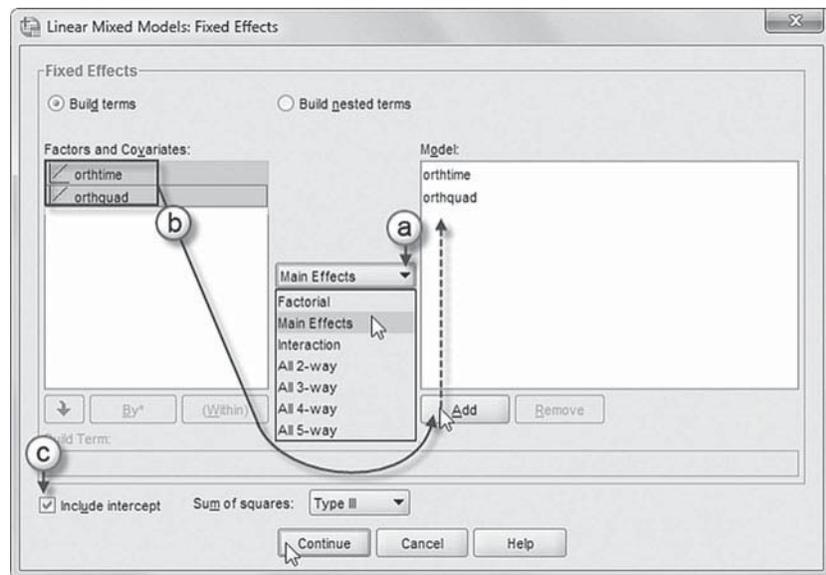


Once the predictor variables have been specified, we may now proceed to define fixed effects for the variables.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
  - b. Click to select *orthtime* and *orthquad* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
  - c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.

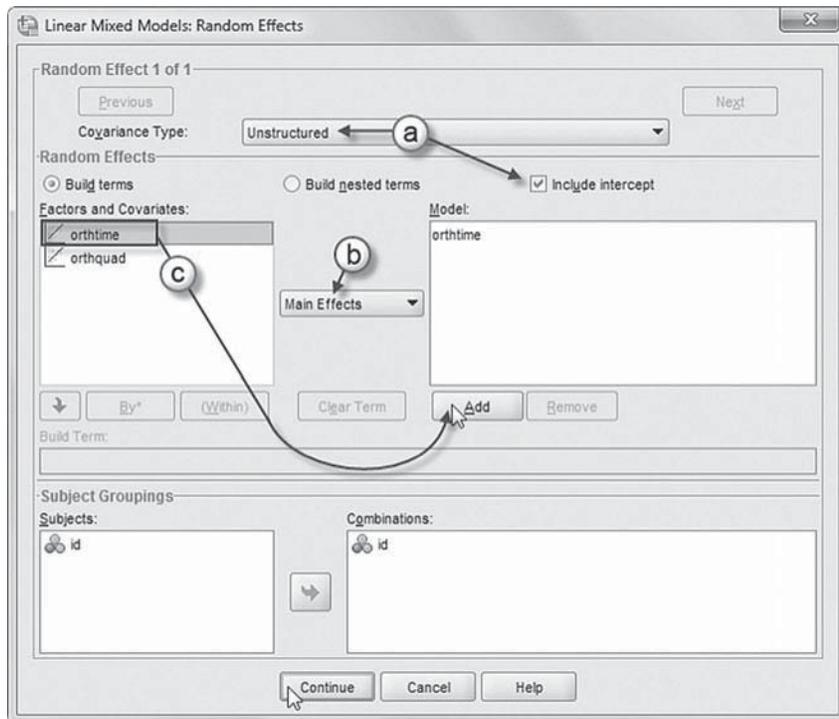


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

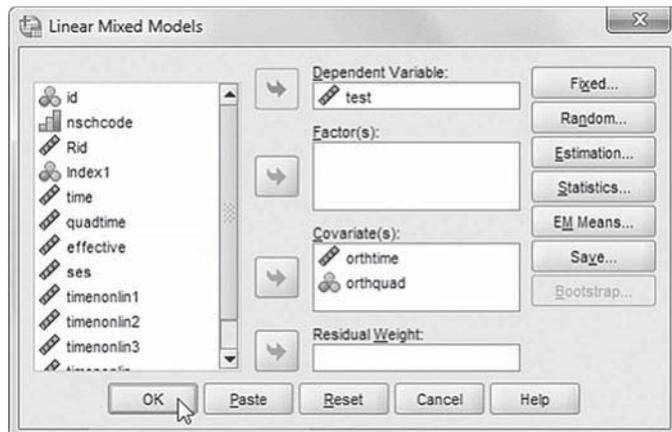
Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

- 5a. Within the *Linear Mixed Models: Random Effects* box, the covariance type and intercept are set to the default settings of the prior model.
- b. The *Main Effects* option is preselected.
- c. Click to select *orthtime* from the *Factors and Covariates* box, and then click the **ADD** button to move the variable into the *Model* box.

Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.



6. Finally, in the *Linear Mixed Models* dialog box, click the **OK** button to run the model.



### Interpreting the Output From Model 1.2

When we rerun the model using these transformed polynomials, we obtain the fixed-effects estimates summarized in Table 5.10. We can observe that both contrasts are significant and, therefore, should be retained in subsequent analyses. We can note that the intercept is now the grand mean of the growth trend instead of the initial status mean (see Table 5.2). Adding the growth components to the intercept provides an estimate of ending achievement (57.095) with a slight difference due to rounding (see Table 5.2 on page 174). We note that transforming the polynomials places the interpretation on the overall growth trend rather than change within any particular interval (Hox, 2010).

TABLE 5.10 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	52.945	0.081	8,669	655.630	.000	52.786	53.103
orthtime	4.231	0.061	8,669.000	69.291	.000	4.111	4.351
orthquad	-.081	0.033	8,669	-2.485	.013	-0.145	-0.017

<sup>a</sup> Dependent variable: test.

TABLE 5.11 Repeated Measures ANOVA Tests of Within-Subjects Contrasts

Measure:test								
Source	Time	Type III Sum of Squares	df	Mean Square	F	Sig.	Noncent. Parameter	Observed Power <sup>a</sup>
Time	Linear	310,416.221	1	310,416.221	4,801.239	.000	4,801.239	1.000
	Quadratic	344.115	1	344.115	6.176	.013	6.176	0.700
Error(time)	Linear	560,479.947	8,669	64.653				
	Quadratic	483,033.589	8,669	55.720				

<sup>a</sup> Computed using alpha = 0.05.

Readers may wish to verify that transforming the polynomials does succeed in creating no correlation between them. Adding the coefficients will provide the estimate of ending achievement in Table 5.2 (57.095, with a slight difference due to rounding). We can compare the results in Table 5.10 to the similar specifications using repeated measures ANOVA (we include the syntax in Appendix A). We provide the within-subjects contrasts from the same repeated measures ANOVA analysis in Table 5.11. Readers will notice that if we take the square root of the  $F$  tests for the time-related contrasts (4,801.239) for the linear contrast and for the quadratic contrast (6.176), we obtain the  $t$  tests for the contrasts in Table 5.10. This suggests that the MIXED solution with orthogonal polynomial linear and quadratic contrasts is consistent with the repeated measures ANOVA solution for the time-related effects.

### Specifying the Level 1 Covariance Structure

We also need to consider the nature of the Level 1 (within individuals) covariance matrix in a repeated measures design. For repeated measures ANOVA, Mauchly's sphericity test assumes a specific structure for the repeated measures covariance matrix. We note that the associated test for sphericity was significant (Mauchly's  $W = 0.977$ , 2  $df$ ,  $p < .001$ ), which in a similar two-level model (with only random intercept) amounts to not accepting compound symmetry. In MIXED, there is no specific test conducted for the covariance matrix between repeated measures. We can, however, begin by examining whether the Level 1 covariance matrix is consistent with a more simplified covariance structure. We will assume that the Level 1 error covariance matrix is scaled identity (ID) and specify an unstructured (UN) covariance structure for the Level 2 random effects.

In Table 5.12, we present the covariance parameters for the Model 1.2. First, we can see that the linear contrast varies randomly across individuals (Wald  $Z = 42.001$ ,  $p < .001$ ). Second, we can also see that with transformed polynomials the covariance between the intercept and slope

**TABLE 5.12** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Variance	55.720	0.846	65.837	.000	54.085	57.403
Intercept + orthtime [subject = id]	UN (1, 1)	37.965	0.904	42.001	.000	36.235	39.779
	UN (2, 1)	3.220	0.460	6.993	.000	2.317	4.122
	UN (2, 2)	4.467	0.648	6.891	.000	3.361	5.936

<sup>a</sup> Dependent variable: test.

(UN 2, 1) is significant and positive, which contrasts with the similar parameter using untransformed polynomials (see Table 5.9 on page 204).

### Investigating Other Level 1 Covariance Structures

As we suggested previously, we can test the fit of the scaled identity covariance matrix for the repeated measures against other possible covariance structures. We summarize several of these alternative structures in Table 5.13. As we noted previously, often there will be little difference in terms of the fixed effects associated with different covariance structures. How can we then decide which structure to use? For nested models, one way is to conduct a likelihood ratio test using the difference in deviance ( $-2$  times the log of the likelihood  $[-2LL]$ ) between models. For models that may not be nested, we can compare the Akaike information criterion (AIC) index for each model. The AIC index ( $D + 2p$ ) is computed by multiplying the number of parameters ( $p$ ) by 2 and adding this product to the deviance statistic, computed using FIML. The addition of  $2p$  to the deviance statistic provides a penalty based on the model's complexity. We can also use the Bayesian information criterion (BIC), which is defined as the sum of the deviance and the product of the natural log ( $\ln$ ) of the sample size and the number of parameters  $[D + \ln(n)p]$ . We prefer the smallest AIC or BIC among models compared, regardless of the number of parameters.

For Model 1, with seven estimated parameters (summarized as Model 1.2 in Table 5.12), the AIC is 189,290.45. After examining Models 2–4, we might select Model 3, with diagonal covariance structure at Level 1 and unstructured covariance structure at Level 2 as a reasonable choice based on available model-fitting information. For this model, the AIC is 189,125.70.

In Table 5.14, we can see that this model more adequately accounts for the different variances between occasions than a model that assumes constant variance across occasions. We also

**TABLE 5.13** Comparing Models, AIC Index, and Number of Parameters

Model	Model Description	AIC	Parameters
Model 1	Identity Covariance Matrix, Level 1; Unstructured Matrix, Level 2	189,290.45	7
Model 2	Diagonal Covariance Matrix, Level 1; Diagonal Covariance Matrix, Level 2	189,295.75	8
Model 3	Diagonal Covariance Matrix, Level 1; Unstructured Covariance Matrix, Level 2	189,125.70	9
Model 4	Autoregressive Errors (AR1), Level 1; Diagonal Covariance Matrix, Level 2	189,282.16*	8

\*Note: Model did not converge.

TABLE 5.14 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [Index1 = 1]	63.583	2.026	31.385	.000	59.734	67.681
	Var: [Index1 = 2]	58.779	1.158	50.756	.000	56.552	61.093
	Var: [Index1 = 3]	35.619	1.980	17.987	.000	31.942	39.720
Intercept + orthtime [subject = id]	UN (1, 1)	38.985	0.946	41.216	.000	37.175	40.884
	UN (2, 1)	7.881	0.610	12.926	.000	6.686	9.076
	UN (2, 2)	7.526	0.964	7.805	.000	5.855	9.675

<sup>a</sup> Dependent variable: test.

investigated a number of other possibilities at Level 1 (unstructured and autoregressive) but found that the model did not converge. We suggest, therefore, that models should generally be judged on various criteria including model fit indices, as well as their substance and sensibility in relation to the study's research purposes.

### Defining Other Level 1 Covariance Structures Using IBM SPSS Menu Commands

(Examples in this section are based on Model 1.2, with scaled identity covariance structure at Level 1 and unstructured covariance structure at Level 2, but illustrate changes to the Level 1 and Level 2 covariance structures as summarized in Table 5.13.)

Model 1: ID (Level 1), UN (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

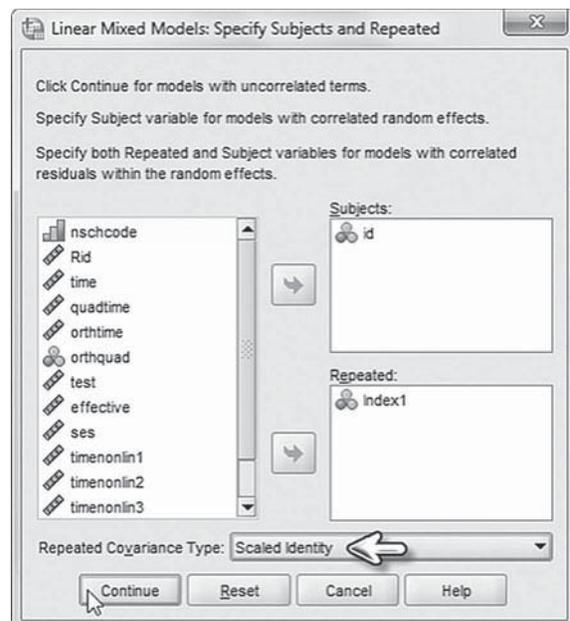
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

#### Scaled Identity Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 1.2. Note that the *Repeated Covariance Type* function is set as *Scaled Identity*.

The *Scaled Identity* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



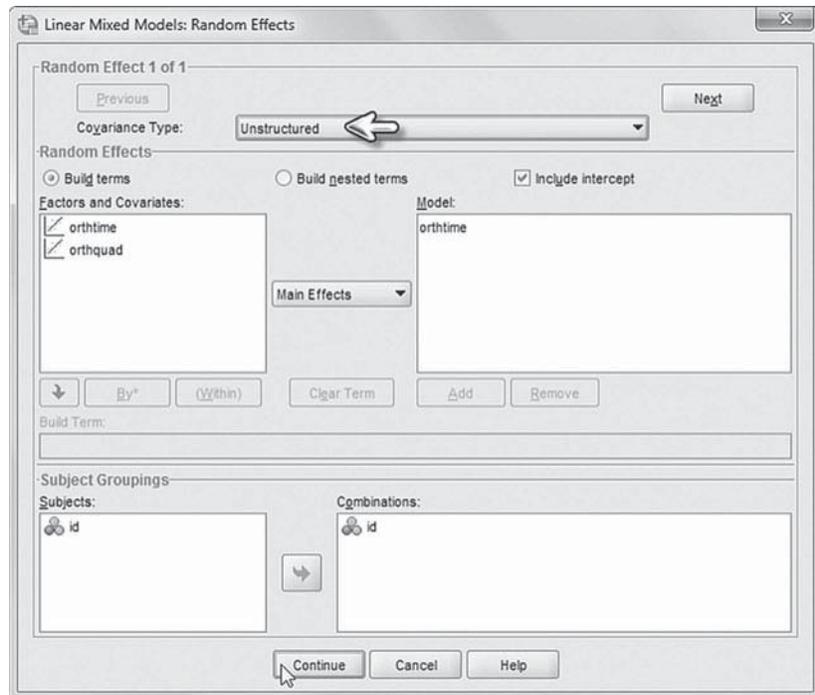
### Unstructured Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 1.2. Note that the *Covariance Type* function is set as *Unstructured*.

*Unstructured* is a completely general covariance matrix (IBM Corporation, 2012).

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 1 results. Compare the output's AIC and model parameters to those in Table 5.13.

(Settings default to Model 1.)



### Model 2: DIAG (Level 1), DIAG (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

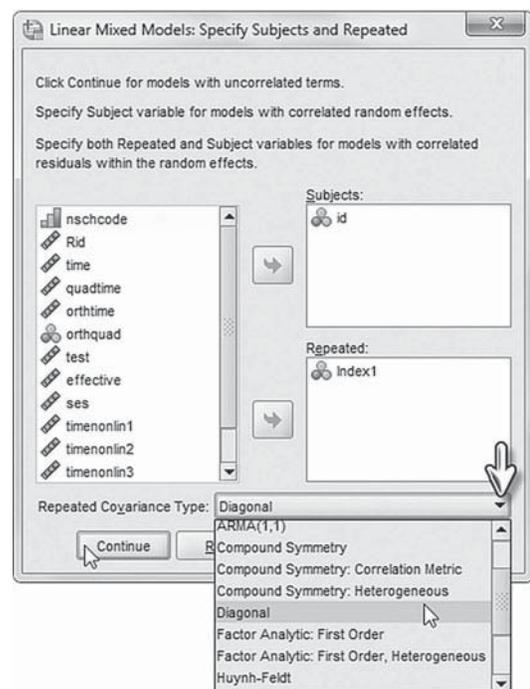
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

### Diagonal Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 1. Change the covariance setting by clicking the pull-down menu to select *Diagonal*.

The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

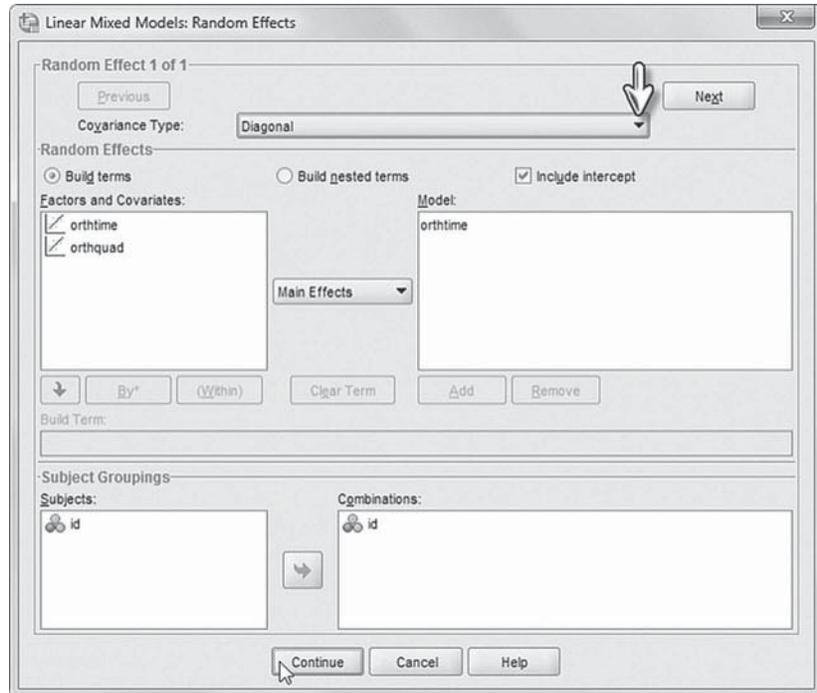
Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



### Diagonal Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 1. Change the *Covariance Type* by clicking the pull-down menu to select *Diagonal*.

The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012). Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 2 results. Compare the output's AIC and model parameters to those in Table 5.13.



(Settings default to Model 2.)

### Model 3: DIAG (Level 1), UN (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

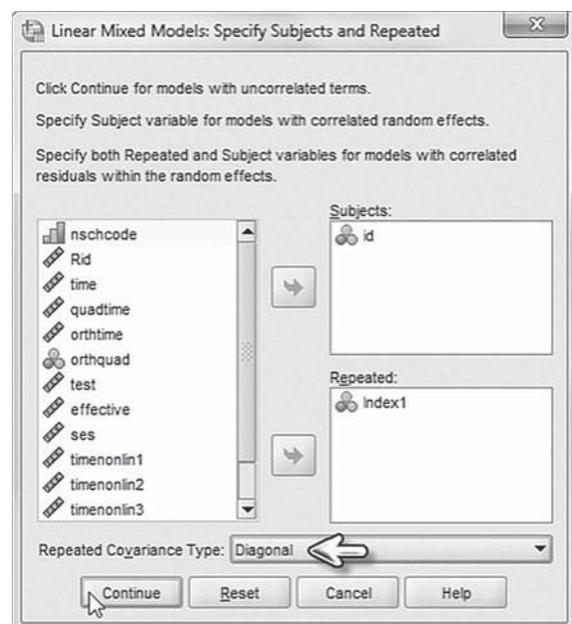
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

### Diagonal Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 2. Note that the *Repeated Covariance Type* is set to *Diagonal*.

The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



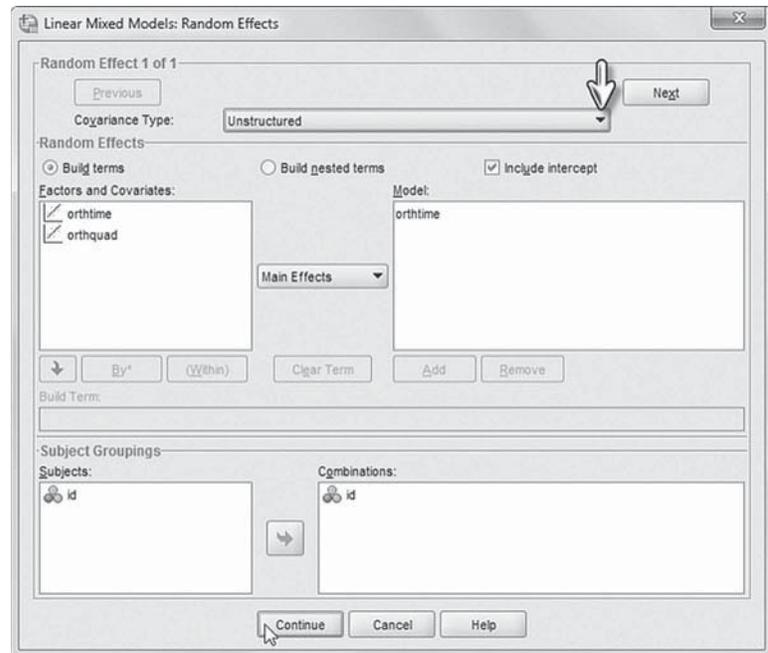
### Unstructured Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 2. Change the *Covariance Type* by clicking the pull-down menu to select *Unstructured*.

*Unstructured* is a completely general covariance matrix (IBM Corporation, 2012).

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 3 results. Compare the output's AIC and model parameters to those in Table 5.13.

(Settings default to Model 3.)



### Model 4: AR1 (Level 1), DIAG (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

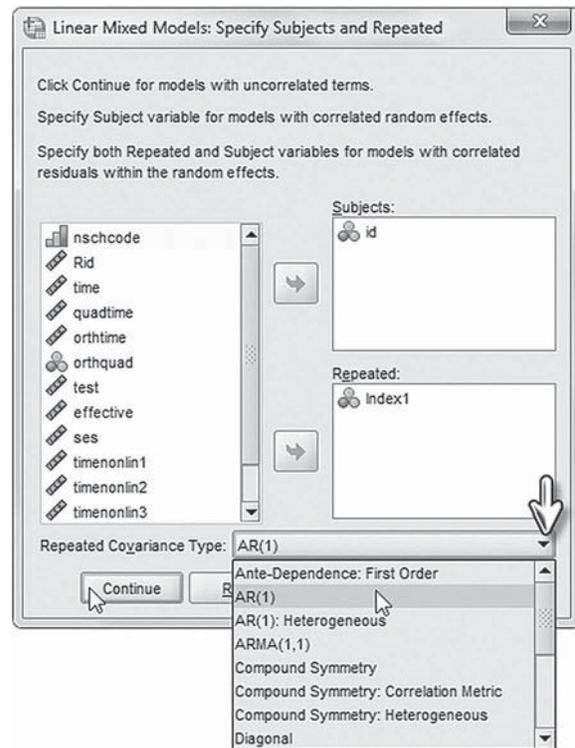
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

### Autoregressive Errors (AR1) Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 3. Change the covariance setting by clicking the pull-down menu to select *AR(1)*.

*AR(1)* is a first-order autoregressive structure with homogenous variances. The correlation between any two elements is equal to  $\rho$  for adjacent elements,  $\rho^2$  for elements that are separated by a third, and so on. We note that  $\rho$  is constrained so that  $-1 < \rho < 1$  (IBM Corporation, 2012).

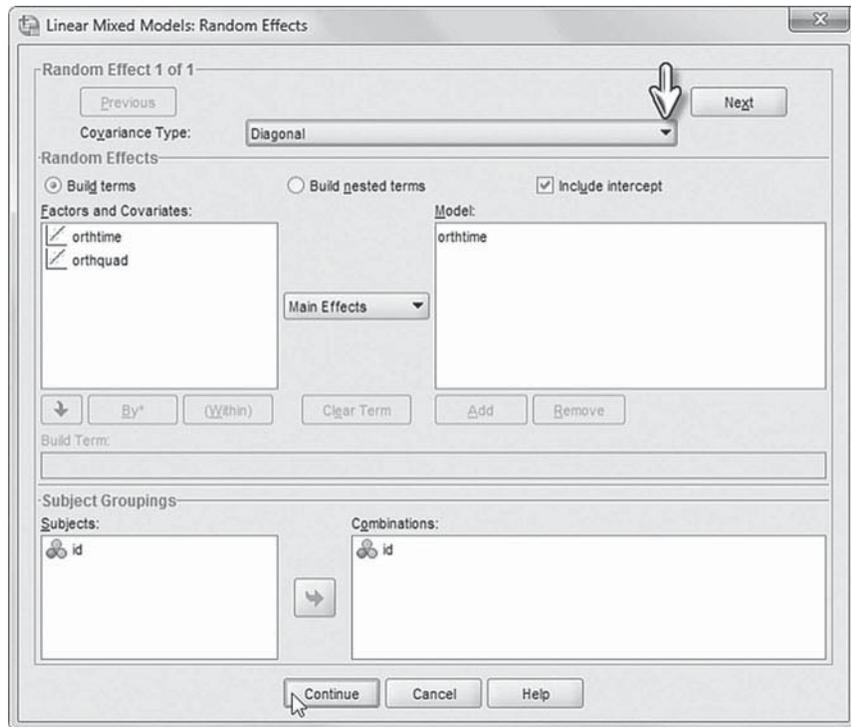
Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



## Diagonal Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 3. Change the *Covariance Type* by clicking the pull-down menu to select *Diagonal*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 4 results, which shows that the model does not converge. Compare the output's AIC and model parameters to those in Table 5.13.



### Model 1.3: Adding the Between-Subjects Predictors

The initial results presented previously suggest that intercepts and linear growth rates vary across individuals. We can next proceed with explaining variability in the random parameters across individuals (at Level 2). This part of the analysis addresses the third research question: Are there differences in development across groups of individuals? A first issue to be addressed is which of the polynomial coefficients should be used to build our successive models concerning differences in growth trajectories by student background factors (e.g., gender and socioeconomic status)? The best advice is that it depends on the specifics of the data set. We suggest trying to build a model that is parsimonious but does justice to the particular features of the data set. This will usually require some preliminary analyses. This part of specifying the model can be challenging since it may not be immediately clear which components may be randomly varying and on which polynomial contrast (or contrasts) we should actually build the explanatory model. Of course, we have more flexibility regarding the number of time-related random effects that can be specified if we have more time points in the study.

In this case, since the quadratic component is fixed within individuals, it seems clear that we should build the Level 2 models on the randomly varying linear component; that is, we seek to explain its variability across individuals. We will propose that students' perceptions of their teachers' effectiveness (coded 1 = effective vs. 0 = average or below) and students' SES might explain differences in their math intercepts and linear growth rates. In order to examine the parallelism hypothesis regarding variability in linear growth rates for different student subgroups (i.e., that growth rates are the same for students having effective vs. not effective teachers), or to consider the impact of an interval variable such as SES on student growth rates, we need to create cross-level interaction terms.

Cross-level interactions involve the effects of Level 2 (between-individual) variables such as teacher effectiveness on a Level 1 slope coefficient—that is, students' growth rates. As we have shown in previous chapters, cross-level interaction terms can be created within SPSS MIXED in the menu command format. We reiterate that for this exercise, we define students' perceptions of teacher effectiveness as a between-student factor; that is, it is simply used as a means of identifying subsets of students (e.g., similar to gender) who are likely to have different growth trajectories. In this example, we do not link groups of students to their individual teachers, so we will not consider the more complicated nesting of students within teachers. For Level 2, the following equations can be formulated:

$$\pi_{0i} = \beta_{00} + \beta_{01}ses_i + \beta_{02}effective_i + u_{0i}, \quad (5.18)$$

$$\pi_{1i} = \beta_{10} + \beta_{11}ses_i + \beta_{12}effective_i + u_{1i}, \quad (5.19)$$

where  $u_{0i}$  and  $u_{1i}$  represent variation associated with estimating the intercept and slope parameters between individuals. The quadratic component is specified as fixed at Level 2 ( $\pi_{2i} = \beta_{20}$ ) as in Equation 5.12. When we substitute the intercept and slope equations (Eqs. 5.18, 5.19, and 5.12) into the Level 1 model (Eq. 5.1 using the transformed components), we obtain the following combined equation:

$$Y_{it} = \beta_{00} + \beta_{01}ses_i + \beta_{02}effective_i + \beta_{10}orthtime_{it} + \beta_{11}ses_i * orthtime_{it} + \beta_{12}effective_i * orthtime_{it} + \beta_{20}orthquad_{it} + u_{1i} orthtime_{it} + u_{0i} + \epsilon_{it}. \quad (5.20)$$

We note that the cross-level interaction terms in Equation 5.20 are created within SPSS MIXED in the menu command format. We reiterate that the Level 1 repeated measures covariance matrix in Model 1.3 is diagonal (as in Eq. 5.5) and the Level 2 covariance matrix is unstructured (Eq. 5.17). The dimensionality of the Level 2 matrix depends on the number of random effects. This makes a total of 13 parameters to estimate (i.e., seven fixed effects, three Level 2 random effects, and three Level 1 residual variances).

If we wished to examine possible differences in the quadratic component (even though we will treat this component as fixed across individuals), we could also add the Level 2 predictors to that portion of the model, but the random component ( $u_{2i}$ ) would remain 0 (Raudenbush & Bryk, 2002):

$$\pi_{2i} = \beta_{20} + \beta_{21}ses_i + \beta_{22}effective_i \quad (5.21)$$

This equation could then also be substituted into the Level 1 equation.

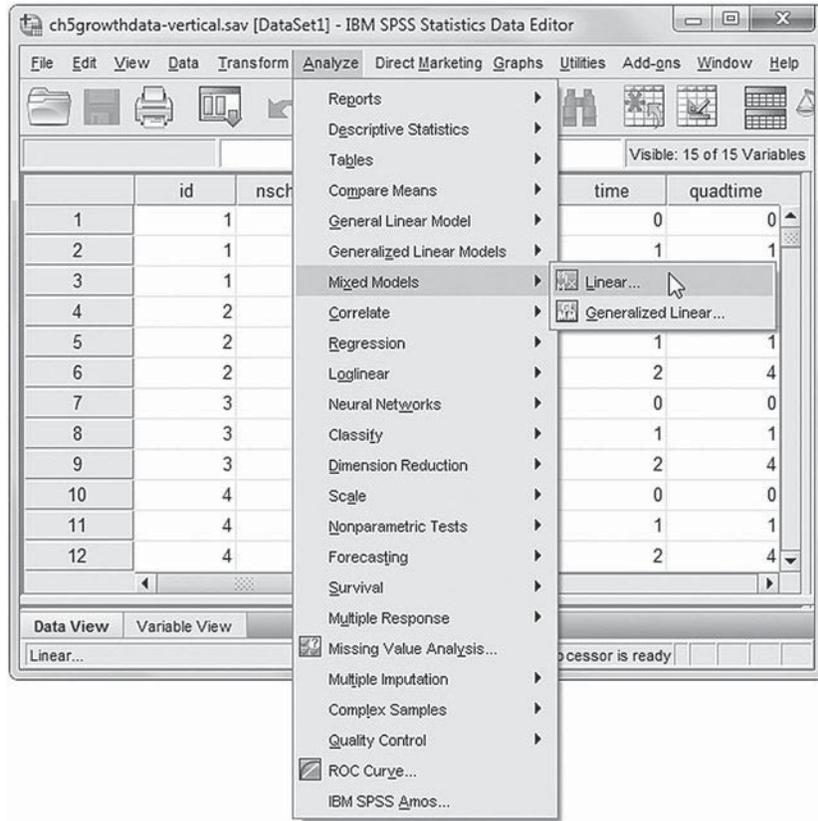
### **Defining Model 1.3 with IBM SPSS Menu Commands**

In this model, we proposed that students' initial status intercept ( $\pi_{0i}$ ) will vary across individuals, and this variation in intercepts will in part be explained by students' socioeconomic status and teacher effectiveness. We also proposed that variation in students' average linear growth rates ( $\pi_{1i}$ ) will be explained by the same predictors.

Continue using the *ch5growthdata-vertical.sav* data. If continuing from the “Investigating Other Level 1 Covariance Structures” examples, please change settings to those used for Model 1.2 before proceeding.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

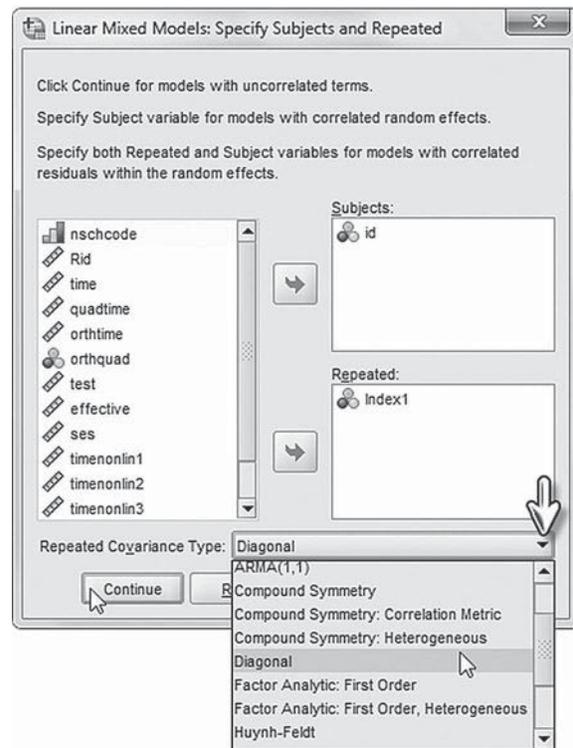
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



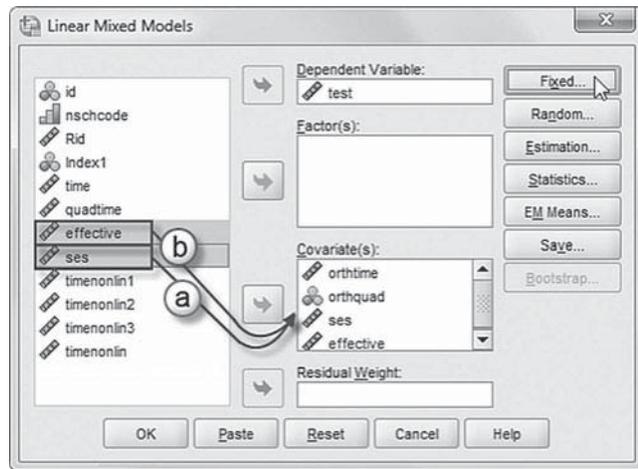
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from Model 1.2.

Change the covariance setting by clicking the *Repeated Covariance Type* pull-down menu to select *Diagonal*.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box



3. The *Linear Mixed Models* dialog box displays *test* in the *Dependent Variable* box with *orthtime* and *orthquad* in the *Covariate(s)* box.
  - a. We will add two variables (*ses* and *effective*) into the model. Entering variables in sequential order helps facilitate reading of the output tables. So first click to select *ses*, and then click the right-arrow button to add the variable the *Covariate(s)* box.
  - b. Click to select *effective*, and then click the right-arrow button to add the variable the *Covariate(s)* box.

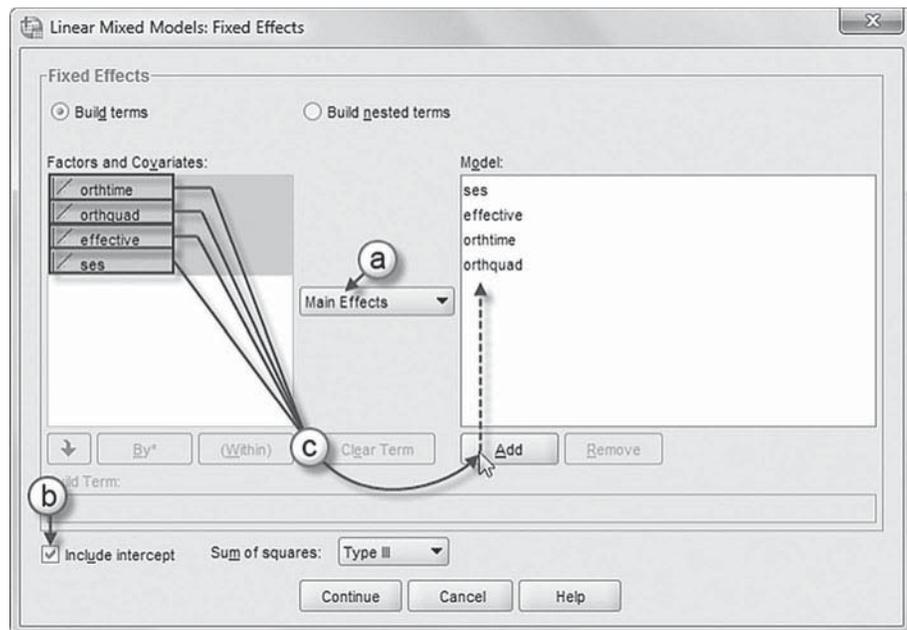


The variable sequence is *orthtime*, *orthquad*, *ses*, and *effective*.

**Note:** An alternative method is to select both variables simultaneously and then “drag” them into the *Covariate(s)* box. The variables may be rearranged by clicking to select a variable and then dragging it upward or downward to change the order of the sequence.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, confirm that *Main Effects* is selected.
- b. Confirm that *Include intercept* is selected.
- c. To facilitate reading the output tables, click to select a variable, and then click the ADD button to move it into the *Model* box in the following order: *ses*, *effective*, *orthtime*, and *orthquad*.

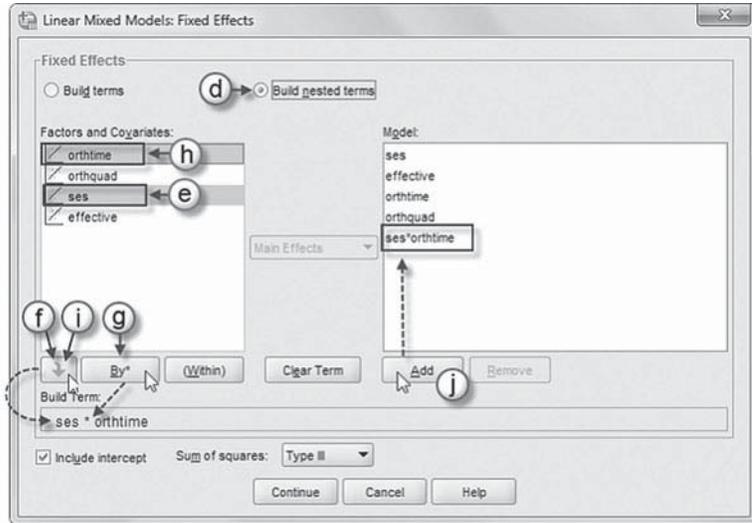


These variables are the “main effects” in the model and will modify the intercept.

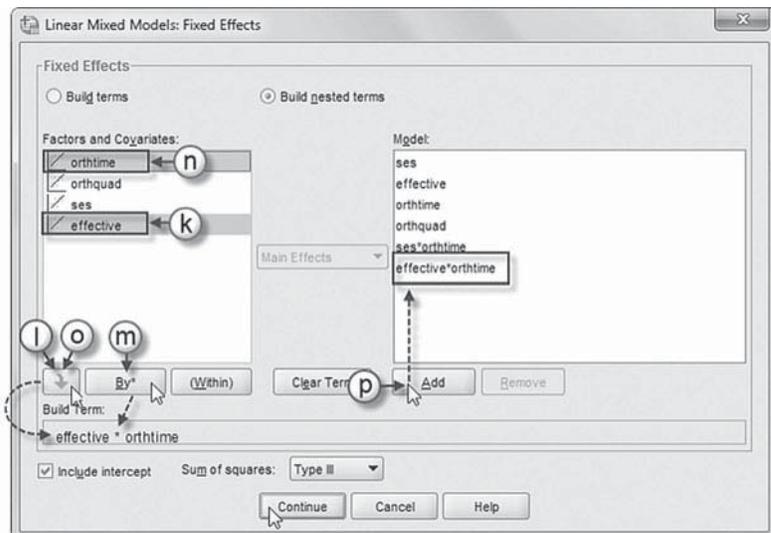
Two cross-level interactions (or nested terms) will be created and added to the model: *ses\*orthtime* and *effective\*orthtime*. These interactions will tell us if the growth trajectories are parallel for different groups of students.

Add First Cross-Level Interaction to Model 1.3: *ses\*orthtime*

- d. Click to select *Build nested terms*.
- e. Click to select the variable *ses* from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves *ses* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button, which will insert the computation command symbol: *ses\**.
- h. Click to select *orthtime* from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move *orthtime* into the *Build Term* box and complete the interaction term: *ses\*orthtime*.
- j. Click the ADD button to transfer the interaction into the *Model* box.

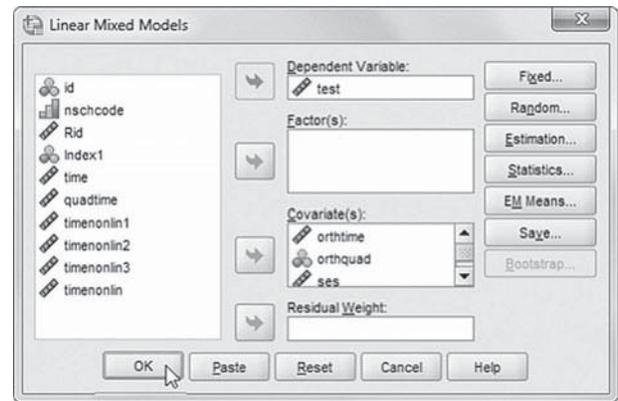
Add Second Cross-Level Interaction to Model 1.3: *effective\*orthtime*

- k. Click to select the variable *effective* from the *Factors and Covariates* box.
- l. Next, click the arrow button below the *Factors and Covariates* box. This moves *effective* into the *Build Term* box.
- m. Now click the BY\* button, which will insert the computation command symbol: *effective\**.
- n. Click to select *orthtime* from the *Factors and Covariates* box.
- o. Click the arrow button below the *Factors and Covariates* box to move *orthtime* into the *Build Term* box and complete the interaction term: *effective\*orthtime*.
- p. Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 1.3

Results of the second model test are presented next. Once again, it is useful to examine the random effects and total parameters estimated summarized in Table 5.15. There are seven fixed effects estimated. In addition, there are six variance-covariance parameters to be estimated. At Level 1 (within individuals), there are three variance parameters (i.e., the variances for each occasion). These are listed in the diagonal covariance matrix at Level 1. In addition, there are three random effects at Level 2 (i.e., the intercept, the linear time slope, and the covariance between them). This suggests 13 total parameters to be estimated.

The fixed-effect estimates are summarized in Table 5.16. Students' achievement intercept ( $\beta_{00}$ ) is 49.672. This can be described as students' true grand-mean achievement adjusted for SES and perceptions of teacher effectiveness. The intercept in this case can be interpreted as the grand-mean test score for students who perceived that they did not have an effective teacher (coded 0) and whose SES status was 0.00 (since SES was defined as a z-score).

The first question we can ask is whether the predictors are related to differences in average achievement. We can see that perceived teacher effectiveness is related to students' achievement level ( $p < .001$ ). The coefficient for effectiveness ( $\beta_{02} = 5.944$ ) suggests that students with effective teachers would have an estimated grand-mean achievement level of about 55.616 ( $49.672 + 5.944$ ). In this model, student SES is not a significant predictor of average achievement level ( $p > .10$ ).

The second question we can ask is whether there are differences in student growth rates related to the predictors. The average linear growth rate (*orthtime*) increases significantly over time ( $p < .001$ ). Keep in mind that the actual polynomial contrast coefficients do not have direct

TABLE 5.15 Model Dimension<sup>a</sup>

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	ses	1		1		
	effective	1		1		
	orthtime	1		1		
	orthquad	1		1		
	orthtime * ses	1		1		
	orthtime * effective	1		1		
Random Effects	Intercept + orthtimea	2	Unstructured	3	id	
Repeated Effects	Index 1	3	Diagonal	3	id	8,670
Total		12		13		

<sup>a</sup> Dependent variable: test.

TABLE 5.16 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	49.672	0.111	8,701.861	447.802	.000	49.455	49.890
ses	0.074	0.095	8,667.000	0.784	.433	-0.112	0.260
effective	5.944	0.149	8,667.000	39.838	.000	5.652	6.237
orthtime	2.307	0.086	8,795.556	26.693	.000	2.138	2.477
orthquad	-0.081	0.033	8,669.000	-2.485	.013	-0.145	-0.017
orthtime * ses	-0.154	0.074	8,667.000	-2.090	.037	-0.298	-0.010
orthtime * effective	3.508	0.116	8,667.000	30.324	.000	3.281	3.735

<sup>a</sup> Dependent variable: test.

meaning regarding a particular time interval after they have been transformed. Because the linear contrast was specified as a random effect, we are primarily interested in whether it varies between individuals in the study. The variation in the size of the within-individual growth parameter across individuals (UN 2, 2) can be examined by referring to Wald Z test (Wald  $Z = 8.547$ ,  $p < .001$ ) in the variance components table (Table 5.17). The significant test suggests we can reject the null hypothesis that the population growth parameter is 0, and we can infer that growth varies significantly across the population of individuals.

Regarding variables that might explain variability in math growth rates between individuals, we can see that the linear interaction (*orthtime\*ses*) is significant at  $p < .05$  ( $\beta_{11} = -0.154$ ,  $p = .037$ ). This coefficient can be interpreted as students at higher SES levels demonstrate slightly less growth over time compared with students at the grand mean for SES. The test for students' perceptions of teacher effectiveness is also significant ( $\beta_{12} = 3.508$ ,  $p < .001$ ). This suggests that students with teachers they perceived as effective have a higher growth rate over time compared to their peers who perceive their teachers to be average or below in effectiveness (since they are coded 0). The quadratic polynomial is also significant, which implies that student growth rates slow slightly over time ( $\beta_{20} = -0.081$ ,  $p < .001$ ). In this model, we did not propose any cross-level interaction related to explaining the slowing of math growth rates.

After the addition of the predictors, Table 5.17 suggests that there is still significant residual variance in intercepts to be explained (Wald  $Z = 37.906$ ,  $p < .001$ ). There is also significant residual variance in slopes left to be explained across individuals (Wald  $Z = 8.547$ ,  $p < .001$ ). The covariance between the intercept and slope is positive (4.159) and also significant ( $p < .001$ ). If desired, we could select an unstructured covariance matrix with a correlation for the slope parameter (UNR). In this case, the correlation between students' initial status (i.e., intercept) and growth of time is 0.27 (not tabled). We reiterate that while this correlation is often of interest in growth models, it can be different depending on how the intercept and slope are defined and the other variables added to the model.

For comparative purposes in Table 5.18, we also provide the repeated measures ANOVA solution that we would obtain for the polynomial contrasts. The relevant output is the tests of within-subject contrasts. That solution also has additional tests of contrasts for the quadratic time-related component and the predictors in the model (i.e., SES and teacher effectiveness). We can see that the linear effect is also significant for individual SES in the ANOVA formulation (but the quadratic effect is not), and the linear and quadratic effects are both significant for *effective* ( $p < .001$ ). We could of course provide this same set of model tests using MIXED by adding the two quadratic contrasts to the fixed-effect model.

TABLE 5.17 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [Index1=1]	61.400	2.004	30.641	.000	57.595	65.456
	Var: [Index1=2]	61.448	1.156	53.158	.000	59.224	63.756
	Var: [Index1=3]	27.124	1.727	15.708	.000	23.942	30.729
Intercept + orthtime [subject = id]	UN (1, 1)	31.503	0.831	37.906	.000	29.915	33.174
	UN (2, 1)	4.159	0.566	7.343	.000	3.049	5.269
	UN (2, 2)	7.465	0.873	8.547	.000	5.935	9.389

<sup>a</sup> Dependent variable: test.

TABLE 5.18 Tests of Within-Subjects Contrasts

Measure:test								
Source	Time	Type III Sum of Squares	df	Mean Square	F	Sig.	Noncent. Parameter	Observed Power <sup>a</sup>
time	Linear	283,779.090	1	283,779.090	4,795.547	.000	4,795.547	1.000
	Quadratic	637.493	1	637.493	11.551	.001	11.551	0.925
time * ses	Linear	244.668	1	244.668	4.135	.042	4.135	0.529
	Quadratic	1.270	1	1.270	0.023	.879	0.023	0.053
time * effective	Linear	47,359.971	1	47,359.971	800.330	.000	800.330	1.000
	Quadratic	4,712.361	1	4,712.361	85.386	.000	85.386	1.000
Error(time)	Linear	512,874.459	8,667	59.176				
	Quadratic	478,319.939	8,667	55.189				

<sup>a</sup> Computed using alpha = 0.05.

In Table 5.19, we provide a summary of the various models we have tested using untransformed and transformed polynomials. Model 1 summarizes the initial results for time with the polynomial contrasts untransformed. In Model 1, without transformed time-related contrasts, the intercept (48.632) corresponds to student achievement at the beginning of the study. Model 3 is the same model as Model 1 but with orthogonal polynomial contrasts. In Model 3, the intercept (52.945) now corresponds to the grand mean for the trend, which is somewhere between interval 1 and interval 2. We can see that the polynomial components are significant ( $p < .05$ ), again suggesting they should be retained in subsequent analyses. Because the intercept is now the grand mean, however, the time-related estimates no longer correspond to any particular time interval in the study.

Readers can see that the pattern of results is the same despite the strong correlation between the linear and quadratic polynomial components in Model 1. Model 2 in Table 5.19 summarizes the Level 2 effects built only on the randomly varying linear time effect. Model 4 presents those same results but with transformed time-related components. Once again, the pattern of results is almost the same for Models 3 and 4 (with only slight differences in the variance parameters). Model 5 represents the MIXED specification of the repeated measures ANOVA analysis, which is in the last column of the table. Regarding the ANOVA results in the last column, as noted previously, the predictors enter in as interactions with growth over time (i.e., *linear effect\*SES*,

**TABLE 5.19 Comparing Results of Untransformed Polynomial Models (1–2) and Orthogonal Polynomial Models (3–5) with Repeated Measures ANOVA**

Variables	Model 1	Model 2	Model 3	Model 4	Model 5	Within-Subjects ANOVA
Intercept	48.632*	47.284*				
time	4.719*	2.795*				
time quadratic	-0.244*	-0.244*				
SES		0.228				
effective		2.436*				
time*SES		-0.154*				
time*effective		3.508*				
Intercept			52.945*	49.672*	49.741*	NA
orthtime			4.231*	2.307*	2.409*	*
orth quadratic			-0.081*	-0.081*	-0.414*	*
SES				0.074	0.076	NA
effective				5.944*	5.820*	NA
orthtime*SES				-0.154*	-0.152*	*
orth quadratic*effective				3.508*	3.322*	*
orth quadratic*SES					-0.006	
orth quadratic*effective					0.605*	*
AIC	189,123.5	186,832.7	189,125.7	186,834.9	186,758.0	NA

\* $p < .05$ ; NA = parameter is not applicable to the ANOVA model.

*linear effect\*teacher effectiveness, quadratic effect\*SES, and quadratic effect\*teacher effectiveness*). As shown in the last column, five of the six effects specified are significant, which exactly matches the MIXED results in Model 5, as well as the repeated measures within-subject contrasts summarized in Table 5.18.

### Graphing the Results

We can summarize the initial difference in growth trajectories with a graph of different growth rates by teacher effectiveness (Figure 5.8). The trajectories are best interpreted as not parallel over time; that is, the initial observed gap in test learning between students with effective and ineffective teachers widens over time. This graph will look slightly different from the one produced in *Repeated Measures* (MANOVA) since there is no control for student SES in the latter graph. This graph can be produced using the following IBM SPSS commands.

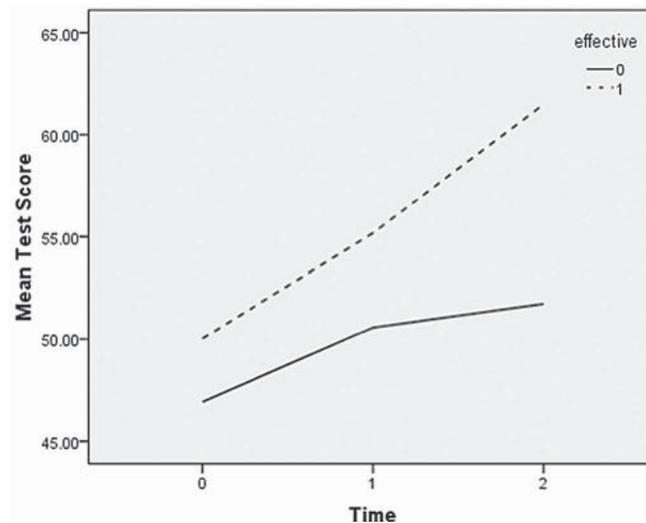
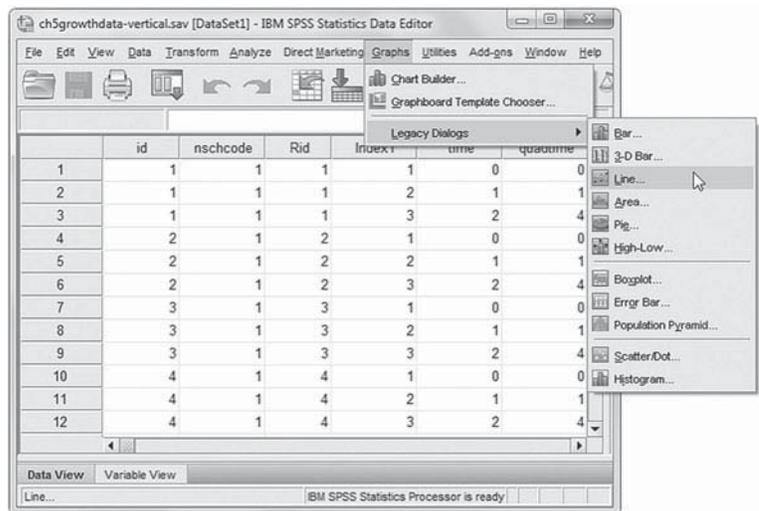


FIGURE 5.8 Individual growth trajectories differentiated by teacher effectiveness.

### Graphing the Growth Rate Trajectories with SPSS Menu Commands

1. Go to the toolbar and select **GRAPHS, LEGACY DIAGNOSTICS, LINE**.

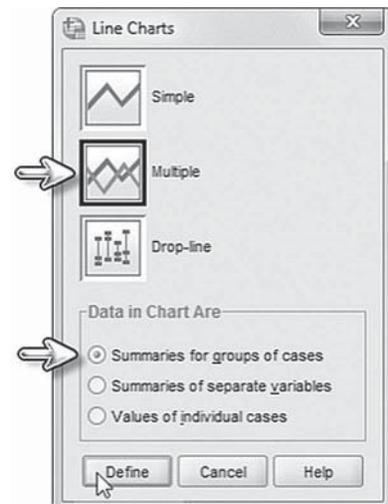
This command will open the *Line Charts* dialog box.



2. In the *Line Charts* dialog box, click to select *Multiple*.

Confirm that *Summaries for groups of cases* is selected.

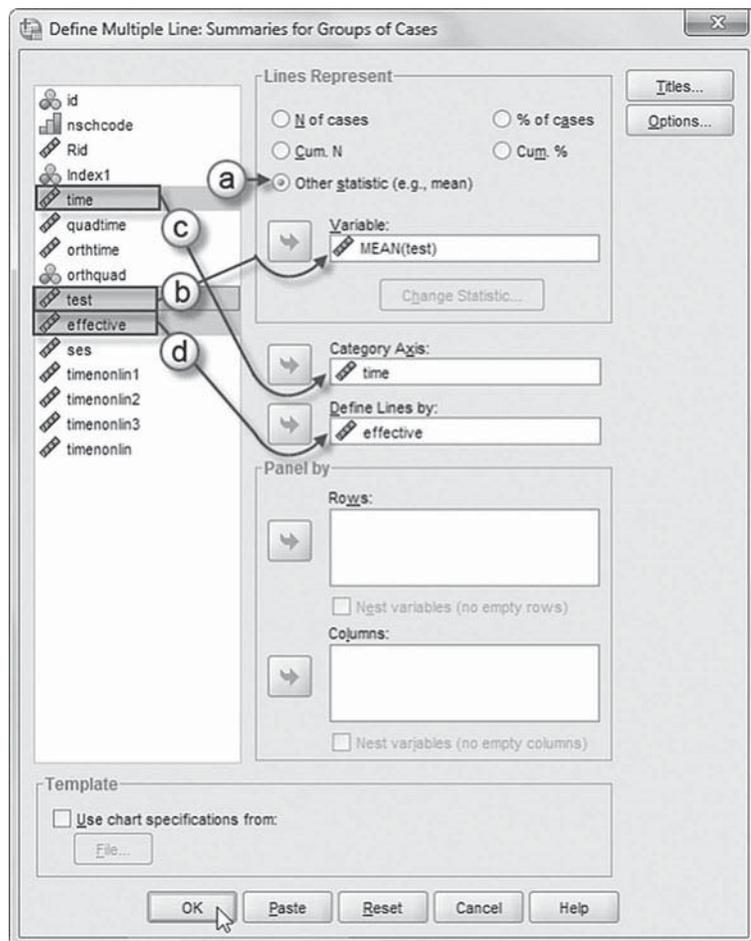
Click the **DEFINE** button to continue, which will open the *Define Multiple Line: Summaries for Groups of Cases* dialog box.



- 3a. Within the *Define Multiple Line: Summaries for Groups of Cases* dialog box, click to select *Other statistic (e.g., mean)*.
- b. Click to select the variable *test* from the left column, and then click the right-arrow button to move *test* into the *Variable* box. The setting *MEAN(test)* defines the use of mean values for *test*.
- c. Click to select *time* from the left column. Then click the right-arrow button to move the variable into the *Category Axis* box.
- d. Click to select *effective* from the left column. Next, click the right-arrow button to move the variable into the *Define Lines by* box.

Click the OK button, which will generate the plot graph.

**Note:** The resulting plot graph's labels and lines may be edited or changed through the *Chart Editor*. To activate the *Chart Editor*, double-click on the plot graph in the output (Fig. 5.8).



### Examining Growth Using an Alternative Specification of the Time-Related Variable

Since we know that the growth trajectory slows slightly between the second and third measurement occasions, we might consider a different manner of coding the repeated measures to represent the growth occurring over the entire period under consideration. To determine the growth that occurs over the entire trend, we can code the first measurement as 0 and the last one as 1. Since the meaning of the slope is the change in  $Y$  for a unit change in  $X$ , coding the time-related variable in this manner will capture the growth over the entire trend regardless of whether the growth dips or spikes between the beginning and ending intervals. Steps for coding four variations (i.e., *timenonlin1*, *timenonlin2*, *timenonlin3*, and *timenonlin*) of the time-related variable are provided at the end of this section. This is similar to the level-and-shape approach discussed previously for estimating a latent growth factor. With a little trial and error, it is possible to obtain a variety of growth curves that may match the actual data quite well. More specifically, if we see that the relationship is linear (perhaps by inspecting several individuals' growth trajectories or the average individual trajectory), we could code the middle interval as 0.5 (i.e., 0.0, 0.5, 1.0). We can then test this hypothesized linear formulation against the data. If we specify the time-related variable in this way (*timenonlin1*), we obtain an AIC coefficient of 189,125.5. We provide the model syntax in Appendix A.

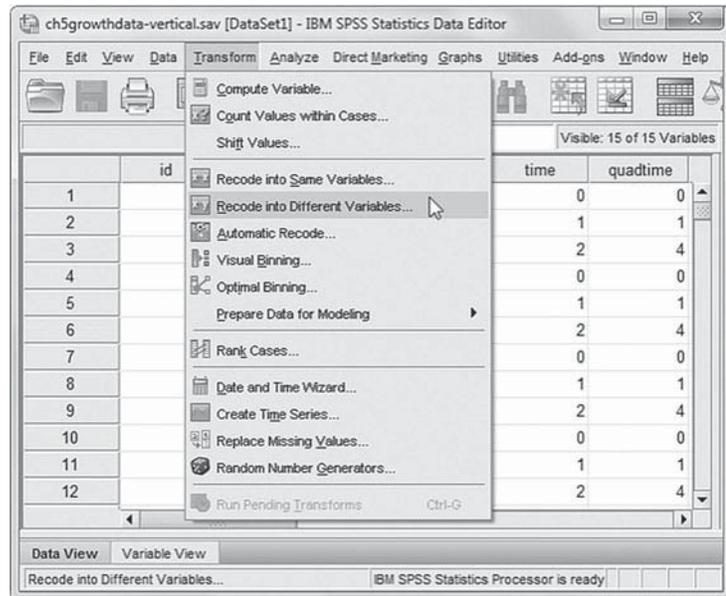
### Coding Time Interval Variables (time to timenonlin Variations) with IBM SPSS Menu Commands

Continue using the *ch5growthdata-vertical.sav* data.

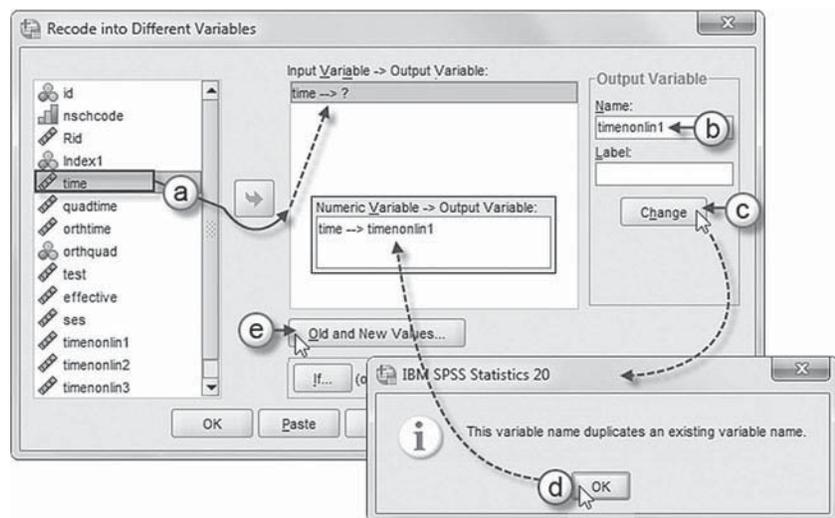
1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.

**Note:** If continuing from performing the prior coding example (*time* to *orthtime* or *orthquad*), click the RESET button before proceeding to clear the default settings.



- 2a. The *Recode into Different Variables* box enables creating a new variable using a variable from the current data set. First, click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.



- b. Now enter the new variable name by typing *timenonlin1* into the *Output Variable, Name* box.
- c. Then click the CHANGE button, which will add *timenonlin1* and complete the RECODE command for *time* → *timenonlin1*.

**Note:** A warning message appears as *timenonlin1* is an existing variable in the data set.

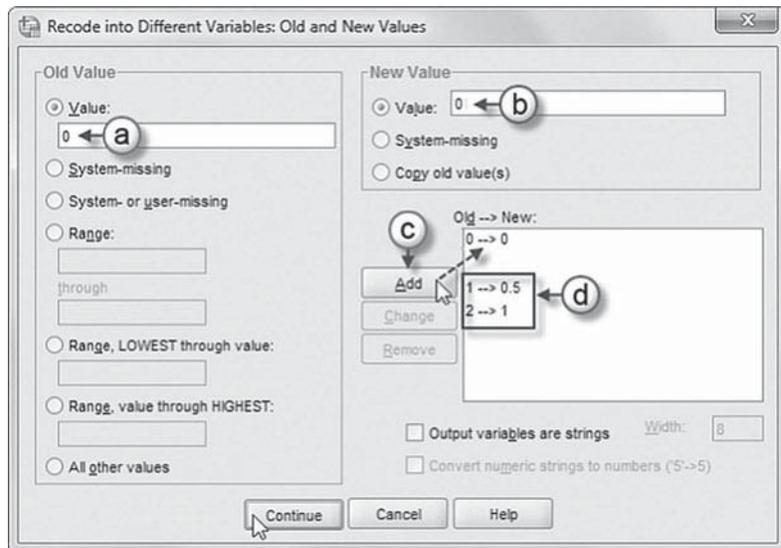
- d. Click OK to continue, which will overwrite the preexisting *timenonlin1* variable. If you prefer not to overwrite the original variable, rename the output variable (e.g., *timenonlin1a*).
- e. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

3. Within the *Recode into Different Variables: Old and New Values* dialog box, we will begin changing the *time* values (0, 1, 2) to reflect *timenonlin1* (0, 0.5, 1).

- Begin by entering the first value for *time* (0) in the *Value* (old) box.
- Next, enter the new value (0) for *timenonlin1* in the *Value* (new) box.
- Click the ADD button to place the first command  $0 \rightarrow 0$  into the *Old → New* box.
- Repeat steps 3a to 3c to complete the remaining coding changes for *timenonlin1* values:

$1 \rightarrow 0.5$

$2 \rightarrow 1$



Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

Click the OK button to generate the recoded variable *timenonlin1* and corresponding time values (0, 0.5, 1).

**Note:** To generate variations of *timenonlin*, repeat all steps but change the output variable and *Old → New* values as follows:

***timenonlin2* (0, 0.6, 1)**

$0 \rightarrow 0$

$1 \rightarrow 0.6$

$2 \rightarrow 1$

***timenonlin3* (0, 0.7, 1)**

$0 \rightarrow 0$

$1 \rightarrow 0.7$

$2 \rightarrow 1$

***timenonlin* (0, 0.53, 1)**

$0 \rightarrow 0$

$1 \rightarrow 0.53$

$2 \rightarrow 1$

**TABLE 5.20** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	48.725	.097	8,669.000	500.287	.000	48.534	48.916
timenonlin1	8.421	.121	8,669.000	69.589	.000	8.184	8.658

<sup>a</sup> Dependent variable: test.

Because we have some evidence that the growth is slightly greater between the first and second intervals and slows between the second and third intervals, we might instead try coding the middle interval as 0.6 (i.e., 0.0, 0.6, 1.0). This coding will represent a slightly slowing trend (*timenonlin2*). When we code the time variable like this, we obtain an AIC coefficient of 189,157.5 (model syntax is provided in Appendix A). If, instead, we thought the slowing might be a bit more extreme, we could code the middle interval as 0.7 (0.0, 0.7, 1.0). In this case (*timenonlin3*), however, we obtain an AIC of 189,335.3 (model syntax is provided in Appendix A). So clearly, coding the middle interval as 0.7 does not fit the data as well as either the first (linear) or second (slightly slowing) coding schemes. In Table 5.20, we provide the estimates of the linear example where we coded the time-related variable as 0.0, 0.5, 1.0. The initial status intercept is estimated to be 48.725, which is a bit higher than the initial intercept of 48.632 in Table 5.2. The growth over the entire trend is estimated as 8.421. In this case, then, we would estimate the ending achievement level as 57.146 ( $48.725 + 8.421 = 57.146$ ), which is close to the observed intercept of 57.094 in Table 5.2. The linear model estimates in Table 5.20, therefore, fit the data quite well.

## Estimating the Final Time-Related Model

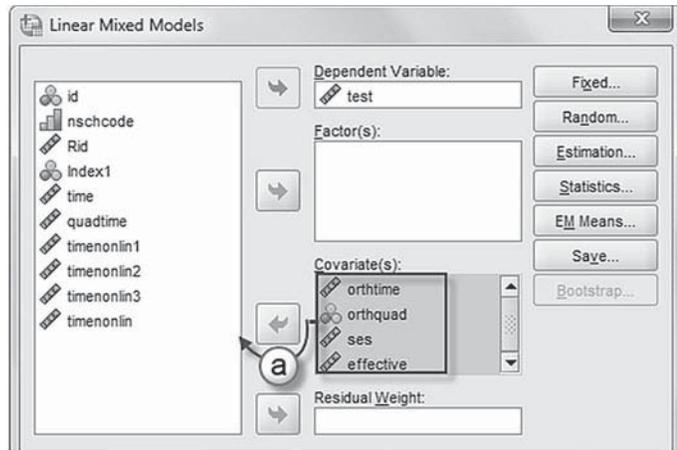
We note that the optimal estimate for the middle interval is actually 0.529 (which does suggest a bit of slowing over time). We obtained this optimal estimate using SEM software, as the level-and-shape LCA approach provides the exact, model-estimated parameter for the middle interval of the latent growth factor (see comparison in Appendix B). In this final IBM SPSS model, the initial intercept was estimated as 48.632, and the time-related estimate (*timenonlin*) was 8.462. The key modeling changes for this model are illustrated in Model 2.1. If we add those, we obtain the intercept estimate for Time 3 in Table 5.2 (57.094). This model also had the lowest AIC coefficient compared with the other coding schemes.

## Defining Model 2.1 with IBM SPSS Menu Commands

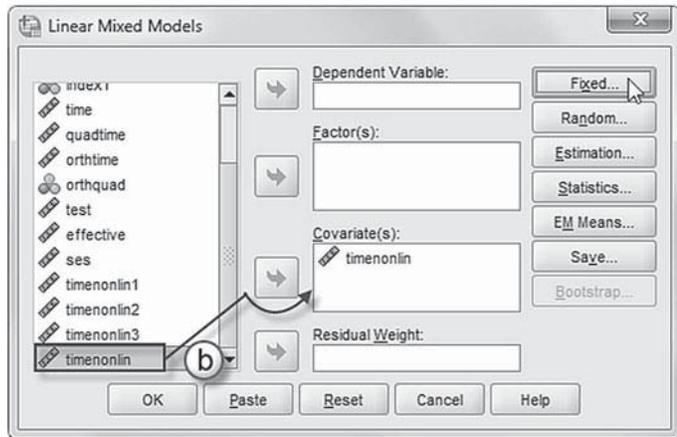
Continue using the *ch5growthdata-vertical.sav* data. Settings default to those used for Model 1.3.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.
2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from Model 1.3. Place the variables *id* and *time* within the *Subjects* and *Repeated* boxes. The *Repeated Covariance Type* is specified as *Diagonal*. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

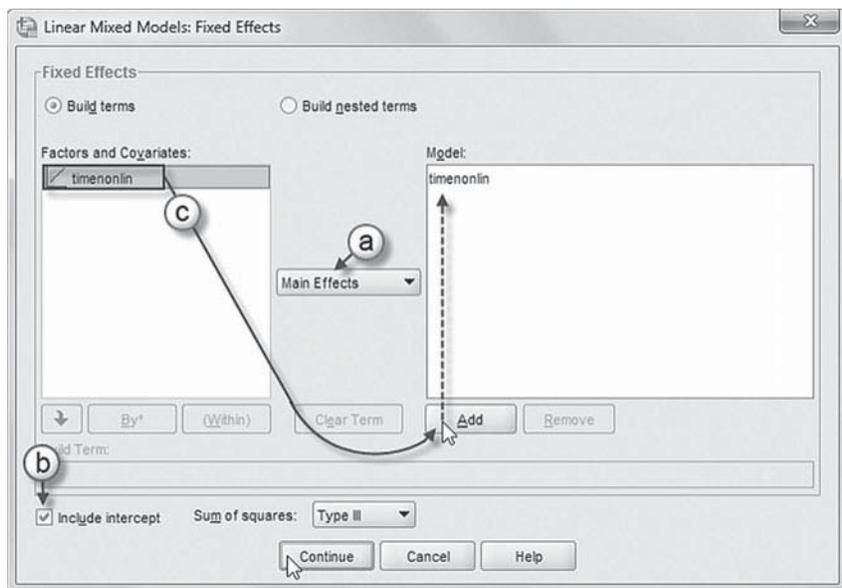
3. The *Linear Mixed Models* dialog box displays *test* in the *Dependent Variable* box with *orthtime*, *orthquad*, *ses*, and *effective* in the *Covariate(s)* box.
  - a. Remove all variables from the *Covariate(s)* box by clicking to select them and then clicking the left-arrow button.
  - b. Click to select *timenonlin*, and then click the right-arrow button to move the variable into the *Covariate(s)* box.



Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, confirm that *Main Effects* is selected.
- b. Confirm that *Include intercept* is selected.
- c. Click to select *timenonlin*, and then click the ADD button to move it into the *Model* box.



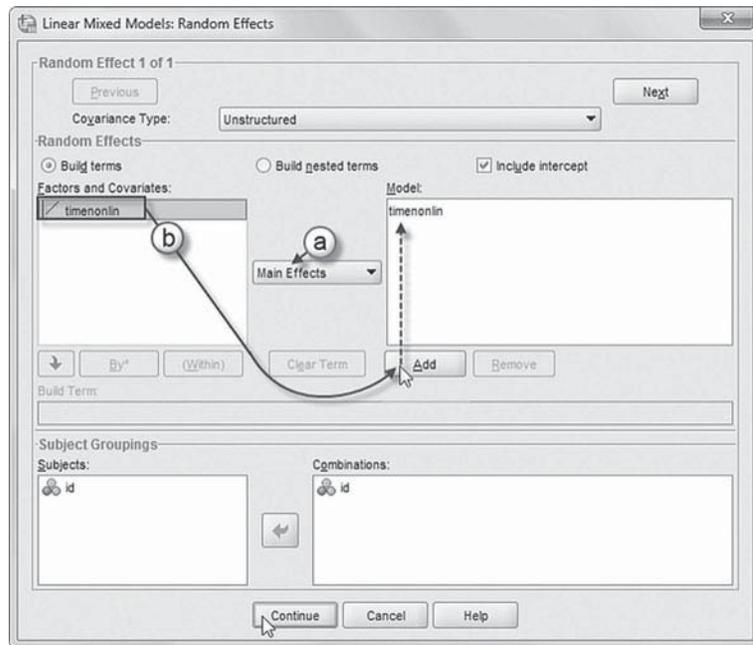
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. Within the *Linear Mixed Models: Random Effects* box, the covariance type and intercept are set to the default settings of the prior model.
  - a. The *Main Effects* option is preselected.
  - b. Click to select *timenonlin* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Adding the Two Predictors

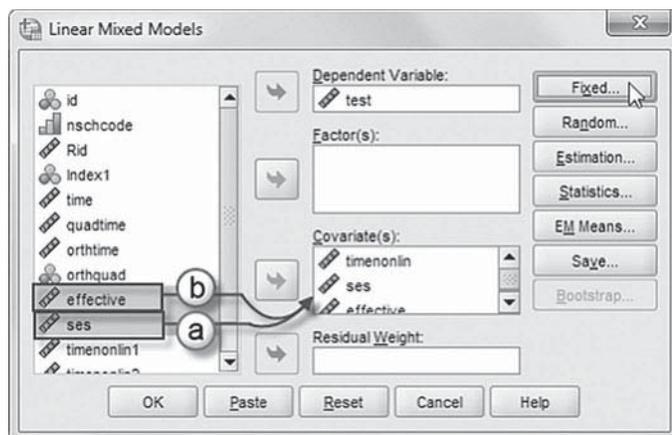
Finally, we add the two between-individual predictors. In this case, we use the optimal estimate for Time 2 (0.529). Key modeling changes are illustrated in Model 2.2.

#### Defining Model 2.2 with IBM SPSS Menu Commands

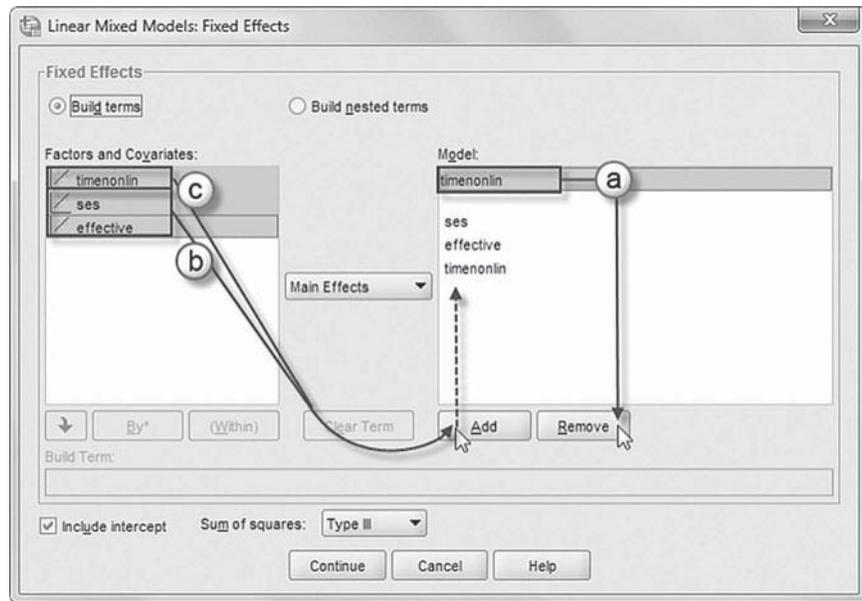
Continue using the *ch5growthdata-vertical.sav* data. Settings default to those used for Model 2.1.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.
2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from Model 2.1. Place the variables *id* and *time* within the *Subjects* and *Repeated* boxes. The *Repeated Covariance Type* is specified as *Diagonal*. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.
3. The *Linear Mixed Models* dialog box displays the default settings for Model 2.1.
  - a. Click to select *ses*, and then click the right-arrow button to move the variable into the *Covariate(s)* box.
  - b. Click to select *effective*, and then click the right-arrow button to move the variable into the *Covariate(s)* box.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



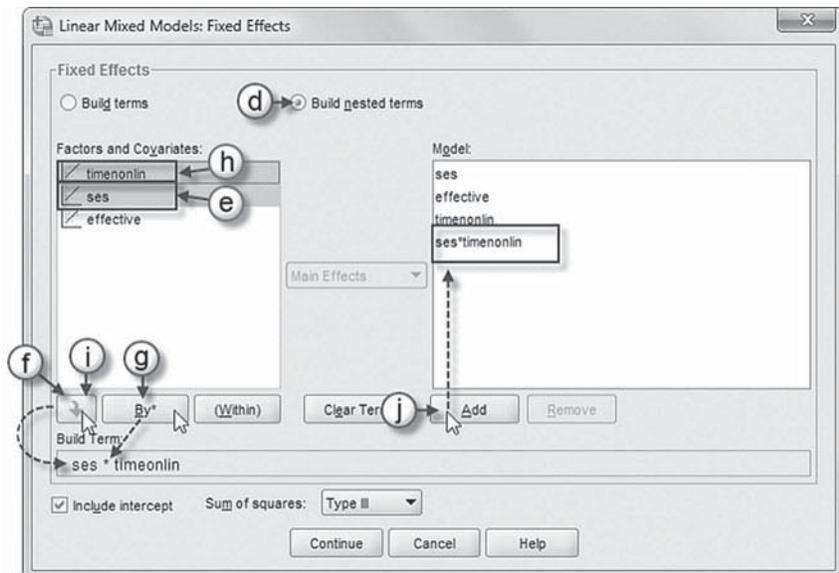
- 4a. To facilitate reading of the output tables, we will change the sequence order of the model variables. Click to select *timenonlin*, and then click the REMOVE button.
- b. Click to select *ses* and *effective*, and then click the ADD button to move the variables into the *Model* box.
- c. Click to select *timenonlin*, and then click the ADD button to move it into the *Model* box.



Two cross-level interactions (or nested terms) will be created and added to the model:  $ses*timenonlin$  and  $effective*timenonlin$ . These interactions will tell us if the growth trajectories are parallel for different groups of students.

#### Add First Interaction to Model 2.2: $ses*timenonlin$

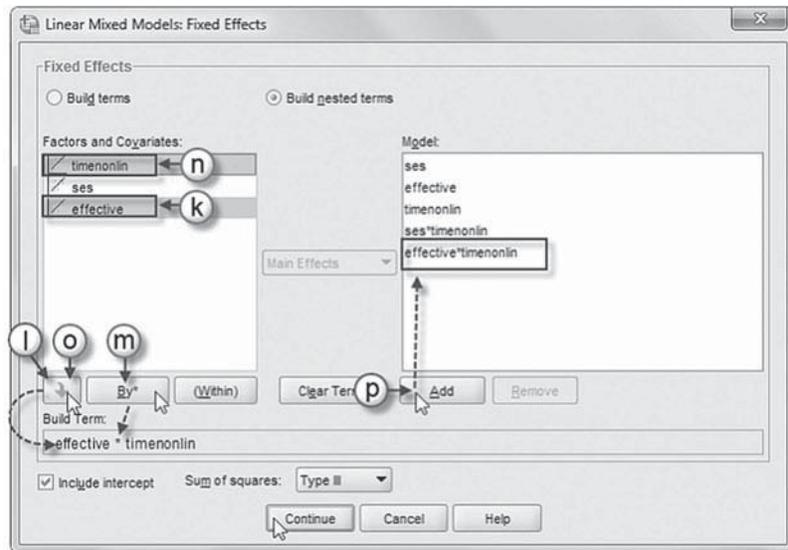
- d. Click to select *Build nested terms*.
- e. Click to select the variable *ses* from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves *ses* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button, which will insert the computation command symbol:  $ses*$ .



- h. Click to select *timenonlin* from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move *timenonlin* into the *Build Term* box and complete the interaction term:  $ses*timenonlin$ .
- j. Click the ADD button to transfer the interaction into the *Model* box.

Add Second Interaction to Model 2.2: *effective\*timenonlin*

- k. Click to select the variable *effective* from the *Factors and Covariates* box.
- l. Next, click the arrow button below the *Factors and Covariates* box. This moves *effective* into the *Build Term* box.
- m. Now click the BY\* button, which will insert the computation command symbol: *effective\**.
- n. Click to select *timenonlin* from the *Factors and Covariates* box.
- o. Click the arrow button below the *Factors and Covariates* box to move *timenonlin* into the *Build Term* box and complete the interaction term: *effective\*timenonlin*.
- p. Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Finally, click OK to run the model.

**Interpreting the Output From Model 2.2**

The fixed effects for the MIXED solution are presented in Table 5.21. The adjusted initial status intercept is estimated as 47.300. We can see that perceived teacher effectiveness affects the intercept (2.406,  $p < .001$ ). Student SES is again not significantly related to initial status ( $p > .05$ ). Turning to growth, we can see that the *ses\*timenonlin* effect is negative and significant ( $-.308$ ,  $p < .05$ ), once again suggesting that students with higher SES demonstrated less growth over the time of the study. Similar to the polynomial model, we can also note that students with teachers who are rated as effective demonstrated more growth over time (6.976,  $p < .001$ ), compared to their peers who rated their teachers as average or ineffective.

**TABLE 5.21** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	47.300	0.145	8,667.000	325.943	.000	47.015	47.584
ses	0.231	0.124	8,667.000	1.856	.063	-0.013	0.475
effective	2.406	0.196	8,667.000	12.302	.000	2.022	2.789
timenonlin	4.637	0.173	8,667.000	26.861	.000	4.298	4.975
ses * timenonlin	-0.308	0.148	8,667.000	-2.084	.037	-0.598	-0.018
effective * timenonlin	6.976	0.233	8,667.000	29.989	.000	6.520	7.432

<sup>a</sup> Dependent variable: test.

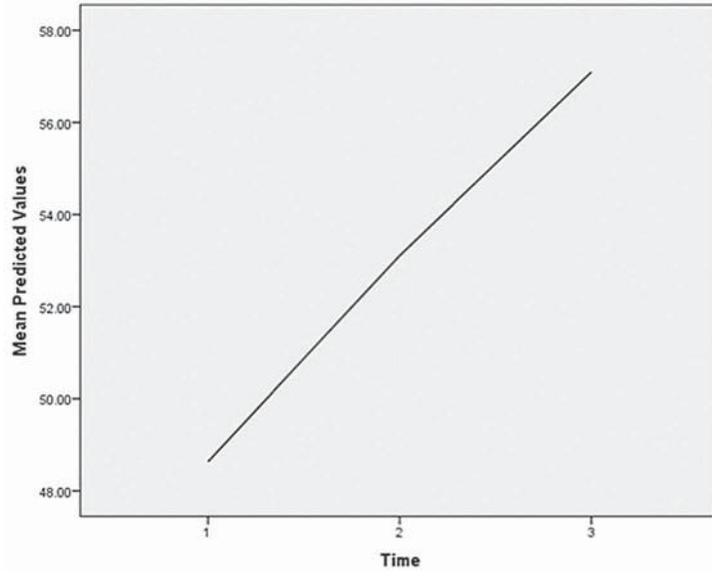


FIGURE 5.9 Mean predicted values from nonlinear growth model.

We can also save mean predicted values from the fixed-effect model in the data set. In Figure 5.9, we can see that the average predicted trajectory appears to slow a bit between the second and third intervals.

We also provide the variance components in Table 5.22. We can see that there is still significant variation in both intercepts and slopes left to be explained between individuals. In this formulation, we note that the covariance between the intercept and slope is significant and negative ( $-6.917, p = .001$ ). In this case, this negative relationship represents the well-known tendency for students who start with higher achievement to demonstrate less growth over time and vice versa. We again make the point that it is typically the case in growth modeling that when the coding of the growth parameters is changed, the relationship between the intercept and slope also changes.

We also draw attention to the fact that this latter solution using SPSS MIXED produces results almost identical to the level-and-shape specification estimated using Mplus. We note that there are some advantages to defining growth models in this manner. One of the main advantages is that it is easier to build models on one randomly varying time slope parameter, as opposed to sometimes building them on two or more polynomials (depending on the number of

TABLE 5.22 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [Index1 = 1]	61.090	2.107	28.993	.000	57.097	65.363
	Var: [Index1 = 2]	61.343	1.153	53.212	.000	59.125	63.644
	Var: [Index1 = 3]	28.489	1.633	17.450	.000	25.463	31.876
Intercept + timenonlin [subject = id]	UN (1, 1)	30.968	1.943	15.934	.000	27.384	35.022
	UN (2, 1)	-6.917	2.154	-3.211	.001	-11.139	-2.695
	UN (2, 2)	28.798	3.506	8.214	.000	22.685	36.559

<sup>a</sup> Dependent variable: test.

random effects that can be supported by the repeated measures). This alternative to polynomial growth models tends to simplify the model-building process. It does, however, take some work in finding a coding of the time-related variable that is consistent with the observed data.

### An Example Experimental Design

Before we leave the individual growth-modeling approach, we provide an example illustrating a comparison of individual growth trajectories using an experimental design. Consider a study to examine whether or not students' participation in a treatment designed to target their deficiencies in solving math problems helps increase their math scores over time. The variables are summarized in Table 5.23. The data in this study consist of 55 middle-school students who were randomly assigned to either a control ( $N = 30$ ) group or a treatment ( $N = 25$ ), where they received individualized instruction with a math tutor over a semester.

The students were measured on four occasions (one pretest) regarding their math achievement during the period of the study. The design is specified as follows:

$$\begin{array}{ccccc} R & O_1 & O_2 & O_3 & O_4 \\ R & O_1 X & O_2 & O_3 & O_4 \end{array}$$

The Rs refer to random assignment. Fixed effects include the treatment (coded = 1) versus control (coded 0), time (coded 0–3 for the four occasions), and the treatment-by-time interaction.

**TABLE 5.23** Data Definition of *ch5experimentaldesigndata.sav* ( $N = 55$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>id</i>	Individual	Student identifier (55 students across four time [test] occasions).	Integer	Ordinal
<i>treatment</i>	Individual	Two-category predictor variable representing students assigned for nontreatment (control) or treatment.	0 = No Treatment, (No Tutor), 1 = Treatment (Tutored)	Scale
<i>time</i>	Within Individual	Variable representing four linear occasions in time (0, 1, 2, 3) measuring students math achievement.	0 = First Time, 1 = Second Time, 2 = Third Time, 3 = Fourth Time	Scale
<i>timenonlin</i>	Within Individual	Recoded <i>time</i> variable from four occasions in time (0, 1, 2, 3) into a time sequence variation representing the whole four time occasions (0.00, 0.50, 0.70, 1.00) measured from 0 to 1.	0.00 = First Time, 0.50 = Second Time, 0.70 = Third Time, 1.00 = Fourth Time	Scale
<i>math</i>	Within Individual	The dependent variable representing each students individual scores on the repeated math measurements.	523 to 856	Scale
<i>orthtime</i>	Within Individual	Recoded <i>time</i> variable from four occasions in time (0, 1, 2, 3) into a different time sequence (-3, -1, 1, 3).	-3 = First Time, -1 = Second Time, 1 = Third Time, 3 = Fourth Time	Scale
<i>orthquad</i>	Within Individual	Recoded <i>time</i> variable from four occasions in time (0, 1, 2, 3) into a time sequence variation(1, -1, -1, 1).	1 = First Time, -1 = Second Time, -1 = Third Time, 1 = Fourth Time	Scale
<i>orthcubic</i>	Within Individual	Recoded <i>time</i> variable from four occasions in time (0, 1, 2, 3) into a time sequence variation (-1, 3, -3, 1).	-1 = First Time, 3 = Second Time, -3 = Third Time, 1 = Fourth Time	Scale

<sup>a</sup> Individual = Level 2; Within individual = repeated measures, Level 1.

The initial  $O_1$  coefficients refer to students' initial status achievement. We expect no initial difference between the treatment and control groups. This is captured by the relationship between the treatment variable and the students' initial status intercept at the beginning of the study (where time is coded 0). After implementing the treatment after the initial pretest measurement, we propose that the treatment group will increase more in math achievement over time than the control group. This will be tested by a treatment-by-time interaction; that is, we would expect different growth trends for the control and treatment groups.

We can specify the Level 1 model for individual  $i$  measured at time occasion  $t$  as follows:

$$Y_{it} = \pi_{0i} + \pi_{1i}time_{it} + \varepsilon_{it}. \quad (5.22)$$

Since the time period for the study is short (i.e., one semester), we might assume a linear model as fitting reasonably well. We note that we could fit the model to a higher order polynomial function. One complication with that approach, however, is that transforming higher order polynomials will shift the intercept to the grand mean. This will obscure the achievement level where each group started before the intervention. We illustrate the two different growth trajectories in Figure 5.10. The figure suggests that the two groups are close before the treatment begins (Time 0). We can see that the treatment growth is closer to a linear growth model over time; however, the control group definitely slows in its progress over the semester.

We first provide results using the model with time specified as linear to describe growth over the short period of time of the study. At Level 2, we assume that the intercept varies between subjects:

$$\pi_{0i} = \beta_{00} + \beta_{01}treatment_i + u_{0i}. \quad (5.23)$$

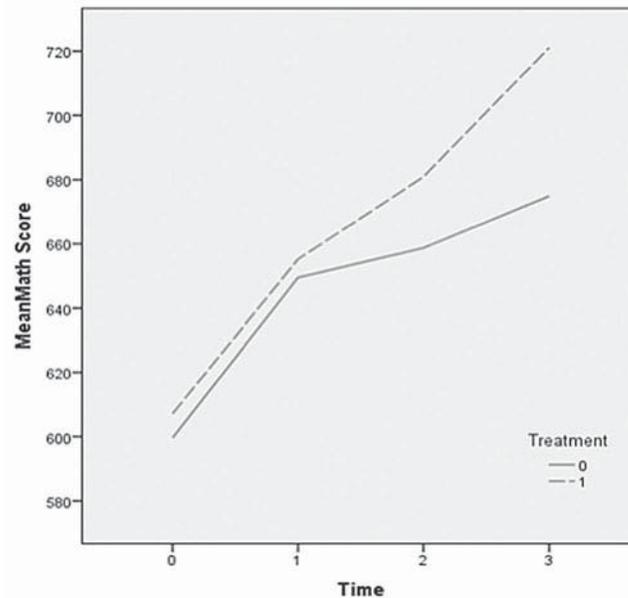


FIGURE 5.10 Examining treatment and control growth trends.

We will also model that the time slope is randomly varying:

$$\pi_{1i} = \beta_{10} + \beta_{11} \text{treatment}_i + u_{1i}. \quad (5.24)$$

After substituting the Level 2 equations into the Level 1 equation, the combined model will then be the following:

$$Y_{it} = \beta_{00} + \beta_{01} \text{treatment}_i + \beta_{10} \text{time}_{it} + \beta_{11} \text{time}_{it} * \text{treatment}_i + u_{1i} \text{time}_{it} + u_{0i} + \varepsilon_{it}. \quad (5.25)$$

We can see that the key parameter in this simple illustration is the *time\*treatment* interaction. This is used to determine if there are different growth trajectories for individuals in the treatment and control groups. We will also specify a diagonal matrix for the residual error structure at Level 1:

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \quad (5.26)$$

At Level 2, we will specify an unstructured covariance matrix:

$$\begin{bmatrix} \sigma_{1,1}^2 & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2}^2 \end{bmatrix} \quad (5.27)$$

We present the fixed-effects estimates in Table 5.24 and syntax to replicate the analysis in Appendix A. We note in passing that we investigated several different covariance structures for the Level 1 repeated measures, but we decided to stay with a diagonal structure, which fit slightly better than the simple scaled identity covariance structure since the within-subject covariance structure was really not the focus of the study. The  $-2LL$  for this model with 11 parameters was 2,054.67, and the AIC was 2,068.67.

The fixed-effect estimates suggest that the students in the control group started with a mean score of 609.057. This linear model seems to slightly overestimate the control group's starting achievement in Figure 5.10 (as well as its ending achievement). Over each interval, individuals'

**TABLE 5.24** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	609.057	6.636	51.556	91.782	.000	595.738	622.376
time	23.813	2.500	47.331	9.525	.000	18.785	28.842
treatment	0.664	9.843	51.556	0.067	.946	-19.091	20.419
time * treatment	12.996	3.708	47.331	3.505	.001	5.538	20.454

<sup>a</sup> Dependent variable: math.

scores in the control group increased by 23.813 points on average. We can also confirm that there was no difference between the treatment and control groups initially ( $\beta = 0.664, p = .946$ ). Over time, however, we can observe that students who received the targeted intervention increased their scores over each interval at a greater rate (12.996 points) than their peers in the control group.

We present the covariance parameter estimates in Table 5.25. We can see that the correlation between initial status and growth was negative ( $-0.266$ ), suggesting that students who started higher in achievement demonstrated a bit less growth over time (and vice versa), but it was not significant ( $p > .05$ ).

One problem with this specification, however, is that we do not take in the curvilinear nature of the growth occurring over time. Alternatively, as we mentioned earlier, if we were to treat the time variable as categorical, we could also specify the model to determine the difference between treatment and control group at *each* occasion, instead of assuming a polynomial growth curve over the entire temporal sequence. This model, summarized in Table 5.26, fits the data slightly better than the previous one, using AIC as an index of model fit (2,013.91 to 2,068.67, respectively).

**TABLE 5.25** Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Repeated Measures	Var: [time = 0]	305.773	188.462	1.622	.105	91.362	1,023.373
	Var: [time = 1]	421.087	104.377	4.034	.000	259.048	684.485
	Var: [time = 2]	150.015	55.704	2.693	.007	72.454	310.604
	Var: [time = 3]	251.289	118.592	2.119	.034	99.646	633.701
Intercept + time [subject = id]	Var(1)	1,099.161	290.476	3.784	.000	654.807	1,845.054
	Var(2)	131.754	37.409	3.522	.000	75.523	229.851
	Corr(2, 1)	-0.266	0.179	-1.480	.139	-0.572	0.106

<sup>a</sup> Dependent variable: math.

**TABLE 5.26** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	674.900	8.595	94.019	78.524	.000	657.835	691.965
[time = 0]	-75.286	6.832	58.889	-11.019	.000	-88.958	-61.614
[time = 1]	-25.400	6.412	51.339	-3.961	.000	-38.270	-12.530
[time = 2]	-16.167	7.879	74.257	-2.052	.044	-31.864	-0.469
[time = 3]	0.000 <sup>b</sup>	0.000	.	.	.	.	.
Treatment	46.220	12.748	94.019	3.626	.000	20.908	71.532
[time = 0] * treatment	-38.685	10.134	58.889	-3.817	.000	-58.964	-18.405
[time = 1] * treatment	-40.480	9.510	51.339	-4.256	.000	-59.569	-21.391
[time = 2] * treatment	-24.033	11.686	74.257	-2.057	.043	-47.317	-0.750
[time = 3] * treatment	0 <sup>b</sup>	0	.	.	.	.	.

<sup>a</sup> Dependent variable: math.

<sup>b</sup> This parameter is set to 0 because it is redundant.

In this case, since IBM SPSS uses the last category of *time* as the reference group, we now have the intercept defined as “ending” math achievement status (674.900) for the control group (coded 0). This ending achievement level for the control group is consistent with Figure 5.10. The treatment group would be estimated as 721.120. We can see that students demonstrated considerable (but differing) growth at each time occasion as represented by the *time* effects. The time coefficients refer to how much lower the control group was at each interval preceding the ending status intercept. We can use the coefficients to estimate achievement for each group at any occasion. For example, the control group’s achievement at the beginning of the study will be  $674.900 - 75.286$ , or 599.614. As we might expect, Table 5.26 also suggests that growth for the control group slows considerably over successive intervals. We can also note that at the *end* of the study, there is a positive effect for being in the treatment group (46.220,  $p < .001$ ). We can also see that the amount of difference at each occasion between the treatment and control groups, which is modeled as the *time\*treatment* interactions, is significant for each occasion ( $p < .05$ ). In this case, the coefficients for each interaction suggest a differing achievement “advantage” for the treatment group at each of the preceding intervals, rather than assuming a *constant linear* effect over time, as summarized in Table 5.24. The information about the treatment can be used along with the other relevant time-related information to calculate where the treatment group is at any occasion in the study. The pattern of results observed in Table 5.26 reflects the different coding scheme between using initial status in the previous model (see Table 5.24 on page 235, where there was no difference between the two groups) and ending status (Table 5.26), where it is obvious in Figure 5.10 that there is considerable difference in achievement between the treatment and control groups.

We also provide results using an alternative coding for time in Table 5.27. In this case, we defined the time-related variable as 0, 0.5, 0.7, 1.0 (*timenonlin*), in order to capture the change taking place over the whole study. This is one way we can easily deal with the curvilinear shape of the average growth trajectories in each group. We can simply substitute *timenonlin* for *time* in Equation 5.22. This solution has the advantage of having an initial status (602.058) for the control group and illustrates that the treatment growth was not significantly different at the beginning of the study (3.217,  $p = .742$ ). Over the time period studied, the treatment group demonstrated considerably higher growth compared to the control group (31.428,  $p < .01$ ). The covariance parameters were similar to the previous linear growth model in Table 5.25, so we do not reproduce them here. We note also that this model fit better than the first model summarized in Table 5.27 (AIC = 2,048.35 to 2,068.67, respectively), but not as well as the previous model summarized in Table 5.26 (AIC = 2,048.35 to 2,013.91, respectively). Substantively, however, all three models provide the same interpretation regarding the positive effect of the educational treatment under consideration. At the end of the study, for example, we would estimate the math achievement of the control group as  $602.058 + 79.072$ , or 681.130. We would estimate the treatment group’s math achievement as  $602.058 + 3.217 + 79.072 + 31.428$ , or 715.775.

These are slightly different from the previous model but certainly consistent with it substantively.

**TABLE 5.27** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	602.058	6.552	50.555	91.885	.000	588.901	615.215
timenonlin	79.072	6.738	47.201	11.735	.000	65.518	92.626
treatment	3.217	9.719	50.555	0.331	.742	16.298	22.732
timenonlin * treatment	31.428	9.995	47.201	3.145	.003	11.324	51.532

<sup>a</sup> Dependent variable: math.

## Summary

In this chapter, we presented a basic two-level model for investigating individual change. Longitudinal analysis represents a rapidly growing application of basic multilevel-modeling techniques. In comparison to ANOVA, we suggested that multilevel modeling of growth trajectories is a more flexible approach because of its ability to handle a wide range of data situations (incomplete data and varying occasions of measurement). The model provides considerably more information about students' initial status and growth rates. The individual growth model can easily be extended to include successive grouping structures above the individual level. We investigate models with individual and group components in the next chapter. As we noted previously in this chapter, there are a considerable number of ways to investigate changes in individuals over time. We provided several different coding possibilities throughout the chapter. We encourage readers to consult a number of introductory sources that provide overviews of the assumptions, uses, and programming of longitudinal models (e.g., Duncan, Duncan, & Strycker, 2006; Raudenbush, Bryk, Cheong, & Congdon, 2004; Raykov & Marcoulides, 2008; Singer & Willett, 2003).

## CHAPTER 6

# Applications of Mixed Models for Longitudinal Data

In this chapter, we expand on our introduction to methods that can be used to examine changes in individuals over time. As we noted in the last chapter, time is often a key factor in understanding how developmental processes may unfold, as well in observing their impact. Longitudinal data collection facilitates the investigation of proposed relationships whose effects become more apparent over time. Presently, there is a wider set of options available for including a temporal dimension in studies, depending on the specific goals of the research and the data structure used to investigate proposed theoretical models. We saw in the last chapter that the mixed-model approach for examining individual change is quite straightforward for repeated measures that are continuous. We examined growth in students' achievement scores related to differences in their backgrounds and the perceived effectiveness of their teachers. In such two-level models of growth, the unit of analysis is the individual student. If we had linked students to their teachers or to their schools, this would have necessitated a third level in the model. In fact, if we included both teachers and schools, we would require a four-level model to examine variability in their achievement—with differences likely due to within-individual growth rates, between-individual variables, between-teacher variables, and between-school variables. SPSS MIXED can actually facilitate this type of complex analysis, although it may take a considerable amount of time and some effort to reduce the computer space it takes to complete the analysis.

In this chapter, we consider several useful research applications of the two-level growth model provided in the last chapter. First, we extend the basic two-level growth model to include a grouping structure as a third level. Second, we provide an example of a multilevel formulation of a regression discontinuity design, which can be used to investigate the effect of an intervention in a treatment versus a control group. This research design is useful in examining changes that occur as a result of an intervention when prior random assignment of individuals to groups is not possible. In this case, we use an assessment of math skills as a means of assigning individuals to the treatment and control groups and then examine the effects of the treatment on students' subsequent achievement. Third, we develop a piecewise growth model, which provides a means of examining two or more different growth trends within one model. In this illustration, we develop a model to investigate institutional trends in admitting freshman student athletes before and after a policy to raise admission standards is introduced.

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### Examining Growth in Undergraduate Graduation Rates

Consider a national study to investigate whether undergraduate degree completion levels are rising over time. In this example, we will consider an application of random-coefficients growth modeling to examine between-state differences in the proportion of undergraduates who graduate

from public universities and changes in the proportion of graduates over an 11-year period. In our example study, summarized in Figure 6.1, the within-subjects data (Level 1) consist of repeated measures on the proportion of undergraduate students who graduate nested within institutions. The between-subjects data (Level 2) consist of institutional variables that may explain differences in graduation levels between institutions, and the group-level data (Level 3) consist of state-level information such as financial resources appropriated to support public higher education.

## Research Questions

The three-level growth formulation facilitates investigating a number of different types of multilevel relationships. As the figure indicates, within institutions (Level 1), we can examine relationships involving various time-varying covariates that might influence institutional graduation rates over time. Time-varying covariates provide a way of accounting for temporal variation in predictors that may increase (or decrease) the value of an outcome predicted by the individual's growth trajectory (Raudenbush & Bryk, 2002). For example, we could focus on how fluctuation in tuition levels might affect graduation rates. Other examples of time-varying covariates that might influence graduation rates include the proportion of the institution's budget allocated to instructional support or the proportion of students receiving financial aid.

In contrast, we could consider the time-varying covariates as "static" variables (i.e., the average tuition level or the average proportion receiving financial aid), which requires only one value for the covariate. In this type of formulation, we would enter the static covariate as a between-institution variable (i.e., Level 2). We note, however, that one of the advantages of the Level 1 (time-varying) formulation is that the slope of the time-varying covariate (e.g., institutional tuition) on graduation rates can then be modeled as a random parameter at Level 2 (between institutions) and perhaps at Level 3 (between states). In the conceptual model, possible cross-level interactions are shown with arrows extending from a higher level to a lower level. For example, we might propose that differences in student selectivity moderate the effect of resources spent to support student learning on graduation rates, or that differences in student composition moderate the effect of student tuition on graduation rates.

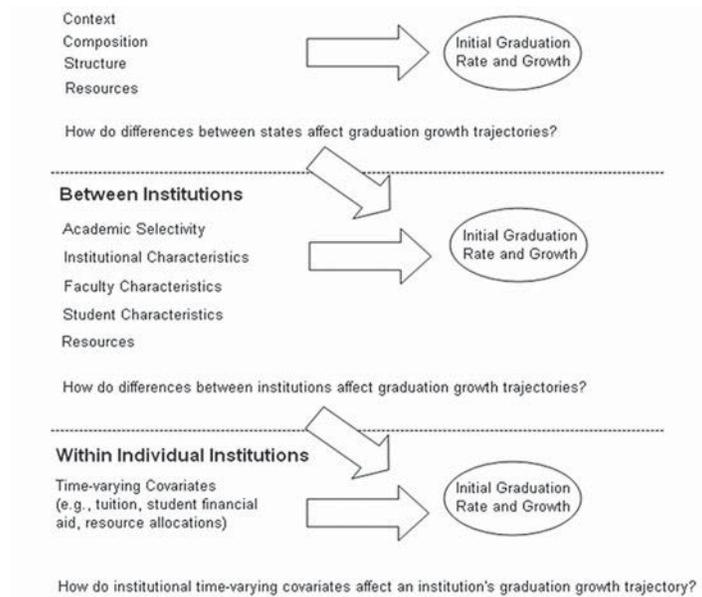
Between institutions (Level 2), we can investigate how various institutional characteristics (e.g., mission, selectivity of admissions, student composition, and faculty characteristics) are related to graduation rates or changes in graduation rates over time. As noted previously, we could also investigate a random covariate–outcome slope that might be of theoretical interest (e.g., tuition–graduation rate or financial aid–graduation rate). Finally, at the state level (Level 3), we can examine how different patterns of state economic and political activity might affect graduation outcomes. For example, we might examine how family share of higher education costs or legislative appropriation for higher education is related to graduation rates and change in graduation rates over time. We could also examine whether a particular type of state higher education policy (such as merit-based tuition support) might influence graduation rates.

The multilevel model implies several research questions such as the following:

- What is the average level and shape of institutions' graduation trajectories over time?
- Which within- and between-institutional variables explain institutional graduation levels?
- How do graduation trajectories vary according features of states?

## The Data

In this example, we focus on a subset of data from a larger study of financing public higher education and changes in student graduation rates over time (Heck, Lam, & Thomas, 2012). The subset consists of undergraduate (6-year) graduation rates from 649 public 4-year institutions within the 50 states. The graduation rates were compiled over an 11-year period. The variables in the example are summarized in Table 6.1.



**FIGURE 6.1** Proposed three-level graduation trajectory model.

**TABLE 6.1** Data Definition of *ch6graduationdata.sav* (*N* = 8,670)

Variable	Level <sup>a</sup>	Description	Values	Measurement <sup>2</sup>
<i>id</i>	Within Individual	Student identifier (8,670 students).	Integer	Ordinal
<i>Rid</i>	Within Individual	A within-group level identifier <sup>b</sup> representing a sequential identifier for each student within 649 schools within 11 time periods within 50 states.	1 to 46	Ordinal
<i>state</i>	State	State identification number.	Integer	Ordinal
<i>time</i>	Within Individual	Variable representing 11 linear occasions (years) in time measuring undergraduate graduation rates.	0 = Time 0 1 = Time 1 2 = Time 2 3 = Time 3 4 = Time 4 5 = Time 5 6 = Time 6 7 = Time 7 8 = Time 8 9 = Time 9 10 = Time 10	Scale
<i>quadtime</i>	Within Individual	Recoded time variable from 11 occasions in time (0, 1, 2, ..., 10) into a "squared" quadratic time sequence (0, 1, 4, ..., 100) to capture any changes (acceleration or deceleration) in the rate of change that might occur over the 11 measurement occasions.	0 = Time 0 1 = Time 1 4 = Time 2 9 = Time 3 16 = Time 4 25 = Time 5 36 = Time 6 49 = Time 7 64 = Time 8 81 = Time 9 100 = Time 10	Scale

(Continued)

TABLE 6.1 (Continued)

Variable	Level <sup>a</sup>	Description	Values	Measurement <sup>2</sup>
<i>time1</i>	Within Individual	Recoded time variable from 11 occasions in time (0, 1, 2, ..., 10) into a time sequence variation that captures the whole 11-year period of time measured from 0 to 1.	0.00 = Time 0 0.10 = Time 1 0.20 = Time 2 0.30 = Time 3 0.40 = Time 4 0.50 = Time 5 0.60 = Time 6 0.70 = Time 7 0.80 = Time 8 0.90 = Time 9 1.00 = Time 10	Scale
<i>quadtime1</i>	Within Individual	Recoded time variable from 11 occasions in time (0, 1, 2, ..., 10) into a "squared" quadratic time sequence that captures the whole 11-year period of time measured from 0 to 1.	0.00 = Time 0 0.01 = Time 1 0.04 = Time 2 0.09 = Time 3 0.16 = Time 4 0.25 = Time 5 0.36 = Time 6 0.49 = Time 7 0.64 = Time 8 0.81 = Time 9 1.00 = Time 10	Scale
<i>gradproportion</i>	School	Proportion of graduating students.	0.00 to 1.00	Scale
<i>mathselect</i>	Within Individual	Predictor interval variable (z-score) measuring percentage of students admitted with SAT or ACT math scores in the 75th percentile or above.	-2.30 to 2.94	Scale
<i>percentFinAid</i>	Within Individual	Predictor interval variable (z-score) measuring the percentage of students within schools receiving financial aid.	-5.69 to 1.67	Scale
<i>tuition</i>	Within Individual	Predictor interval variable (z-score) measuring the average tuition level at the school.	-1.57 to 1.64	Scale
<i>percentFTfaculty</i>	School	Predictor interval variable (z-score) measuring percentage of full-time faculty.	-3.56 to 1.83	Scale
<i>aveRetention</i>	State	Predictor interval variable (z-score) measuring average percentage of freshman retained in the state.	-2.76 to 1.55	Scale
<i>aveFamilyshare</i>	State	Predictor interval variable (z-score) measuring the percentage share that families have to pay for higher education in the state.	-2.41 to 1.87	Scale

<sup>a</sup> Within individual schools = Level 1; School = Level 2; State = Level 3.

<sup>b</sup> Results from ranking student cases (*id*) with the school group identifier (not tabled).

<sup>2</sup> Measurement icon settings displayed in subsequent model screenshots may differ from Tables 6.1, 6.14, and 6.19 but will not affect the output.

## Defining the Model

In modeling changes in institutional graduation rates over time, we assume that a number of institutions have been sampled and measured on one or more variables over several occasions.

Because multilevel modeling does not require balanced data, it is not a problem if all measurements are not available on all participants. This model can be extended to include situations where the timing and spacing of the measurements differ across participants. As we illustrated in the last chapter, in random-coefficients growth modeling, within- and between-individual changes are typically represented through a two-level univariate model (see also Hox, 2010; Raudenbush & Bryk, 2002; Singer & Willett, 2003). A third level (and successive levels) can be added to model changes between higher order units.

### Level 1 Model

The Level 1 part of the model represents the change each member of the population is expected to experience during the time period under study (Singer & Willett, 2003). In this case, each institution's successive graduation measurements can be represented by an individual growth trajectory (or growth curve) and random error. Following Raudenbush and Bryk's (2002) notation, the systematic growth for each individual can be represented as a polynomial of degree  $P$ , with the Level 1 model for the proportion of undergraduates who graduate at time  $t$  for institution  $i$  in state  $j$  written as

$$y_{ij} = \pi_{0ij} + \pi_{1ij}a_{tij} + \pi_{2ij}a_{tij}^2 + \dots + \pi_{p_{ij}}a_{tij}^{p_{ij}} + \varepsilon_{ij}, \quad (6.1)$$

where  $\pi_{0ij}$  is the intercept parameter (which is defined as the level of the institution's "true" status at some relevant point in the series of measurement occasions);  $a_{tij}$  is a time-varying variable of interest, with individual  $i$  measured on  $t$  occasions; and  $\pi_{p_{ij}}$  is the growth trajectory parameter  $p$  for subject  $i$  in group  $j$  associated with the polynomial of degree  $P$  (i.e.,  $p = 0, \dots, P$ ). One or more time-varying covariates ( $X_t$ ) can also be added to the Level 1 model as needed.

The growth (slope) parameters are the most important parameters because they represent the rate at which institution  $i$  within state  $j$  changes over time. The polynomial curves facilitate the representation of several different growth trajectories including, for example, linear ( $a$ ), quadratic ( $a^2$ ), and cubic ( $a^3$ ). In terms of interpretation, however, if one considers that the linear trajectory model represents a *constant* rate of change over time and the quadratic trajectory models represent *change* (accelerating or decelerating) in the rate of change over time, it becomes increasingly more difficult to interpret models of higher polynomial degrees. The specification of the general Level 1 model implicitly assumes that all the change trajectories have a common algebraic form, but not every individual institution has the same trajectory (Singer & Willett, 2003). Each institution, therefore, draws its parameter values (i.e., intercept and slope) from an unknown (underlying) bivariate distribution of intercepts and slopes.

As we noted in the last chapter, a simple error structure is often assumed for  $\varepsilon_{ij}$ , represented as

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2), \quad (6.2)$$

which suggests that the errors are independent and normally distributed ( $N$ ), with a mean of 0 and constant variance across time (Raudenbush & Bryk, 2002). The residual variance parameter ( $\sigma_\varepsilon^2$ ) represents the variation in residuals around each individual institution's true change trajectory (Singer & Willett, 2003). Restrictions about the residuals can be relaxed, however. We reiterate that other types of repeated measures covariance structures (e.g., autocorrelated) can be considered where there are many measurements per subject.

### Level 2 Model

At Level 2, a set of between-institution predictors (e.g., student selectivity and student composition) can be added to the model to explain variation in institutions' intercepts (often defined as initial status) and growth rates. Specifically, for each of the  $P + 1$  individual growth parameters,

$$\pi_{p_{ij}} = \beta_{p0j} + \sum_{q=1}^{Q_p} \beta_{pq} X_{qij} + r_{p_{ij}}, \quad (6.3)$$

where  $X_{qij}$  might include institutional characteristics,  $\beta_{pq}$  represents the effect of  $X_{qij}$  on the  $p$ th growth parameter, and  $r_{p_{ij}}$  is a matrix of random effects. The set of  $P + 1$  random effects for institution  $i$  can be contained in one of several different types of covariance matrices at Level 2. The exact dimensionality of this covariance matrix depends on the number of random Level 2 coefficients in the model. Common choices are diagonal, with variances for random effects only and no covariances between intercepts and slopes, or a completely unstructured covariance matrix, with variances for each random effect and covariances between each pair of random effects. If the choice is an unstructured covariance matrix, it is dimensioned  $(P + 1) \times (P + 1)$  and is assumed to be multivariate normally distributed, with means of 0, and some variances and covariances between Level 2 residuals.

### Level 3 Model

At Level 3, we can model differences in growth between states using the general modeling framework to examine variation in the Level 2 random intercept ( $\beta_{p0}$ ) and slope ( $\beta_{pq}$ ) parameters. The model defining Level 3 relationships can be written as:

$$\beta_{p_{qj}} = \gamma_{p_{q0}} + \sum_{s=1}^{S_{pq}} \gamma_{p_{qs}} W_{sj} + u_{p_{qj}}, \quad (6.4)$$

where  $\gamma_{p_{q0}}$  is the intercept for the state-level model,  $W_{sj}$  represents predictors such as level of state resources for public higher education,  $\gamma_{p_{qs}}$  represents structural parameters for the Level 3 predictors, and  $u_{p_{qj}}$  represents the state-level random effects. There are also similar choices regarding the covariance matrix of residuals from the Level 3 equations (e.g., diagonal and unstructured). Similarly, the dimensionality of the covariance matrix at Level 3 depends on the number of random coefficients at that level.

Figure 6.2 summarizes the average of the institutional graduation rates over time, without regard to the nesting of institutions within states. One can see from the figure that public undergraduate graduation rates appear to increase over the decade by about 4%.

We next provide a small sample of eight individual states' trajectories in Figure 6.3. Readers who wish to develop this graph can do so by using the "select if" command and then defining the subset ( $stateid > 15$  &  $stateid < 24$ ). There is one state trajectory considerably higher than the others in this subset. In general, however, it appears that most of the individual trajectories increase slightly over time.

The first concern in the analysis is whether there is any significant change in graduation rates that took place over the period studied. For example, it might be that there was no increase in graduation rates over time. Second, we might want to examine possible differences in institutions' and states' graduation rates over time due to their resources (e.g., tuition levels and percentages of students receiving financial aid), institutional characteristics (selectivity and faculty characteristics), and state characteristics (e.g., average student retention rates and family share of higher education).

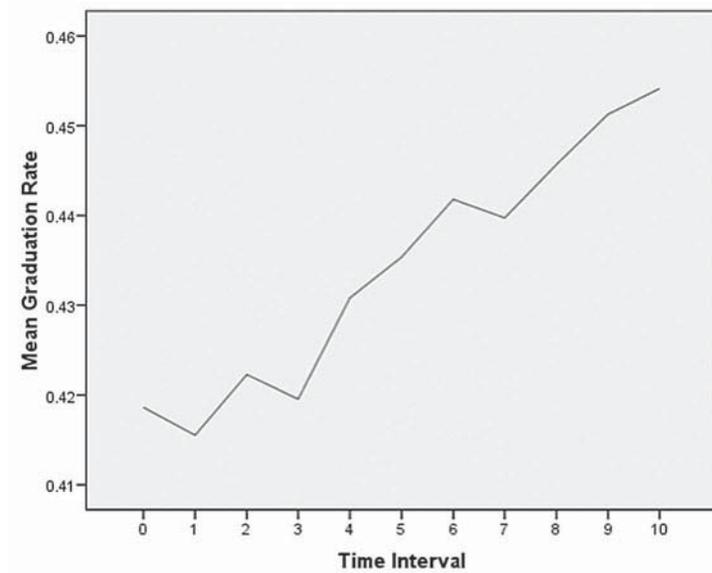


FIGURE 6.2 Public institutional graduation level rates over time.

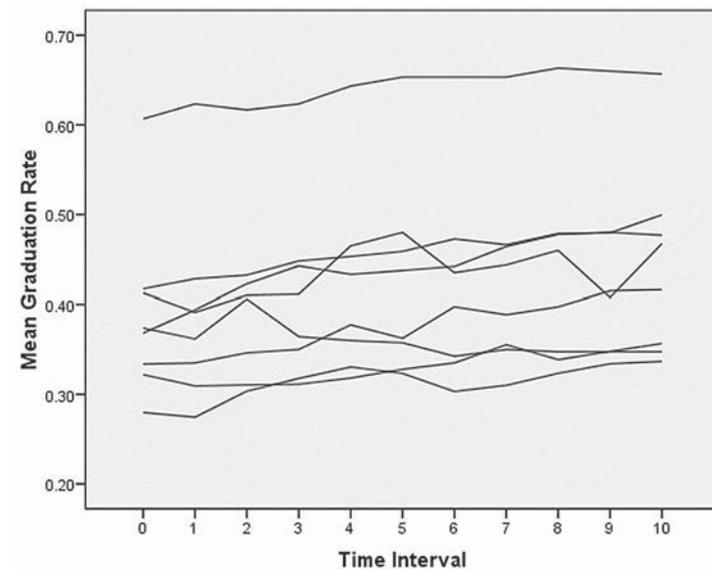


FIGURE 6.3 Graduation rates for selected states.

### The Null Model: No Predictors

As we noted in the last chapter, it is possible to examine an *unconditional means* model first—that is, a simple model consisting only of the repeated measurements of  $y_{ij}$  without the time-related parameter. This model can be used to partition the variance in the grand-mean estimate into its within-individual and between-individual components, regardless of time (see Singer & Willett, 2003, for further discussion). These components can be used to calculate an intraclass correlation (ICC). We begin with a simple unconditional model to examine the variance decomposition of graduation rates across levels of the data hierarchy:

$$y_{tij} = \pi_{0ij} + \varepsilon_{tij}, \quad (6.5)$$

where  $y_{tij}$  is the graduation proportion for institution  $i$  in state  $j$  measured at time point  $t$ . The intercept is  $\pi_{0ij}$  and  $\varepsilon_{tij}$  is the error term. At Level 2, the model is

$$\pi_{0ij} = \beta_{00j} + r_{0ij}, \quad (6.6)$$

where  $\beta_{00j}$  is the intercept, and we will use  $r_{0ij}$  as the error term for the between-individuals model. At Level 3, the model is

$$\beta_{00j} = \gamma_{000} + u_{00j}, \quad (6.7)$$

where  $\gamma_{000}$  is the intercept and  $u_{00j}$  is the error in equations at Level 3.

For this first “no-predictors” model (i.e., with no time-related variable specified), we can examine the decomposition of variance in graduation rates associated with differences within institutions, between institutions, and between states. The proportion of variability (intraclass correlation) in outcomes at Level 3 is defined as

$$\rho = \sigma_{Level 3}^2 / (\sigma_{Level 1}^2 + \sigma_{Level 2}^2 + \sigma_{Level 3}^2) \quad (6.8)$$

For Level 2, the ICC would be

$$\rho = \sigma_{Level 2}^2 / (\sigma_{Level 1}^2 + \sigma_{Level 2}^2 + \sigma_{Level 3}^2) \quad (6.9)$$

and for Level 1, the ICC would be

$$\rho = \sigma_{Level 1}^2 / (\sigma_{Level 1}^2 + \sigma_{Level 2}^2 + \sigma_{Level 3}^2) \quad (6.10)$$

We remind readers that this first estimate of the variance in graduation rates at each level of the data hierarchy is approximate only, owing to estimation assumptions about sampling variability made in the initial estimates of variance at each level (Hox, 2010; Snijders & Bosker, 1994).

### Level 1 Error Structures

The Level 1 residuals ( $\varepsilon_{tij}$ ) represent the variability in measuring individual-level outcomes at each occasion. They are typically assumed to be independent, normally distributed, and with mean of 0 and common variance ( $\sigma_e^2$ ). For repeated measures models, MIXED has a variety of different error structures that can be considered. The default Level 1 structure in MIXED is a diagonal covariance matrix with heterogeneous variances in the diagonals for each measurement occasion, which has no off-diagonal covariances between occasions. Using just the first four occasions for ease of presentation, the diagonal Level 1 covariance matrix would look like the following (with variances  $\sigma_1^2$  to  $\sigma_4^2$  in the diagonals and 0s representing fixed covariances between occasions):

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \quad (6.11)$$

Compound symmetry (CS) assumes one equal variance across all occasions and one equal covariance between pairs of covariances. As we suggested in the last chapter, with a linear longitudinal model with random intercept only, this amounts to accepting homogeneity of variance over the repeated measures similar to the repeated measures analysis of variance (RM-ANOVA) formulation. At the other end of the spectrum, we could consider an unstructured (UN) covariance matrix, which assumes heterogeneous variances for each occasion and heterogeneous covariances for pairs of occasions. This may be overly complex, however, in many research situations since it results in the calculation of many variances and covariances, which might not be of real substantive interest.

Another choice for repeated measures is an autoregressive covariance structure. Since we have reason to believe there are covariances between the off-diagonal elements in the matrix, we might try either a heterogeneous first-order autoregressive structure (i.e., with separate variances in the diagonals and covariances in the off-diagonals) or the more simplified first-order autoregressive structure with constant diagonal variance and an off-diagonal covariance. Once again, using the first four occasions as an example, we have the following (where  $|\rho| \leq 1$ ):

$$\sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix} \quad (6.12)$$

The first-order autoregressive covariance structure therefore requires the estimation of two parameters (i.e., one variance and one autocorrelation). Equation 6.12 implies that the size of the covariance between the repeated measures depends on the number of steps between occasions; that is, the strength of the relationship diminishes as the distance between measurements increases. This provides an advantage over the compound symmetry repeated measures specification, especially where there are more repeated measures in the study.

We investigated several different covariance structures for the repeated-measures estimates at Level 1. We can use restricted maximum likelihood (REML) estimation to examine the fit of various structures since we are only comparing differences in variance components. Because there are 11 years of measurements, it is quite likely that there are considerable correlations between occasions. Therefore, the default diagonal covariance (i.e., with variances only and no covariances) would not likely capture this complexity ( $-2LL = -17,698.122$ ). We note that the model  $-2LL$  (or deviance) in this case is negative, which can occur in model estimation if the probability density is greater than 1, meaning that the log of the likelihood function at the maximum is positive (i.e.,  $> 0$ ), and therefore, when multiplied by  $-2$ , the resulting deviance and Akaike's information criterion [AIC] are negative. The fit of the model with a single variance at Level 1 (scaled identity) fit the data somewhat better ( $-2LL = -17,424.371$ ). The model with compound symmetry (five estimated parameters) did not converge. Similarly, the model with an unstructured covariance matrix was overly complex (i.e., taking more than 2 hours to converge on a solution for 69 estimated parameters). It is likely that we can find a more parsimonious structure for the Level 1 repeated measures.

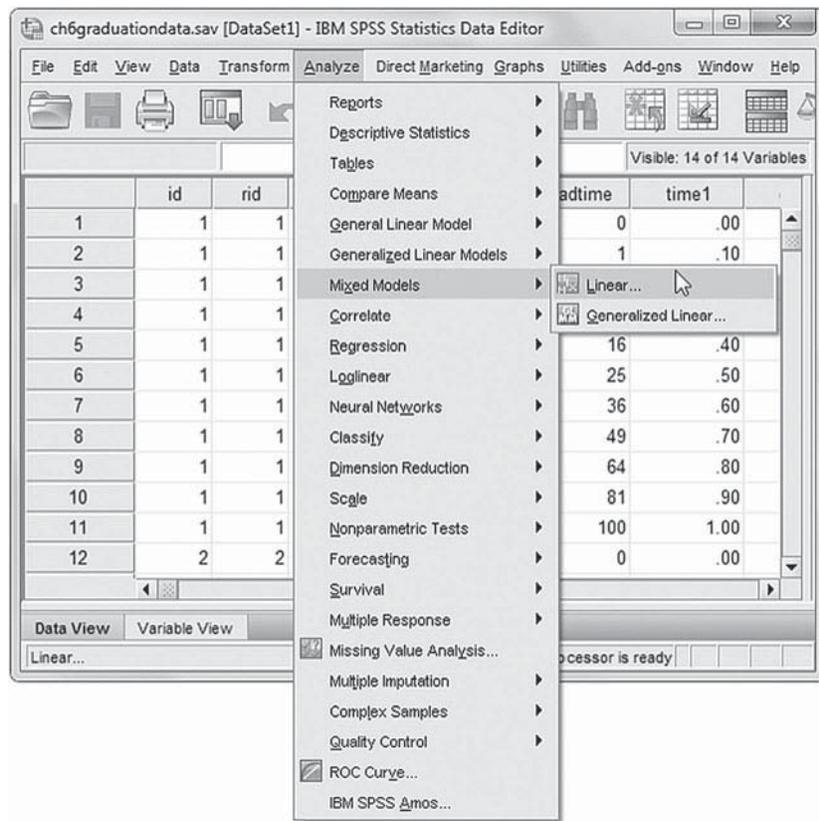
We also compared the autoregressive heterogeneous structure ( $-2LL = -18,988.848$ ) against the autoregressive homogeneous structure ( $-2LL = -18,776.575$ ). We note that in evaluating model fit, the negative sign matters; that is, we prefer the smallest value of fit indices such as  $-2LL$  or AIC. In order, the heterogeneous autoregressive covariance structure (ARH1) provided the best fit; the homogeneous autoregressive structure (AR1), the second best; the diagonal covariance structure (DIAG), the third best; and the identity covariance structure (ID), the worst. We might settle, however, on the autoregressive homogeneous (AR1) structure as being a reasonable compromise between an identity covariance matrix and a more fully specified covariance structure as in the autoregressive heterogeneous structure.

### Defining Model 1.1 (Null) with IBM SPSS Menu Commands

Launch the IBM SPSS program application, and select the *ch6graduationdata.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

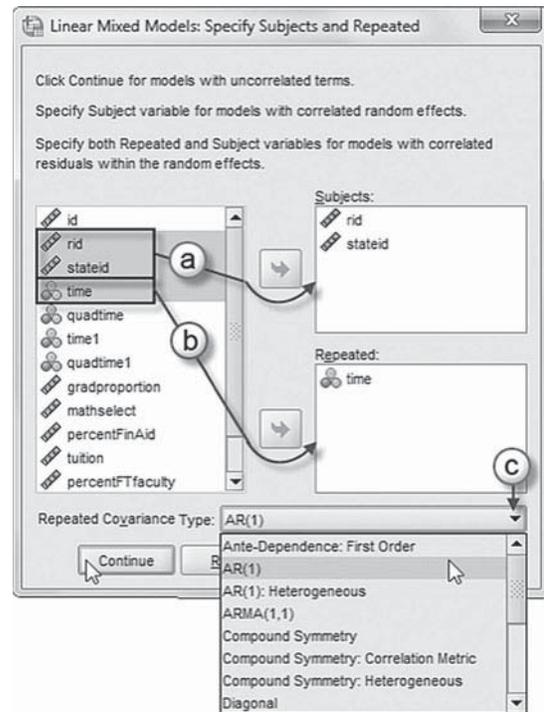


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.

- a. A subject is an observational unit that may be independent of other subjects. For this model, we will designate *rid* (individual identification number reindexed to nest students within 11 time periods within each state to facilitate model processing) and *stateid* (state identification number) as the subject variables. Click to select *rid* and *stateid* from the *Variables* column, and then click the right-arrow button to move the variables into the *Subjects* box.
- b. The *Repeated* box allows specifying variables that identify repeated observations. For this model, *time* identifies repeated observations over 11 time periods. Click to select *time*, and then click the right-arrow button to move the variable into the *Repeated* box.

The combination of values for *rid*, *stateid*, and *time* defines a particular student from a particular state across 11 time periods.

- c. The *Repeated Covariance Type* specifies a model's covariance structure. For this model, we will use the autoregressive covariance matrix, *AR(1)*. Click the pull-down menu to select the autoregressive covariance matrix, *AR(1)*, as the *Repeated Covariance Type*.

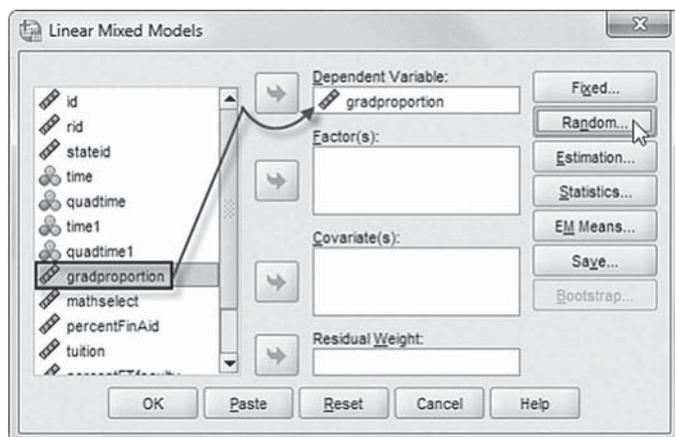


*AR(1)* is a first-order autoregressive structure with homogenous variances. The correlation between any two elements is equal to  $\rho$  for adjacent elements,  $\rho^2$  for elements that are separated by a third, and so on. We note that  $\rho$  is constrained so that  $-1 < \rho < 1$  (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, and covariates, as well as access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

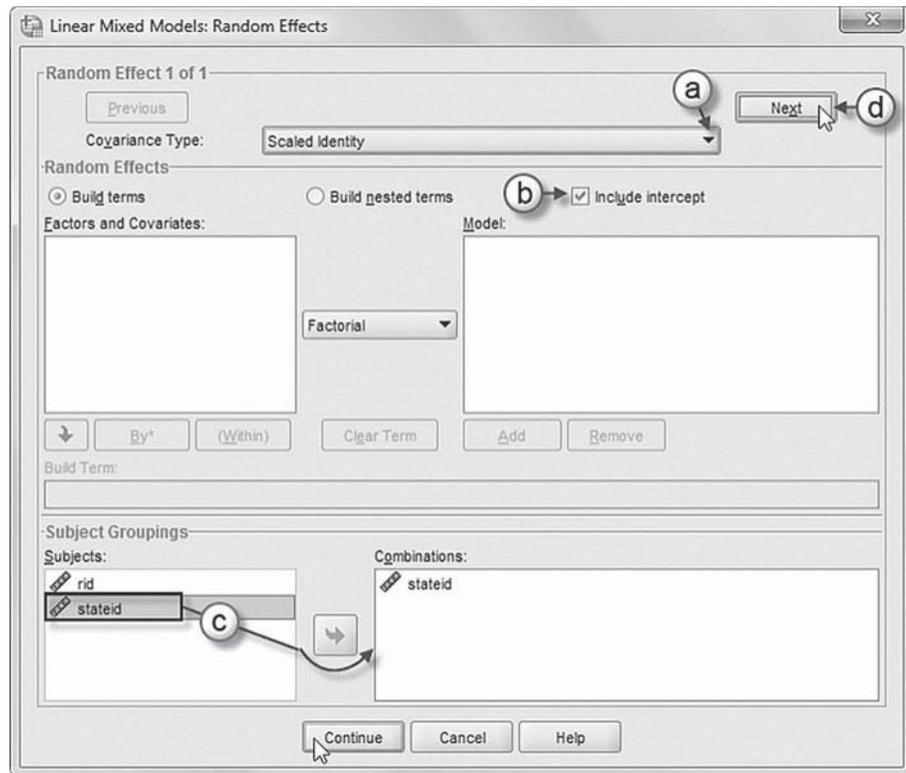
For this model, we will use *gradproportion* as the dependent variable. Click to select the *gradproportion* variable from the left column listing. Then click the right-arrow button to transfer *gradproportion* into the *Dependent Variable* box.



The null model does not have predictors, but we will be designating a random effect. So skip over the *FIXED* button, and click the *RANDOM* button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.

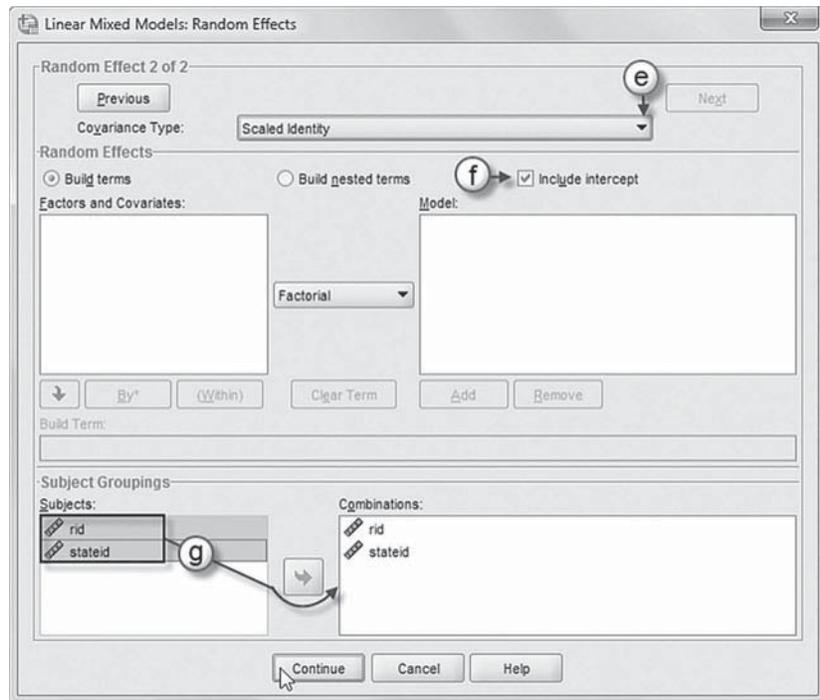
- Begin by specifying the covariance structure from the default variance components (VC) to scaled identity. Click the pull-down menu and select *Scaled Identity* (ID). The scaled identity structure has constant variance and assumes that no correlation occurs between elements.
- We want the intercept to be included in the model, so click *Include intercept*.
- The *Subject Groupings* box displays the *rid* and *stateid* variables that were specified as a subject variable in the *Specify Subjects and Repeated* dialog box show in step 2a. We will specify *stateid* as the subject for the random-effects part of this model. Click to select *stateid*, and then click the right-arrow button to move the variable into the *Combinations* box.
- At the top-right section of the window, click the NEXT button to access the *Random Effect 2 of 2* screen.



**Note:** The NEXT button may not work in earlier or unpatched versions of IBM SPSS when creating multilevel models with random intercepts. An update issued by IBM SPSS for software Version 19 addressed the problem, and Version 20 appears to have resolved the issue. A workaround to activating the NEXT button is to either (a) add or reenter a subject variable into the *Combinations* box or (b) add a variable from the *Factors and Covariates* column to the *Model* box and then remove it before proceeding to the *Random Effect 2 of 2* screen.

The *Random Effect 2 of 2* screen display is similar to the first screen and requires the following changes.

- e. Change the covariance type by clicking on the pull-down menu and selecting *Scaled Identity*.
- f. Click to select the *Include intercept* option.
- g. We will specify *rid* and *stateid* as the subjects for the Level 2 random effects in this model. Click to select *rid* and *stateid*, and then click the right-arrow button to move the variables into the *Combinations* box.

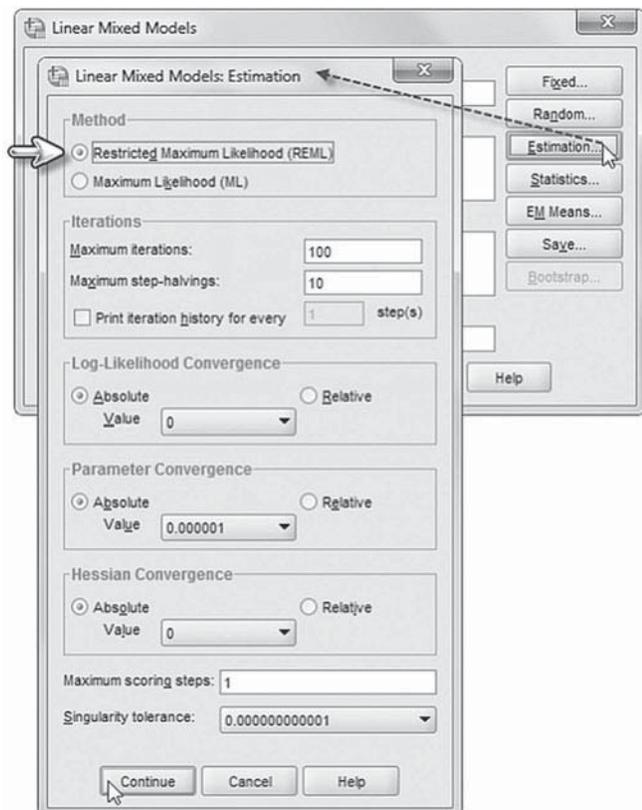


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: maximum likelihood (ML) or REML.

In this chapter, we will use the default setting of REML to estimate the models.

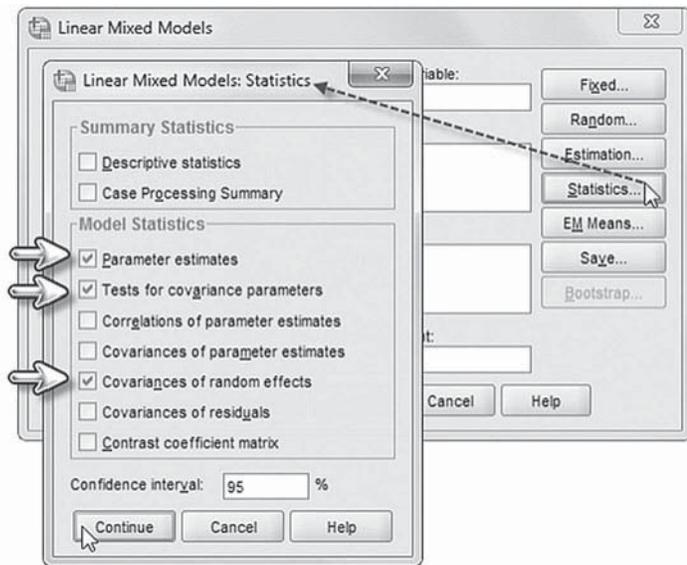
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



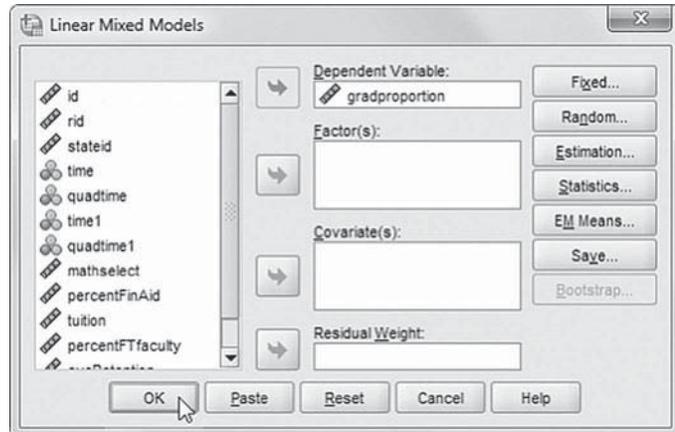
6. In the *Linear Mixed Models* dialog box, click the **STATISTICS** button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the **OK** button to run the model.



### Interpreting the Output From Model 1.1 (Null)

The output for the initial “no predictors” model is summarized in Table 6.2. Averaging across the period of time, the estimates suggest that about 13.7% of the variability in graduation rates is between states ( $0.003255/0.004104 + 0.016397 + 0.003255 = 0.003255/0.023756 = 0.137$ ), about 69% of the variability is between institutions ( $0.016397/0.023756 = 0.690$ ), and about 17.3% of the variability is within institutions ( $0.004104/0.023756 = 0.173$ ). As we mentioned in the last chapter, if we added the time-related variables, we might have a somewhat different estimate of the proportion of variance at each level. At Level 1, rho ( $\rho$ ) represents the correlation between any two consecutive occasions across the time series (i.e., 0.513).

In Table 6.3, the grand mean for graduation rates at the state level across the 11 observations is 0.431, or about 43%.

### Model 1.2: Adding Growth Rates

At the next step, we can add the growth rate indicators. In many situations where we examine change over time, it is sufficient to propose that a linear growth trajectory will describe the data adequately. When the time periods are reasonably short and there are not too many observations

**TABLE 6.2** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	AR1	0.004	0.000	34.806	.000	0.004	0.004
	diagonal						
	AR1 rho	0.513	0.014	36.169	.000	0.484	0.540
Intercept [subject = stateid]	Variance	0.003	0.001	3.254	.001	0.002	0.006
Intercept [subject = rid * stateid]	Variance	0.016	0.001	16.350	.000	0.015	0.018

<sup>a</sup> Dependent variable: gradproportion.

**TABLE 6.3** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	0.431	0.010	47.374	42.668	.000	0.411	0.451

<sup>a</sup> Dependent variable: gradproportion.

per individual (i.e., an individual subject or, in this case, an individual institution), the linear growth model will often provide a good approximation for more complex models that cannot be fully modeled because of the sparse number of observations (Raudenbush & Bryk, 2002). In this case, because we are not sure, we may wish to check for the shape of the growth trajectory first. We can define a quadratic component at Level 1 to take into consideration the idea that the growth rates may accelerate or decelerate over time.

### Level 1 Model

To add a quadratic component to the model, the Level 1 equation can be written as follows:

$$y_{ij} = \pi_{0ij} + \pi_{1ij}a_{ij} + \pi_{2ij}a_{ij}^2 + \varepsilon_{ij}, \quad (6.13)$$

where  $y_{ij}$  is the response variable for institution  $i$  in state  $j$  measured at time  $t$ . The intercept is  $\pi_{0ij}$ , and  $\pi_{1ij}$  is the linear growth rate for institution  $i$  in state  $j$  over the data-collection period, representing the expected change during a fixed unit of time. The quadratic component  $\pi_{2ij}$  represents any change in the rate of change over time. Each institution, therefore, has its own growth trajectory (developed from the intercept and slope), with likely variability present in the random coefficients across the set of institutions.

At Level 2, we will have the intercept vary across institutions, but we will first fix the variances of the time-related components to 0 (shown by removing the variance parameters  $r_{1ij}$  and  $r_{2ij}$ , respectively):

$$\begin{aligned} \pi_{0ij} &= \beta_{00j} + r_{0ij}, \\ \pi_{1ij} &= \beta_{10j}, \\ \pi_{2ij} &= \beta_{20j}. \end{aligned} \quad (6.14)$$

At Level 3, we will again allow the intercept to vary at random across states but again fix the variances of the time-related components to 0.

$$\begin{aligned}\beta_{00j} &= \gamma_{000} + u_{00j} \\ \beta_{10j} &= \gamma_{100} \\ \beta_{20j} &= \gamma_{200}\end{aligned}\tag{6.15}$$

### Coding the Time Variable

It is important to note that the meaning of the intercept parameter ( $\pi_{0i}$ ) depends on the scaling of the time variable. Figure 6.4 illustrates two coding schemes for time. We could code the linear *time* variable in yearly intervals ( $0, 1, 2, \dots, N = t$ ). Most often, researchers code the first measurement occasion as 0, so that the intercept parameter can be interpreted as the true initial status of graduation rates in institution  $i$  in state  $j$  at time point  $a_{ij} = 0$ . Defining the intercept as initial status serves as a baseline for interpreting the subsequent change that takes place over time for each individual institution in the sample. If there is a quadratic term (*quadtime*), the corresponding variable would be scaled in a similar manner (i.e.,  $0, 1, 4, 9, \dots, N = t$ ), as shown in the table. The figure also shows an alternative coding for the linear (*time1*) and quadratic (*quadtime1*) components using 0 and 1 as the end points for the growth trajectory. We note in passing that varying intervals between measurement occasions can also be accommodated (e.g.,  $0, 1, 3, 7, \dots, N$ ).

As Figure 6.4 indicates, the time intervals are again entered in the Level 1 data set in vertical format as a variable (*time*), and each time point corresponds to a specific graduation proportion for each institution. This necessitates 11 data lines in the data set per institution. For example, for institution ID #1, the graduation proportion corresponding to *time* = 0 (the first year) is 0.35, and for *time* = 10 (the last year), it is 0.33. The data are linked to successive levels with unique identifiers (e.g., institution and state). We note that the institutional identifier (*id*) has also been recoded within institutions (*rid*) to reduce the time it takes to run the model.

We reiterate that centering the time-related variables in various ways (i.e., as initial status, end status, and middle status) will affect the level of the intercept, the variance at higher levels (e.g., Level 2 and Level 3), and the covariance between the intercept and slope (see Raudenbush & Bryk, 2002, for further discussion of centering options). For example, if we define the intercept as initial status, the intercept is 0.41, and the correlation between the intercept and slope is negative. Selecting the last time point would result in a factor that describes the institutions' graduation rates at the end of the time series (e.g.,  $a_{it} = -10, -9, \dots, 0$ ). In this case, if we define graduation rate as end status, the intercept is about 0.44, and the correlation between the intercept and slope is positive. Centering in the middle of the time series may also have desirable effects in some instances. We also have to keep in mind the possibility of higher order polynomials in defining the growth trajectories. In general, such decisions should be made with respect to the purposes of the study and characteristics of the model and data.

The choice of the appropriate modeling approach and coding of time should be based on the ease with which model parameter estimates can be interpreted, always keeping in mind the substantive questions of the research study. In this example, it is convenient to code the time variable in the following manner (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0). We will call this variable *time1*, to differentiate it from the yearly *time* variable in the data set. This will allow us to capture the change in graduation growth over the entire period under study (i.e., the change in graduation proportion as *time1* changes from 0 to 1). We can create a similar quadratic component (*quadtime1*) by computing *time1\*time1* and saving it in the database. When we estimated a model with both components, however, we found that the quadratic component was not significant ( $p = .197$ ). We could also recenter the linear and quadratic components at the grand mean (i.e., year 6 mean) of the trend (*time1mid* and *quadtime1mid*), which will remove the correlation

	id	rid	stateid	time	quadtime	time1	quadtime1	gradproportion
1	1	1	1	0	0	.00	.00	.35
2	1	1	1	1	1	.10	.01	.37
3	1	1	1	2	4	.20	.04	.40
4	1	1	1	3	9	.30	.09	.32
5	1	1	1	4	16	.40	.16	.41
6	1	1	1	5	25	.50	.25	.39
7	1	1	1	6	36	.60	.36	.38
8	1	1	1	7	49	.70	.49	.33
9	1	1	1	8	64	.80	.64	.32
10	1	1	1	9	81	.90	.81	.34
11	1	1	1	10	100	1.00	1.00	.33
12	2	2	1	0	0	.00	.00	.33
13	2	2	1	1	1	.10	.01	.33
14	2	2	1	2	4	.20	.04	.35
15	2	2	1	3	9	.30	.09	.34
16	2	2	1	4	16	.40	.16	.38
17	2	2	1	5	25	.50	.25	.38
18	2	2	1	6	36	.60	.36	.39

FIGURE 6.4 Level 1 data structure for graduation percentages.

between them ( $r = 0.008$ ) and often facilitates model estimation (Hox, 2010). When we estimated this model with linear and quadratic trends, however, we confirmed that the quadratic component was again not significant ( $p = .197$ ).

We present the fixed-effect results of this latter model centered at the midpoint of the trend in Table 6.4. Readers should keep in mind that we are actually testing a population of universities and states, so statistical testing is not really needed since we are not making inferences from a sample to a population. For purposes of demonstration, however, we will interpret them as if we were making these types of inferences. The table indicates that the average midpoint graduation proportion is 0.429. As expected, this is between the initial status proportion of 0.41 and the end status proportion of 0.44. The table also suggests that the linear component is necessary in describing graduation growth over time ( $\gamma = 0.039$ ,  $p < .001$ ). The quadratic component, however, is not required ( $\gamma = 0.013$ ,  $p > .10$ ) to model growth over time. Because the quadratic component does not contribute to modeling growth in graduation rates over time, we can drop it from successive models.

TABLE 6.4 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.
Intercept	0.429	0.010	48.633	42.214	.000
time1mid	0.039	0.003	1,391.355	12.407	.002
quadtime1mid	0.013	0.010	2,737.942	1.289	.197

<sup>a</sup> Dependent variable: gradproportion.

We note in passing that centering the time-related components at the beginning of the trend or in the middle results in different estimates of the intercept and linear time component; however, the quadratic component (as the higher order polynomial) is unaffected by the recentering. In this example, a linear growth model using *time1* should be sufficient to capture the change in graduation rates over the period of the study. Since we have settled on the shape of the trajectory, we can also specify the linear growth component (*time1*) as randomly varying across institutions and states. At Level 1, we will have the following equation without the quadratic component:

$$y_{ij} = \pi_{0ij} + \pi_{1ij}time1_{ij} + \varepsilon_{ij}. \quad (6.16)$$

Between institutions (Level 2), we will specify the linear component as randomly varying by adding the respective variance ( $r_{1ij}$ ) at Level 2, as shown in Equation 6.14. At Level 3, we will also specify the linear component as randomly varying ( $u_{10j}$ ), as shown in Equation 6.15. When we substitute the new Equations 6.14 and 6.15 into Equation 6.16, we will obtain the following combined equation:

$$y_{ij} = \gamma_{000} + \gamma_{100}time1_{ij} + u_{10j}time1_{ij} + u_{00j} + r_{1ij}time1_{ij} + r_{0ij} + \varepsilon_{ij}. \quad (6.17)$$

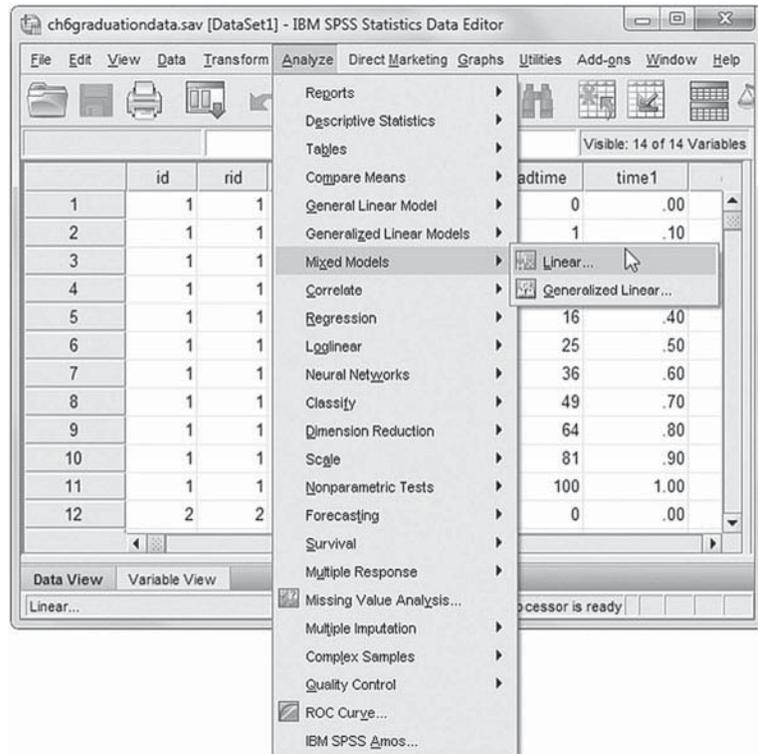
This suggests a total of eight parameters to estimate. This includes two fixed effects, two random effects at Level 3, two random effects at Level 2, and two repeated effects (Level 1) parameters (i.e., the variance and autocorrelation). Note also that we will first assume diagonal covariance matrices for the fixed effects at Levels 2 and 3. We could, of course, define those covariances as unstructured.

### Defining Model 1.2 with IBM SPSS Menu Commands

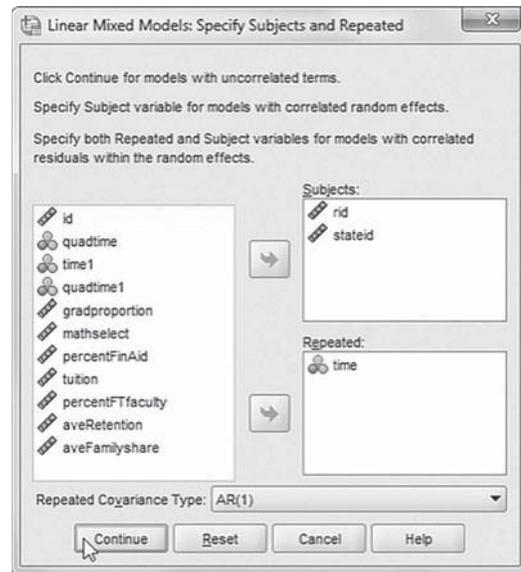
**Note:** IBM SPSS settings will default to those used in Model 1.1.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

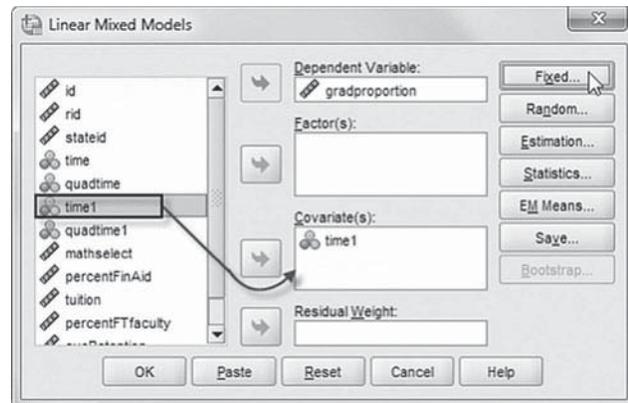


2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main dialog box displays *gradproportion* as the dependent variable.

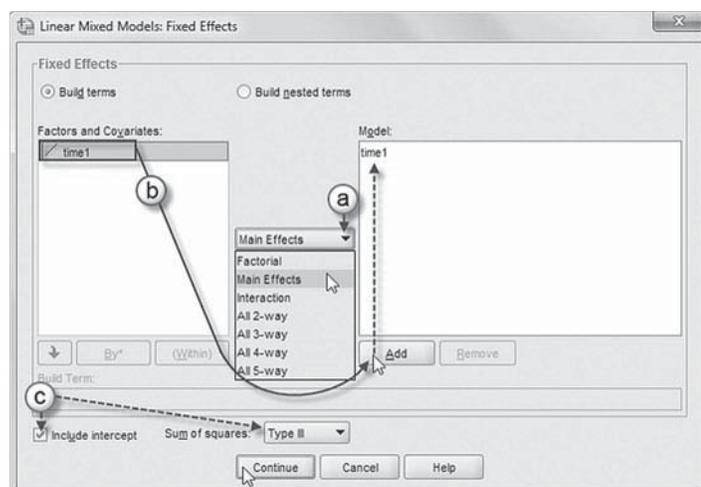
Factors and covariates may be specified in predicting the dependent variable. Factors are categorical predictors that may be numeric or string. Covariates are scale predictors that must be numeric. We will designate a predictor variable (*time1*), which will be used in the model. Locate and click the variable *time1* from the left column listing, and then click the right-arrow button to move the variable into the *Covariate(s)* box.



We may now proceed to define fixed effects for the variable.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
- b. Click to select *time1* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

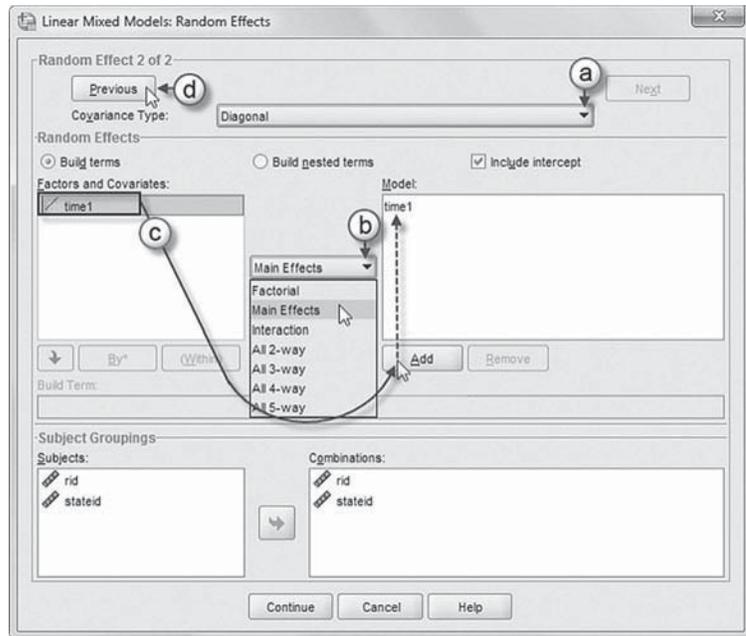
We will now add random effects to this model.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Random Effect 2 of 2* screen display is displayed as it was the last.
  - a. Change the covariance type by clicking on the pull-down menu and selecting *Diagonal*.

The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

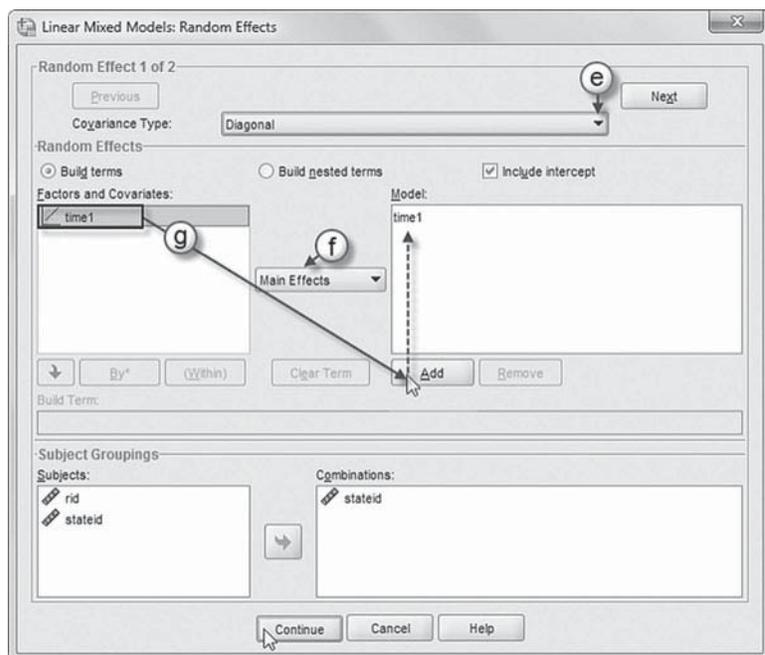
- b. We will specify a Level 2 random effect (*time1*) by first clicking the pull-down menu to change the factorial setting to *Main Effects*.
- c. Next, click to select *time1* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
- d. Click the PREVIOUS button to access the *Random Effect 1 of 2* screen display.



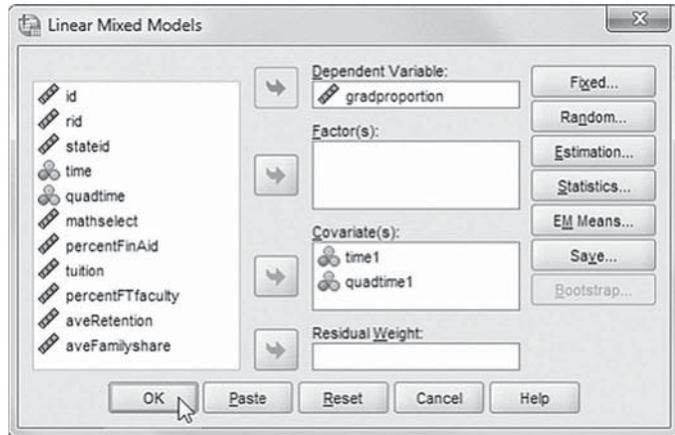
The *Random Effect 1 of 2* screen displays the default settings used for the prior model.

- e. Change the covariance type by clicking on the pull-down menu and selecting *Diagonal*.
- f. We will specify a Level 1 random effect (*time1*) by first clicking the pull-down menu to change the factorial setting to *Main Effects*.
- g. Next, click to select *time1* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



6. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 1.2**

We can confirm the model specification of eight parameters to estimate in Table 6.5.

In Table 6.6, we next present the fixed effects, noting that we now have initial status defined as the beginning of the time trend, suggesting that the initial state 6-year graduation rate was 0.411 (or about 41.1%). Over the period of the study, state graduation rates grew by about 0.04 (or 4%).

Next, it is important to consider the nature of the deviations of the individual growth trajectories from the mean growth trajectory. The estimates of the initial status and slope variances are presented in Table 6.7. Once again, the Wald Z statistic is provided as a test of homogeneity; that is, there is no true variation in individual growth parameters (0). Since the variance cannot be below 0, the Wald Z test should be conducted as a one-tailed test. For initial status, at the state

**TABLE 6.5 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	time 1	1		1		
Random Effects	Intercept + time 1	2	Diagonal	2	stateid	
	Intercept + time 1	2	Diagonal	2	rid * stateid	
Repeated Effects	quadtime	11	First-Order Autoregressive	2	rid * statied	649
Total		17		8		

<sup>a</sup> Dependent variable: gradproportion.

**TABLE 6.6 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	0.411	0.010	48.862	42.029	.000	0.392	0.431
time 1	0.040	0.005	27.359	8.120	.000	0.030	0.050

<sup>a</sup> Dependent variable: gradproportion.

TABLE 6.7 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	AR1 Diagonal	0.003	0.000	37.747	.000	0.003	0.003
	AR1 rho	0.287	0.018	16.071	.000	0.252	0.322
Intercept + time   [subject = stateid]	Var: Intercept	0.003	0.001	3.206	.001	0.002	0.005
	Var: time	0.000	0.000	1.331	.183	0.000	0.002
Intercept + time   [subject = rid * stateid]	Var: Intercept	0.016	0.001	16.357	.000	0.014	0.018
	Var: time	0.005	0.001	9.860	.000	0.004	0.007

<sup>a</sup> Dependent variable: gradproportion.

level the Wald Z is 3.206, one-tailed  $p < .01$ . This leads to rejecting the null hypothesis that there is no variation among states' initial status graduation proportions. At the institutional level, the Wald Z was 16.357, one-tailed  $p < .001$ , also suggesting that we should reject the null hypothesis of no significant variation in institutions' initial graduation proportions. At Level 1, the average correlation between any two consecutive repeated measures across the time series was about 0.287. Regarding growth rates, at the state level, we might have some preliminary evidence that graduation growth rates do vary across states (Wald  $Z = 1.331$ , one-tailed  $p = .0915$ ), keeping in mind that in this study we actually have a population of states instead of a small ( $n = 50$ ) random sample. At the institutional level, the evidence is clearer that growth in graduation rates varies across institutions (Wald  $Z = 9.860$ , one-tailed  $p < .001$ ).

The random-coefficients growth model can also provide an estimate of the covariance between initial status and the growth rate. In this subsequent analysis when we examined an unstructured covariance matrix (not tabled), at the institutional level we found that the covariance was negative ( $p = .052$ ), which suggests some tendency for institutions with higher graduation rates at the beginning of the study to demonstrate less growth over time. At the state level, however, the same relationship was not statistically significant ( $p = .383$ ). We reiterate the importance of giving some attention to the definition and scaling of the model's initial status and growth parameters because this affects the meaning one attaches to the coefficients (for other discussions, see Hox, 2002; and Singer & Willett, 2003). For this reason, we decided not to include the covariance between intercept and time slope in our subsequent models.

### Model 1.3: Adding Time-Varying Covariates

We will next add two time-varying covariates to the model. These are variables that are measured through the 11-year period. In this case, we will use the percentage of students within the institution receiving financial aid and the average tuition level at the institution. We will standardize each variable (mean  $[M] = 1$ , standard deviation  $[SD] = 0$ ).

$$y_{tij} = \pi_{0ij} + \pi_{1ij}time1_{tij} + \pi_{2ij}finaid_{tij} + \pi_{3ij}tuition_{tij} + \varepsilon_{tij}, \quad (6.18)$$

where  $\pi_{0ij}$  is the intercept for institutions,  $\pi_{1ij}$  is the linear growth rate,  $\pi_{2ij}$  represents the coefficient describing the effect of the percentage of students receiving federal financial aid (standardized),  $\pi_{3ij}$  is the coefficient describing the effect of institutional tuition level (standardized), and  $\varepsilon_{tij}$  is the residual variance associated with predicting institutional graduation rates. We will keep

Levels 2 and 3 the same as in the last model (see Eqs. 6.14 and 6.15). When we substitute those equations into Equation 6.18, we will have the following single-equation model:

$$y_{ij} = \gamma_{000} + \gamma_{100}time1_{ij} + \gamma_{200}finaid_{ij} + \gamma_{300}tuition_{ij} + u_{10j}time1_{ij} + u_{00j} + r_{1ij}time1_{ij} + r_{0ij} + \varepsilon_{tij}. \quad (6.19)$$

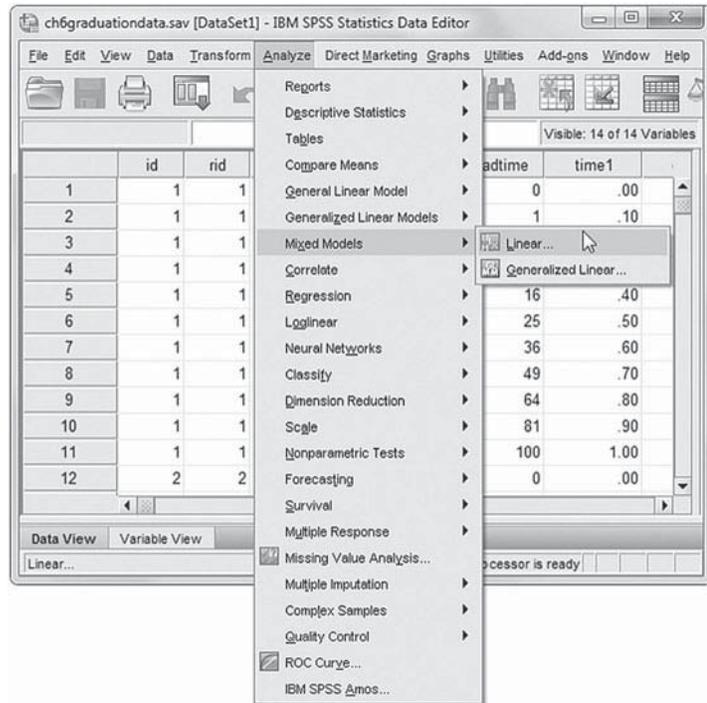
This provides a total of 10 parameters to estimate (four fixed effects, four random effects, and two repeated effects).

### Defining Model 1.3 with IBM SPSS Menu Commands

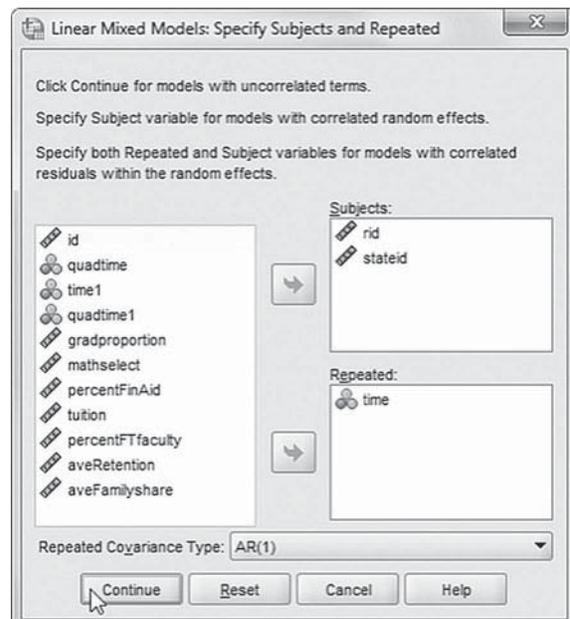
**Note:** IBM SPSS settings will default to those used in Model 1.2.

1. Go to the toolbar and select **ANALYZE, MIXED MODELS, LINEAR**.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



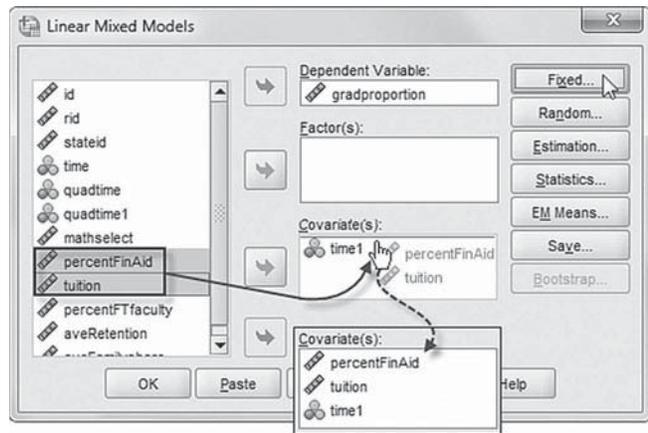
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main dialog box displays default settings from the prior model.

We will introduce two additional predictors to be used in the model (*percentFinAid* and *tuition*). First, click to select *percentFinAid* and *tuition*, and then “drag” both variables to the *Covariate(s)* box above *time1*.

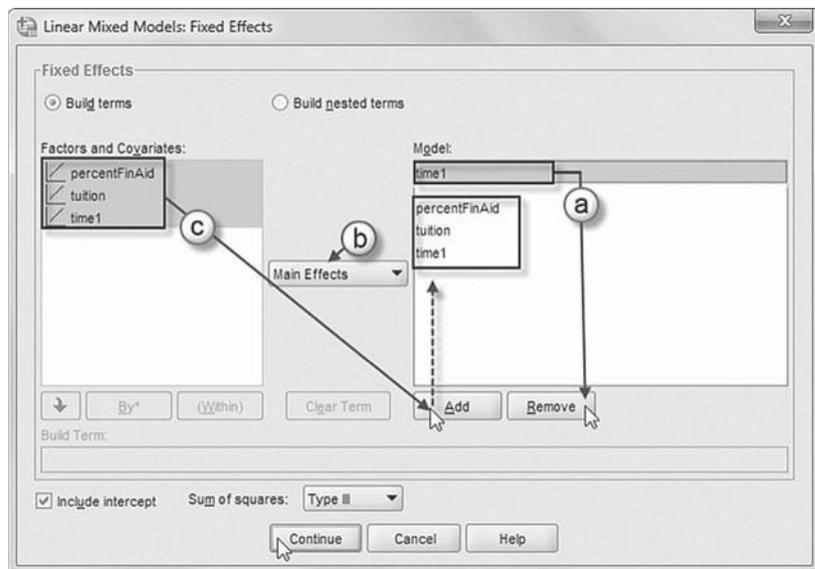
Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading the output tables, we will first remove *time1* by clicking to select the variable and then clicking the REMOVE button.

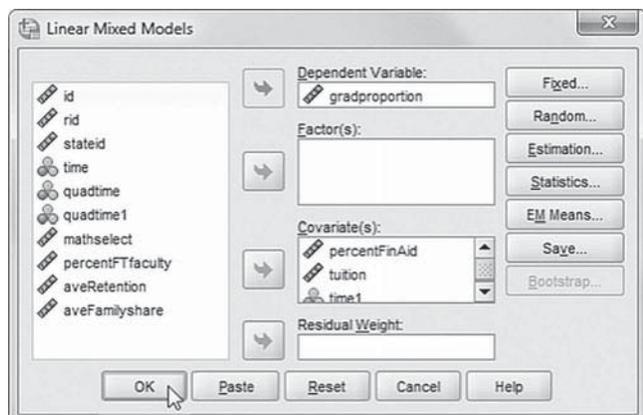
- b. Confirm that the factorial setting is *Main Effects* before adding the predictor variables to the model.

- c. Now click to select the variables *percentFinAid*, *tuition*, and *time1* from the *Factors and Covariates* box. Then click the ADD button to move the variables into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**TABLE 6.8** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	0.469	0.011	98.626	41.737	.000	0.447	0.492
percentFinAid	0.002	0.001	5,693.474	1.779	.075	0.000	0.003
tuition	0.049	0.005	5,044.319	9.269	.000	0.039	0.060
time1	0.016	0.005	59.168	3.063	.003	0.006	0.027

<sup>a</sup> Dependent variable: gradproportion.

**TABLE 6.9** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	AR1 Diagonal	0.003	0.000	38.123	.000	0.003	0.003
	AR1 rho	0.276	0.018	15.391	.000	0.240	0.310
Intercept + time1 [subject = stateid]	Var: Intercept	0.003	0.001	3.143	.002	0.001	0.005
	Var: time1	0.000	0.000	1.419	.156	0.000	0.001
Intercept + time1 [subject = rid * stateid]	Var: Intercept	0.015	0.001	16.276	.000	0.014	0.017
	Var: time1	0.005	0.001	10.076	.000	0.004	0.007

<sup>a</sup> Dependent variable: gradproportion.

### Interpreting the Output From Model 1.3

The fixed-effect estimates are provided in Table 6.8. After adding the time-varying covariates at Level 1, the new state graduation intercept was 0.469 (or about 47%). This can be interpreted as the initial graduate rate for an institution at the grand means (0) of percentage of students receiving financial aid and tuition levels. Table 6.8 suggests that the average tuition level was positively associated with student graduation rates; that is, a 1-*SD* increase in institutional tuition would result in about a 5% (0.050) increase in initial graduation rate. Similarly, percentages of students receiving financial aid was associated with higher graduation rates ( $p < .10$ ). After adjusting for the covariates, state graduation rates appeared to grow about 1.6% (0.016) over the decade.

Again, the variance components table (Table 6.9) suggests that that intercepts vary across institutions and states. Growth rates also varied across institutions (Wald  $Z = 10.076$ , one-tailed  $p < .01$ ). There is some evidence suggesting that they also varied across states (Wald  $Z = 1.419$ , one-tailed  $p = .078$ ).

### Model 1.4: Explaining Differences in Growth Trajectories Between Institutions

Because there was significant variation in each parameter present among individual institutions, both the intercept and growth parameters can be allowed to vary across institutions. The variation in each may likely be partially explained by between-institutional characteristics as in Equation 6.3. In this example, we consider two institutional predictors that may account for systematic variation in the intercept and slope parameters between institutions.

The first is selectivity in student admissions (i.e., defined as the percentage of students admitted with SAT or ACT math scores at the 75th percentile or above), which we standardized

( $M = 0, SD = 1$ ). The other variable is percentage of full-time faculty, which is also standardized. The two Level 2 submodels are now defined as follows:

$$\pi_{0ij} = \beta_{00j} + \beta_{01}mselect_{ij} + \beta_{02}fulltime_{ij} + r_{0ij}, \tag{6.20}$$

$$\pi_{1ij} = \beta_{10j} + \beta_{11}mselect_{ij} + \beta_{12}fulltime_{ij} + r_{1ij}. \tag{6.21}$$

The covariance matrix of random effects at Level 2 can be defined as diagonal or unstructured; however, we will maintain a diagonal covariance matrix at each level.

At Level 3 (between states), we will also add two predictors. The first is the percentage share that families have to pay for higher education in the state, which we standardized. The second variable is the average percentage of freshman retained in the state (which we also standardized), which may serve as one proxy indicator of the quality of the state’s public higher education system.

$$\beta_{00j} = \gamma_{000} + \gamma_{001}avefamilyshare_j + \gamma_{002}averetention_j + u_{00j}. \tag{6.22}$$

For this model, we will consider the growth rate to vary also across states but will not try to explain that variation:

$$\beta_{10j} = \gamma_{100} + u_{10j}. \tag{6.23}$$

When we combine these, we will have 16 parameters to estimate (keeping in mind two repeated measures effects at Level 1).

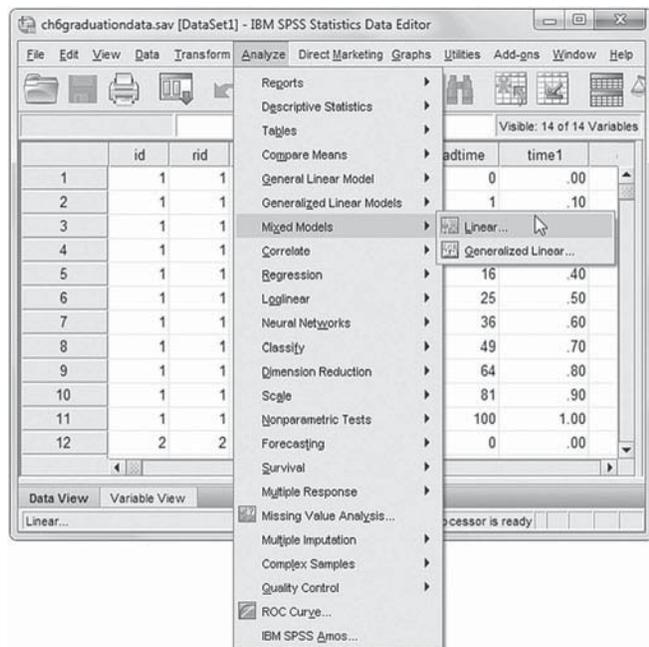
$$\begin{aligned} y_{tij} = & \gamma_{000} + \gamma_{001}avefamilyshare_j + \gamma_{002}averetention_j + \gamma_{010}mselect_{ij} + \gamma_{020}fulltime_{ij} \\ & + \gamma_{100}time1_{tij} + \gamma_{110}mselect_{ij} * time1_{tij} + \gamma_{120}fulltime_{ij} * time1_{tij} + \gamma_{200}finaid_{tij} \\ & + \gamma_{300}tuition_{tij} + u_{10j}timel_{tij} + u_{00j} + r_{1ij}timel_{tij} + r_{0ij} + \epsilon_{tij} \end{aligned} \tag{6.24}$$

**Defining Model 1.4 with IBM SPSS Menu Commands**

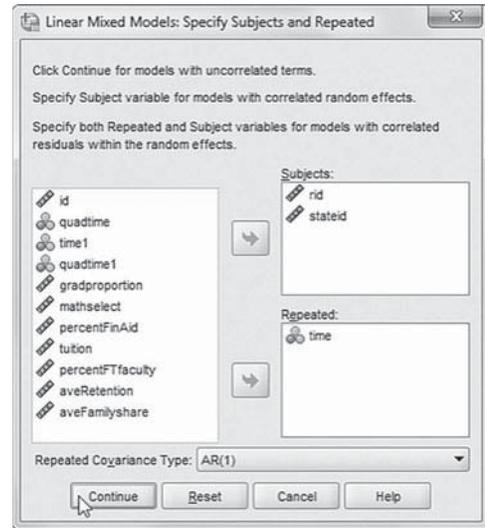
Note: IBM SPSS settings will default to those used in Model 1.3.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

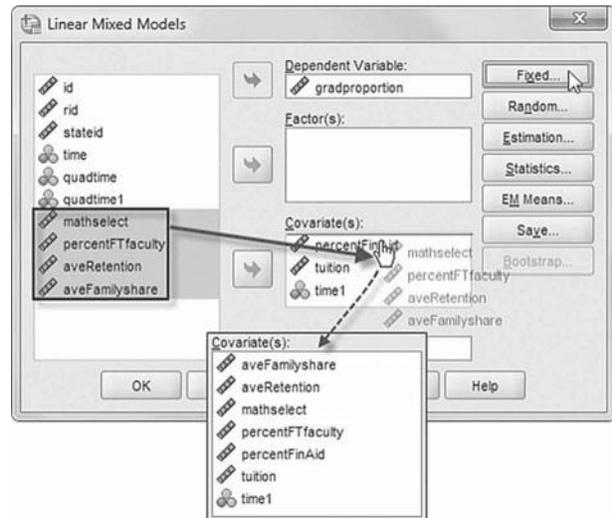


2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



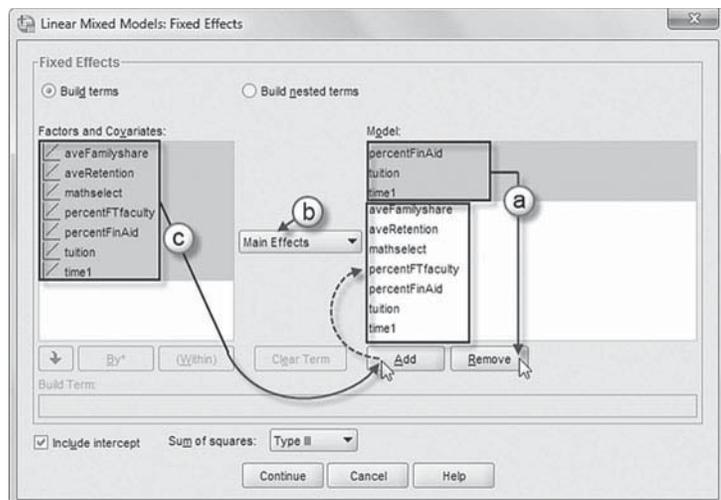
3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

We will introduce four additional predictors to be used in the model (*mathselect*, *percentFTfaculty*, *aveRetention*, and *aveFamilyshare*). First, click to select *mathselect*, *percentFTfaculty*, *aveRetention*, and *aveFamilyshare*, and then “drag” the variables to the *Covariate(s)* box. To facilitate reading of the output tables, we will rearrange the sequence of the variables by dragging them to form the following order: *aveFamilyshare*, *aveRetention*, *mathselect*, *percentFTfaculty*, *percentFinAid*, *tuition*, and *time1*.



Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

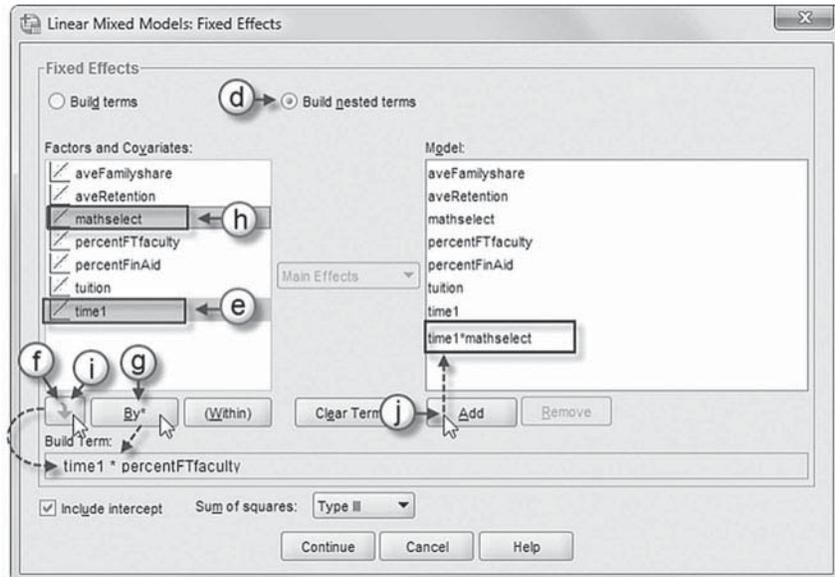
- 4a. We will rearrange the sequence order of the variables to facilitate reading of the output tables. First, click to select *percentFinAid*, *tuition*, and *time1*, and then click the REMOVE button to clear the *Model* box.
- b. Note that *Main Effects* is the default setting.
- c. Now click to select all of the variables in the *Factors and Covariates* column, and then click the ADD button to transfer the variables into the *Model* box.



Two cross-level interactions (or nested terms) will be created and added to the model:  $time1*mathselect$  and  $time1*percentFTfaculty$ . These interactions will tell us if the growth trajectories vary between institutions.

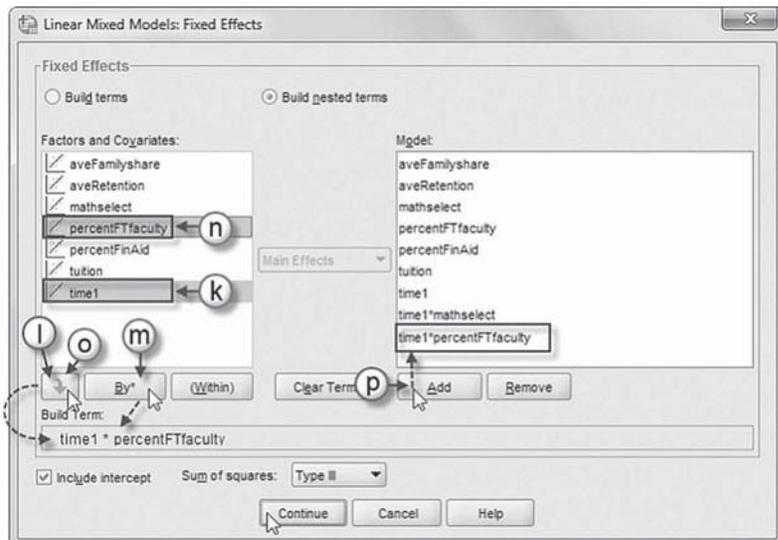
#### Add First Interaction to Model 1.4: $time1*mathselect$

- d. Click to select *Build nested terms*.
- e. Click to select the variable *time1* from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves *time1* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button, which will insert the computation command symbol:  $time1*$ .
- h. Click to select *mathselect* from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move *mathselect* into the *Build Term* box and complete the interaction term:  $time1*mathselect$ .
- j. Click the ADD button to transfer the interaction into the *Model* box.



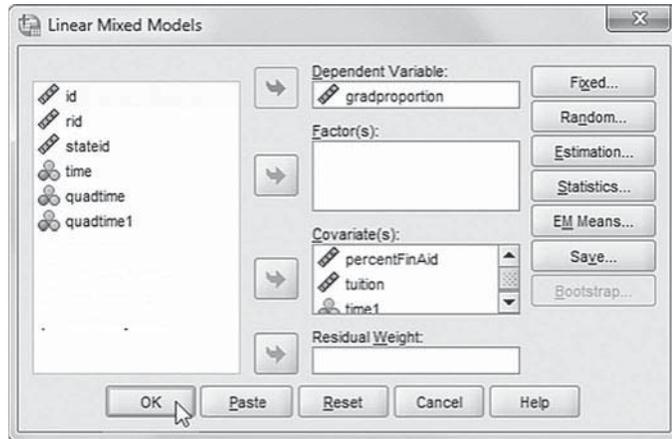
#### Add Second Interaction to Model 1.4: $time1*percentFTfaculty$

- k. Click to select the variable *time1* from the *Factors and Covariates* box.
- l. Next, click the arrow button below the *Factors and Covariates* box. This moves *time1* into the *Build Term* box.
- m. Now click the BY\* button, which will insert the computation command symbol:  $time1*$ .
- n. Click to select *percentFTfaculty* from the *Factors and Covariates* box.
- o. Click the arrow button below the *Factors and Covariates* box to move *percentFTfaculty* into the *Build Term* box and complete the interaction term:  $time1*percentFTfaculty$ .
- p. Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 1.4

As Table 6.10 indicates, after adjusting for the institutional variables, the true initial status graduation rate for states is 0.475 (or about 47.5%). At Level 3, the coefficients suggest that average family share ( $\gamma = 0.020$ ,  $p < .01$ ) and average year 1 retention rates ( $\gamma = 0.046$ ,  $p < .01$ ) are both significant in explaining graduation rates. At Level 2, selectivity ( $\gamma = 0.094$ ,  $p < .01$ ) and percentage of full-time faculty remain significant predictors ( $\gamma = 0.037$ ,  $p < .01$ ) of graduation rates. Within institutions, the percentage of students receiving financial aid ( $\gamma = 0.002$ ,  $p < .05$ ) and tuition levels ( $\gamma = 0.043$ ,  $p < .01$ ) also remain significant predictors of graduation rates. We draw attention to the degrees of freedom in Table 6.10 as a means of checking that variables are defined at their correct levels of the data hierarchy (i.e., keeping in mind that they reflect adjustments used in calculating significance tests, so they may be larger or smaller than the “exact” number of degrees of freedom at each level). We can see, for example, that *family share* and average *freshman retention* are state-level predictors (with degrees of freedom in the range between 40 and 50). Student *selectivity* and percentage of *full-time faculty* are institutional indicators (with

TABLE 6.10 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	0.475	0.008	166.118	57.204	.000	0.459	0.491
aveFamilyshare	0.020	0.006	47.667	3.669	.001	0.009	0.031
aveRetention	0.046	0.005	44.386	8.277	.000	0.034	0.057
mathselect	0.094	0.005	706.719	18.129	.000	0.084	0.104
percentFTfaculty	0.037	0.004	651.637	9.005	.000	0.029	0.045
percentFinAid	0.002	0.001	5,654.256	2.110	.035	0.000	0.004
tuition	0.043	0.005	4,999.898	8.253	.000	0.033	0.053
time1	0.019	0.005	66.645	3.756	.000	0.009	0.030
mathselect * time1	0.006	0.005	678.626	1.316	.189	0.003	0.016
percentFTfaculty * time1	0.019	0.004	536.271	5.131	.000	0.012	0.027

<sup>a</sup> Dependent variable: gradproportion.

**TABLE 6.11 Estimates of Covariance Parameters<sup>a</sup>**

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	AR1 Diagonal	0.003	0.000	34.057	.000	0.003	0.003
	AR1 rho	0.312	0.019	16.269	.000	0.274	0.349
Intercept + time1 [subject = stateid]	Var: Intercept	0.001	0.000	2.362	.018	0.000	0.002
	Var: time1	0.000	0.000	1.602	.109	0.000	0.001
Intercept + time1 [subject = rid * stateid]	Var: Intercept	0.007	0.000	14.951	.000	0.006	0.008
	Var: time1	0.004	0.000	7.984	.000	0.003	0.005

<sup>a</sup> Dependent variable: gradproportion.

degrees of freedom between 600 and 700). The percentage of students receiving *financial aid* and average *tuition* are within-institutional variables (reflecting several thousand repeated measures observations). Regarding the growth parameters, we can also observe that the average growth slope (*time1*) is defined at the state level and the two growth cross-level interactions are specified at the institutional level.

Turning to the growth rate model (*time1*), the adjusted growth in graduation rate for states over time was approximately 2% (0.019). The percentage of full-time faculty was positively related to within-institution growth in graduation rates ( $\beta = 0.019, p < .01$ ). Holding other variables in the model constant, this implies that a 1-SD increase in percentage of full-time faculty would be related to a 1.9% increase in graduation growth rate over the period of the study, compared with institutions at the grand mean of full-time faculty. Student selectivity, however, was not related to growth in state graduation rates ( $\beta = 0.006, p > .10$ ).

The variance components table (Table 6.11) suggests initial graduation rates still varied across states (Wald  $Z = 2.362$ , one-tailed  $p < .05$ ) and institutions (Wald  $Z = 14.951$ , one-tailed  $p < .01$ ) after inclusion of the Level 2 and Level 3 predictors. After addition of the institutional-level predictors, graduation growth rates still varied across institutions (Wald  $Z = 7.984$ , one-tailed  $p < .01$ ). Growth may also still vary across states (Wald  $Z = 1.602$ , one-tailed  $p < .055$ ). We note that because Wald  $Z$  coefficients are likely to be too conservative in small data sets (Hox, 2002), it might be instructive to develop a simple model to examine growth rates across states using the same two predictors at Level 3.

### Model 1.5: Adding a Model to Examine Growth Rates at Level 3

The new model for the state-level growth rate is as follows:

$$\beta_{10j} = \gamma_{100} + \gamma_{101}avefamilyshare_j + \gamma_{102}averetention_j + u_{10j} \tag{6.25}$$

When we combine all the predictors, we will have 18 parameters to estimate (including two repeated measures parameters at Level 1).

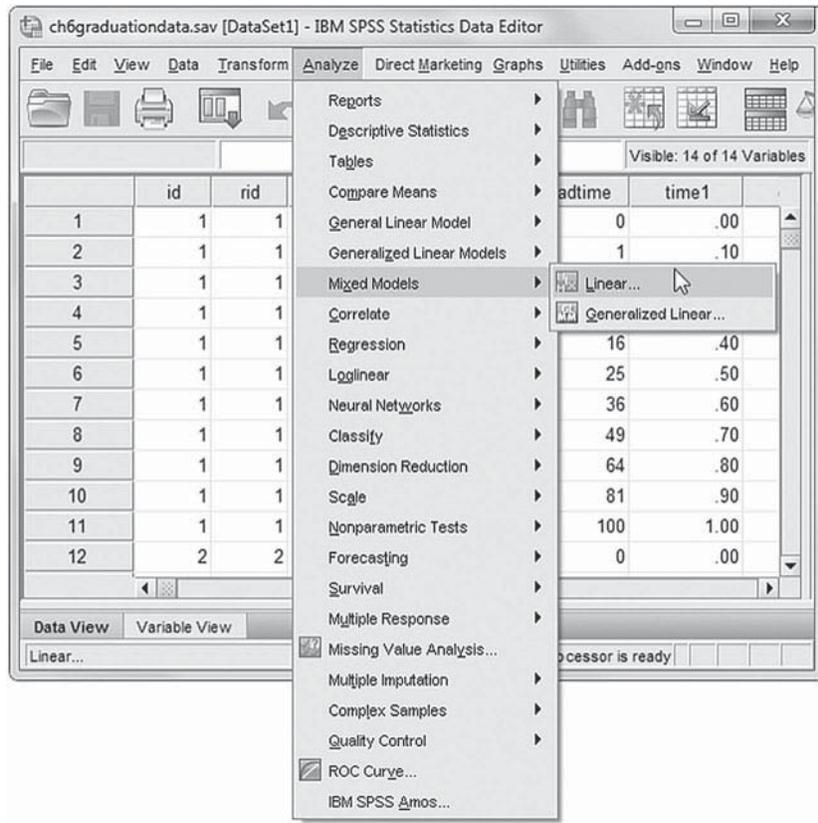
$$y_{ij} = \gamma_{000} + \gamma_{001}avefamilyshare_j + \gamma_{002}averetention_j + \gamma_{010}mselect_{ij} + \gamma_{020}fulltime_{ij} + \gamma_{100}time1_{ij} + \gamma_{101}Zavefamilyshare_j*time1_{ij} + \gamma_{102}averetention_j*time1_{ij} + \gamma_{110}mselect_{ij}*time1_{ij} + \gamma_{120}fulltime_{ij}*time1_{ij} + \gamma_{200}finaid_{ij} + \gamma_{300}tuition_{ij} + u_{10j}time1_{ij} + u_{00j} + r_{1ij}time1_{ij} + r_{0ij} + \epsilon_{ij} \tag{6.26}$$

### Defining Model 1.5 with IBM SPSS Menu Commands

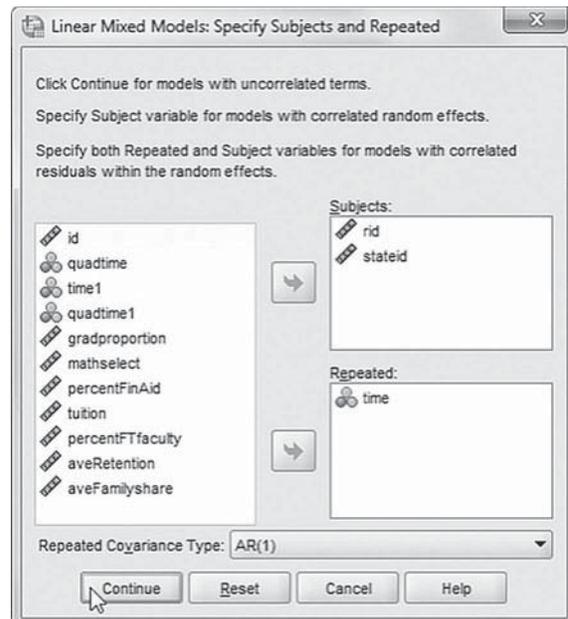
Note: IBM SPSS settings will default to those used in Model 1.4.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



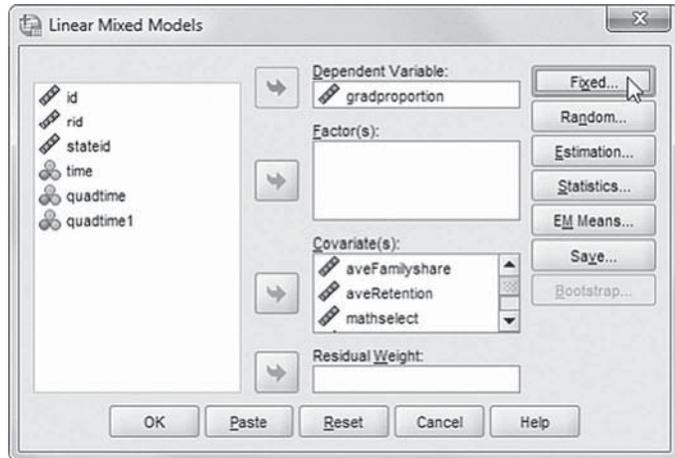
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main screen displays the default settings from the prior model.

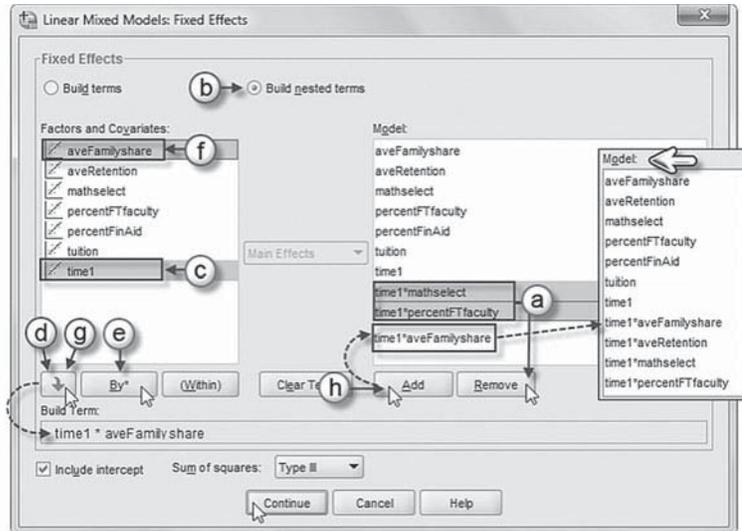
We will add four cross-level interactions as fixed effects to the model, so click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

Four cross-level interactions (or nested terms) will be created and added to the model:  $time1*aveFamilyshare$ ,  $time1*aveRetention$ ,  $time1*mathselect$ , and  $time1*percentFTfaculty$ . These interactions will tell us about the growth trajectories for states. (The final model is shown in the image insert.)



#### Add First Interaction to Model 1.5: $time1*aveFamilyshare$

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading the output tables, we will first remove the two (*percentFinAid* and *tuition*) interactions by clicking to select them and then clicking the REMOVE button.
- b. Click to select *Build nested terms*.
- c. Click to select the variable *time1* from the *Factors and Covariates* box.
- d. Then click the arrow button below the *Factors and Covariates* box. This moves *time1* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- e. Next, click the BY\* button, which will insert the computation command symbol:  $time1*$ .
- f. Click to select *aveFamilyshare* from the *Factors and Covariates* box.
- g. Click the arrow button below the *Factors and Covariates* box to move *aveFamilyshare* into the *Build Term* box and complete the interaction term:  $time1*aveFamilyshare$ .
- h. Click the ADD button to transfer the interaction into the *Model* box.



#### Add Second Interaction to Model 1.5: $time1*aveRetention$

Repeat steps 4c to 4h using *time1* and *aveRetention*.

#### Add Third Interaction to Model 1.5: $time1*mathselect$

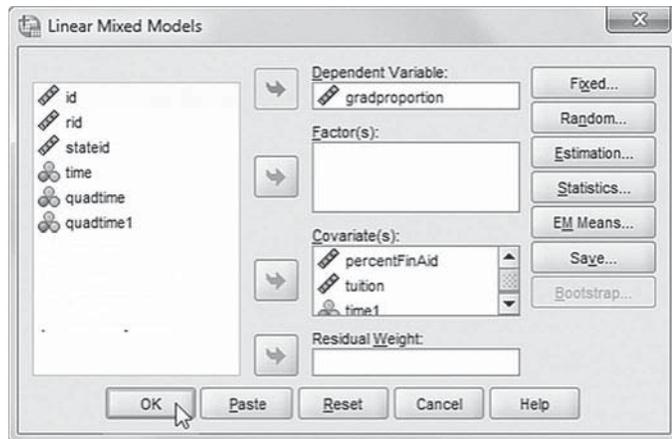
Repeat steps 4c to 4h using *time1* and *mathselect*.

**Add Fourth Interaction to Model 1.5:  $time1 * percentFTfaculty$** 

Repeat steps 4c to 4h using *time1* and *percentFTfaculty*.

The complete model with the four interactions is displayed in the *Model* box insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

**Interpreting the Output From Model 1.5**

Our primary interest is the growth rate portion of the model in Table 6.12. Once again, the average state growth rate is estimated as about 0.020. Average family share of higher education costs did not affect growth in state graduation rates over time ( $\gamma = -0.002, p > .10$ ). In contrast, however, average retention of freshman students was significantly related to increases in graduation growth rates ( $\gamma = 0.011, p < .05$ ). This suggests that a 1-*SD* increase in year 1 student retention rates would result in about a 1% increase in graduation growth rate over time, compared with states at the grand mean for retention rate, holding the *time1* slope and other variables constant. The other fixed effects remained consistent with the previous model.

**TABLE 6.12 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	0.475	0.008	166.959	57.074	.000	0.458	0.491
aveFamilyshare	0.020	0.006	49.395	3.666	.001	0.009	0.032
aveRetention	0.044	0.006	46.191	7.846	.000	0.032	0.055
mathselect	0.094	0.005	707.405	18.178	.000	0.084	0.105
percentFTfaculty	0.037	0.004	653.623	8.960	.000	0.029	0.045
percentFinAid	0.002	0.001	5,655.894	2.136	.033	0.000	0.004
tuition	0.043	0.005	5,100.035	8.186	.000	0.032	0.053
time1	0.020	0.005	60.234	4.015	.000	0.010	0.030
aveFamilyshare * time1	-0.002	0.004	39.421	-0.372	.712	-0.010	0.007
aveRetention * time1	0.011	0.004	40.280	2.575	.014	0.002	0.020
mathselect * time1	0.005	0.005	671.231	1.077	.282	-0.004	0.015
percentFTfaculty * time1	0.020	0.004	514.186	5.303	.000	0.013	0.027

<sup>a</sup> Dependent variable: gradproportion.

TABLE 6.13 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	AR1 Diagonal	0.003	0.000	34.100	.000	0.003	0.003
	AR1 rho	0.311	0.019	16.266	.000	0.273	0.348
Intercept + time1 [subject = stateid]	Var: Intercept	0.001	0.000	2.368	.018	0.000	0.002
	Var: time1	0.000	0.000	1.263	.207	0.000	0.001
Intercept + time1 [subject = rid * stateid]	Var: Intercept	0.007	0.000	14.954	.000	0.006	0.008
	Var: time1	0.004	0.000	7.985	.000	0.003	0.005

<sup>a</sup> Dependent variable: gradproportion.

The variance components table (Table 6.13) suggests there is still Level 3 and Level 2 variance in initial status graduation levels to be explained. After adding the state-level variables to explain growth in graduation rates over time, there is not sufficient variability left to be explained (Wald  $Z = 1.263$ , one-tailed  $p = .1035$ ). There is, however, sufficient variability in growth rates still across institutions (Wald  $Z = 14.954$ , one-tailed  $p < .001$ ). We could include other predictors at the institutional level to model this variability if we desired.

We can also determine how much variance in the random coefficients was accounted for by the predictors, keeping in mind our previous cautions about variance reduction in multilevel models. The proportion of variance explained is the ratio of the difference between the total parameter variance estimated from the unconditional model and the residual parameter variance from the fitted model relative to the total parameter variance. For intercepts, the amount of variance accounted for is reduced from 0.004 (Table 6.2) to 0.003 in Table 6.13 ( $0.004 - 0.003 = 0.001/0.004$ ), or about 25% of the variance accounted for within institutions. At Level 2, the variance component is reduced from 0.016 in Table 6.2 to 0.007 (or  $R^2$  of 0.563), or about 56.3% of the variance accounted for at Level 2. At the state level (Level 3), the variance is reduced from 0.003 to 0.001 (or  $R^2$  of 0.667), or about 67.7% of the variance at that level. Of course, these estimates would be a bit different if we began with a baseline model that included *time1*.

## A Regression Discontinuity Analysis of a Math Treatment

Other types of models to examine change over time can also be formulated in SPSS MIXED. In the remaining part of the chapter, we focus on two types of research designs for investigating effects of interventions over time. Both models involve the observation of a discontinuity, or “jump” in repeated measurements. In the next example, we illustrate how the regression discontinuity (RD) design can be used to examine the impact of a hypothetical classroom treatment in math on students. The design provides a strong, reliable alternative to the randomized experiment (Shadish, Cook, & Campbell, 2002). The RD design is a type of pre- and posttest design, which makes it possible to assess the impact of a treatment accurately by establishing a precise preintervention criterion for assigning individuals to treatment and control groups. In an RD design, individuals above the chosen cut point are assigned to one group, those below to another, and the treatment assigned to either group.

The RD design compares the outcomes of individuals on either side of the selection cutoff point but who are similar on other baseline covariates. The observed treatment effects are expected to be most easily discerned for individuals near the cut point; that is, one can control for confounding factors by simply contrasting participants close to the treatment cut point to nonparticipants

close to the cut point. In most situations, however, researchers look beyond this examination of individuals only around the cut point and use all the data they have (or at least data covering a much broader range of individuals in the study). As Cain (1975, in Pedhazur & Schmelkin, 1991) noted, “The critical difference for avoiding bias is not whether the assignments are random or nonrandom, but whether the investigator has knowledge of and can model this selection process” (p. 304). Thus, estimation bias may be avoided in the RD design, not because of random assignment of subjects, but because the covariate used to assign subjects is known beforehand (see Cook & Campbell, 1979; Pedhazur & Schmelkin, 1991). We encourage readers to investigate additional sources on the RD design for further information about the uses of this design.

## The Data and Design

In our illustration, we have 4,591 students who were assigned to one of two instructional conditions—we will say one being a more traditional instructional approach consisting of teacher lecturing and students reading the supporting text material (coded 0), or an alternative approach, where students work more collaboratively with their peers and the teacher using a lab-based, problem-solving approach. In our example, 53 math teachers implement the traditional and alternative approaches within the basic math courses they teach. The treatment, therefore, is implemented at the individual student level (Level 1) since students assigned to a particular teacher could have received either instructional approach. At the teacher level (Level 2), covariates related to teacher effectiveness and class composition serve as controls for peer and teacher effects that might interact with the treatment effects. The variables used in the example are summarized in Table 6.14.

**TABLE 6.14** Data Definition of *ch6RDdata.sav* ( $N = 4,591$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>subjectid</i>	Individual	Student identifier (4,591 students across three time occasions).	Integer	Ordinal
<i>teachcode</i>	Teacher	Teacher identifier (53 math teachers).	Integer	Ordinal
<i>pretest</i>	Individual	Math placement test score used to divide students into a treatment or control group.	148 to 171	Scale
<i>npretest</i>	Individual	The transformed measure of the student's pretest math placement test score ( <i>pretest</i> ) in relation to the cut point for group assignment.	-11 to 12	Scale
<i>treatment</i>	Individual	Dichotomous variable indicating students assigned to the control group (coded 0) or the treatment group (coded 1).	0 = Control group 1 = Treatment Groups	Scale
<i>nmath</i>	Individual	Dependent variable representing student performance in math.	561 to 856	Scale
<i>teachqual</i>	Teacher	Standardized composite score ( $M = 0$ , $SD = 1$ ) of the teacher's overall teaching effectiveness.	-2.67 to 340	Scale
<i>classcomp</i>	Teacher	Factor score ( $M = 0$ , $SD = 1$ ) representing the weighted percentage of students in each class who are low SES, receiving English language services, and receiving special education services.	-1.11 to 2.16	Scale
<i>npretest2</i>	Individual	Quadratic pretest component ( $npretest * npretest$ )	0 to 144	Scale
<i>npretest3</i>	Individual	Cubic pretest component ( $npretest^2 * npretest$ )	-1331 to 1728	Scale

<sup>a</sup> Individual = Level 1; teacher = Level 2.

Students are assigned to the treatment and control conditions based on a math pretest score. In our example, we set a cut score on the pretest of 159. Therefore, students with scores of 160 or higher are assigned to the treatment, and those with scores of 159 or lower are assigned to the control (although this assignment can also be reversed). If the treatment has no effect (i.e., assuming a linear relationship between the covariate and outcome), a single regression line is expected; that is, there would be no discontinuity between students who scored 159 or below and those who scored 160 and above. If the treatment is effective, a discontinuity in the regression line reflects the size of the treatment effect (Shadish et al., 2002).

Several advantages and cautions regarding the design have been noted (Cook & Campbell, 1979). First, RD can produce an unbiased causal inference, if it is precisely implemented with correct modeling of the relation between the assignment criterion (i.e., age) and the independent variable (i.e., achievement). Its major drawback is the possibility that effects may be biased if the relationship between the assignment covariate and the outcome variable is incorrectly modeled (e.g., nonlinear relationship or possible interactions ignored). A second advantage is that, in principle, controlling various background variables to assess the effect of the treatment is not required. This is because the exact criterion of assignment to treatment or control group is known, and its effect is accounted for in the analysis. We do have the flexibility, however, to include student-level covariates as a check for the robustness of treatment estimates.

Misclassification can sometimes be a problem, if other factors besides the criterion used to assign individuals also influence assignment decisions (e.g., self-selection and individuals who are eligible to participate but choose not to participate). When other factors may potentially influence treatment effects, this is referred to as a *fuzzy* discontinuity. In practice, this potential problem is sometimes addressed by comparing subgroups of eligible versus noneligible participants. Moreover, the RD design identifies the mean impact of the treatment at the cut point for selection (versus an experimental design that provides the average treatment effect for all individuals). The RD design, therefore, does not directly inform us about the impact for individuals who might be further away from the cut point. Individuals at the extreme ends of the distribution on the assignment criterion are unlikely to be comparable, which can make the analyst rely more on covariates to control for the differences between individuals.

### Assumptions of the Design

As Trochim (1984) notes, interpreting results of studies using the RD design depends on the support of three assumptions. These assumptions were met in conducting the analyses presented in the following section.

1. *The assignment of participants according to the cut score must be followed.*
2. *The pattern of pretest scores must be specified correctly by the statistical model used.* Models can be tested initially for possible higher order polynomial effects related to the pretest scores (i.e., quadratic and cubic) and for interactions between the higher order polynomials and the treatment. This amounts to testing for higher order terms above and below the cut point (i.e., including higher order polynomials and interactions between the polynomials and the treatment). As Shadish et al. (2002) note, if interactions or nonlinear effects are suspected, the analysis should be conducted with the higher order terms in the model, and then nonsignificant terms should be dropped from higher to lower order. When there is doubt, the terms should be kept in the model because this overfitting will yield unbiased coefficients.
3. *There is no coincidental factor at the chosen cut score that would result in program effects.* This assumption can be tested by conducting a study over a repeated number of semesters and with situations where students above the cut score were assigned to the treatment group and with situations where students above the cut score were assigned to the control group.

## Steps in the Regression Discontinuity Analysis

In the RD approach, an important first step is to determine the functional relationship between the assignment variable and the outcome variable (Shadish et al., 2002). In this case, the assignment variable is students' scores on a math pretest. Determining the functional relationship (i.e., whether it is linear or curvilinear) between the assignment and outcome variable is important in order to ensure unbiased treatment effects (Shadish et al., 2002). This can be accomplished by initially including higher order (i.e., quadratic and cubic) assignment (pretest) effects and possible interactions with the treatment and then removing the nonsignificant effects (Moss & Yeaton, 2006). In our example, we created the higher order terms but found that none of them were significant in explaining the relationship between the pretest, treatment, and outcome. Therefore, we eliminated them. We retained the *treatment × pretest* interaction to illustrate that this might mean there was an advantage or disadvantage associated with the treatment dependent on students' pretest scores. If this were significant, it would suggest that the treatment worked differentially for students with differing levels of prior math ability.

In the multilevel analysis of a RD design, there are three primary equations to consider: the student-level equation to model individuals' pretest and treatment effects on an achievement intercept, the equation to explain variation in between-teacher intercepts, and the equation to explain variability in between-teacher treatment slopes.

## Predictors in the Models

*Treatment effect.* This variable is simply an indicator of the student being assigned to the control group (coded 0) or the treatment group (coded 1).

*Pretest.* Students took a math placement test before entering the developmental math sequence. The variable *npretest* represents the transformed measure of the student's pretest in relation to the cut point for group assignment. Student math placement test scores were used for purposes of assignment to the traditional or redesigned course sequence. The highest score at the cut point (159) was coded 0, which with respect to the intercept in the model equation represents the achievement of the students with most prior ability in either the control or treatment group, depending on whether the treatment is assigned to the set of students with lower math prior skills or higher prior math skills. In this case, we assigned the students with scores of 159 or lower to the control group. Prior math scores of 160 or above were assigned to the treatment group. The scores were then recoded to be centered on the cut score such that scores below 159 were negatively coded (e.g., 158 = -1; 157 = -2) and scores above 159 are positively coded (e.g., 160 = 1; 161 = 2).

*Classroom composition.* This is the factor score ( $M = 0$ ,  $SD = 1$ ) describing the weighted percentage of students in each class who are low socioeconomic status (SES), receive English language services, and receive special education services.

*Teacher quality.* This is a standardized composite score ( $M = 0$ ,  $SD = 1$ ) of the teacher's overall teaching effectiveness considering several domains (e.g., academic expectations, monitoring of student process, implementation of curriculum standards, and classroom performance evaluations).

## Specifying the Model

Equation 6.27 represents the reduced RD model examined after testing for higher order treatment effects and interactions:

$$Y_{ij} = \beta_{0j} + \beta_1 npretest_{ij} + \beta_2 treatment_{ij} + \varepsilon_{ij}. \quad (6.27)$$

Covariates ( $X$ ) can be added to the model to increase the precision of the treatment by accounting for additional residual variance. It should be noted that in the RD equation, the pretest score

is transformed by taking into consideration the cut point, or highest score that will separate the groups, which is transformed to 0. This defines the intercept in the model as the mean math achievement score for students with the highest pretest score in the control group. The coefficients  $\beta_1$  and  $\beta_2$  represent the potential effects of the pretest and the effect of the treatment, respectively, on math achievement.

At Level 2, we will allow the achievement intercept and the treatment slope to vary at random:

$$\beta_{0j} = \gamma_{00} + u_{0j}, \quad (6.28)$$

$$\beta_{2j} = \gamma_{20} + u_{2j}. \quad (6.29)$$

We will also fix the pretest slope ( $\beta_{1j} = \gamma_{10}$ ). Through substitution, we arrive at the combined model:

$$Y_{ij} = \gamma_{00} + \gamma_{10}npretest_{ij} + \gamma_{20}treatment_{ij} + u_{2j}treatment_{ij} + u_{0j} + \varepsilon_{ij}. \quad (6.30)$$

We note that we also specified the covariance between the intercept and treatment slope Level 2 by specifying a UN covariance matrix. This suggests seven parameters to estimate in the initial model (i.e., three fixed effects, three random effects at Level 2, and one residual).

In Table 6.15, we provide the preliminary models where we examined possible nonlinearities in the effects of students' entrance pretest scores and treatment effects. As those can be removed, it strengthens the case for the difference in effect on either side of the cut point. We note that in the preliminary two models having higher order polynomial terms, no predictor was significant. We then eliminated the cubic and quadratic terms. We can see in the last model (with linear term) that the interaction between the pretest and treatment was nonsignificant, suggesting it could also be removed from subsequent models.

**TABLE 6.15 Examination of Higher Order Polynomial Interactions**

Variables	Cubic Term	Quadratic Term	Linear Term
<i>Between Teachers</i>			
Intercept	672.648*	672.267*	671.466*
treatment	5.563	3.702	4.661**
<i>Student Level</i>			
npretest	1.450	0.900	0.400**
treatment*npretest	-2.501	-0.314	0.125
npretest2	0.179	0.047	
treatment*npretest2	0.112	-0.051	
npretest3	-0.008		
treatment*npretest3	-0.023		
<i>Variance Components</i>			
Level 2 Intercept	148.48*	148.51*	148.17*
Level 2 Slope	25.49**	25.47**	25.45**
Covariance	5.88	5.86	6.08
Residual	1,340.22*	1,339.78*	1,339.32*

\* $p < .05$ ; \*\* $p < .10$ .

## Regression Discontinuity Models to Explain Learning Differences

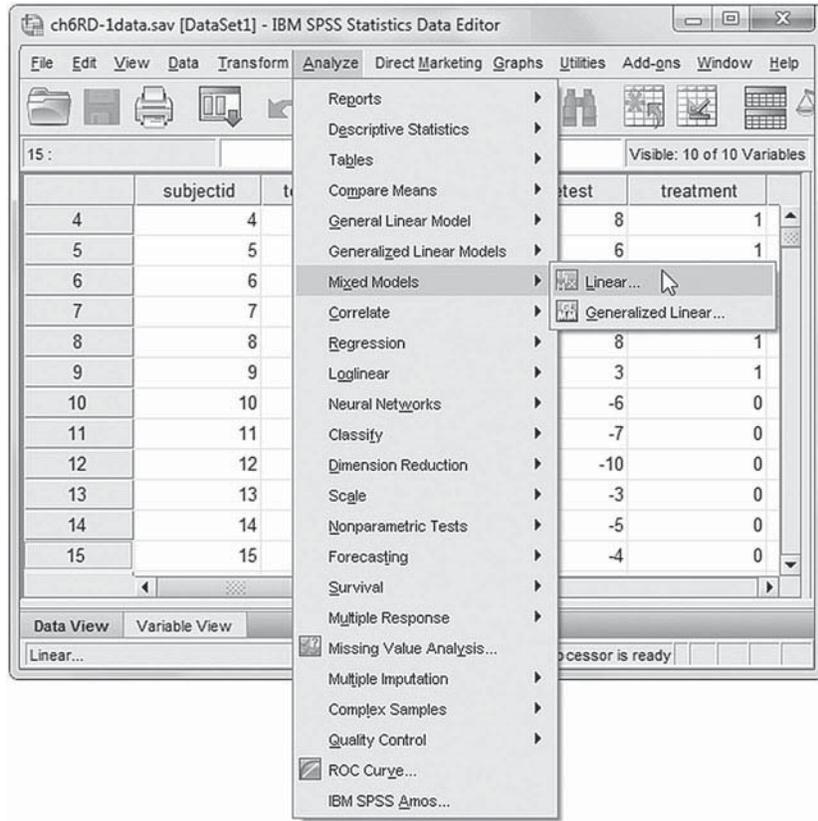
The initial research question asked if there is a difference in learning outcomes between students in traditional and problem-based learning groups.

### Defining Model 2.1 with IBM SPSS Menu Commands

Launch the IBM SPSS program application, and select the *ch6RD-1data.sav* data file.

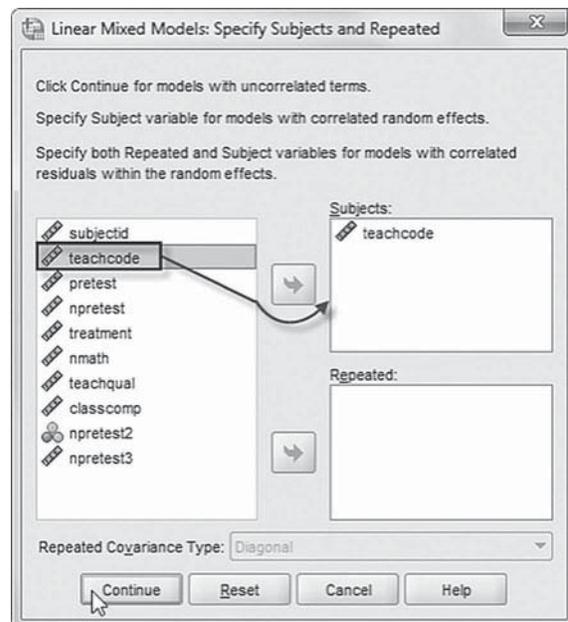
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

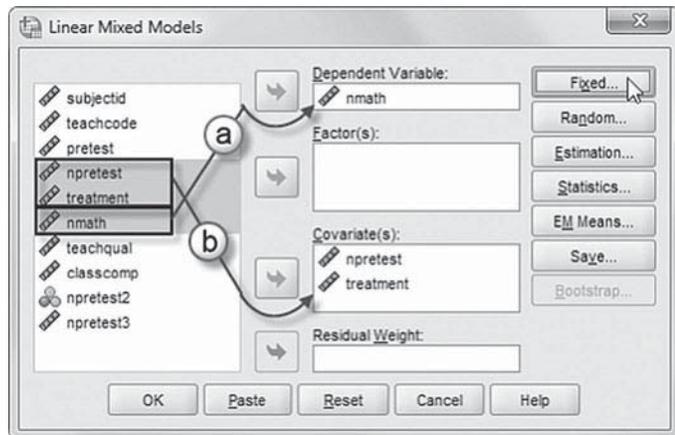


2. Within the *Linear Mixed Models: Specify Subjects and Repeated* dialog box, click to select *teachcode* from the left column. Then click the arrow button to transfer the variable into the *Subjects* dialog box.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

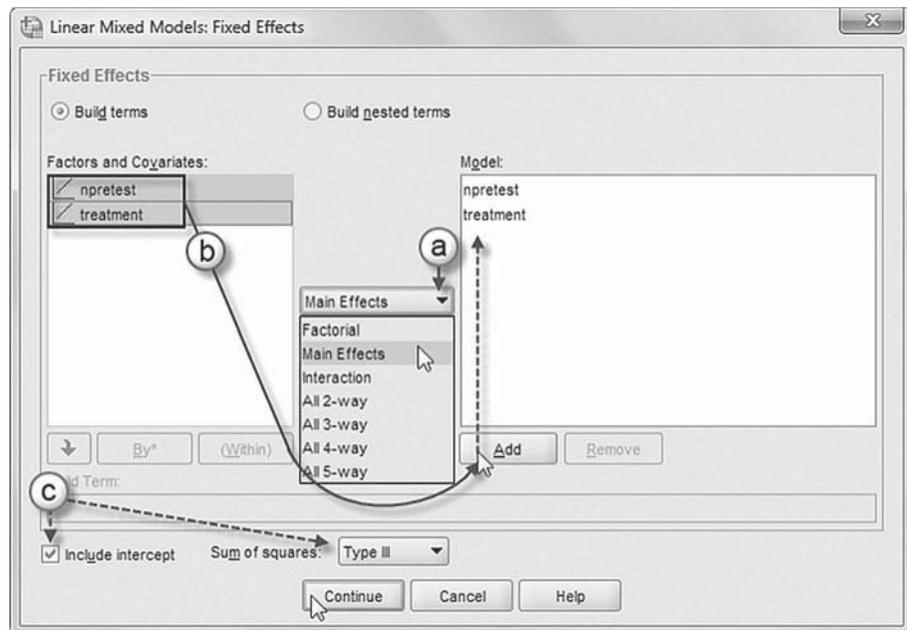


- 3a. In the *Linear Mixed Models* dialog box, click to select the *nmath* variable from the left column listing, and then click the right-arrow button to move it into the *Dependent Variable* box.
- b. Click to select *npretest* and *treatment* and then click the right-arrow button (or “drag them”) to move them into the *Covariate(s)* box.



Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
- b. Click to select *npretest* and *treatment* from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box.
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.

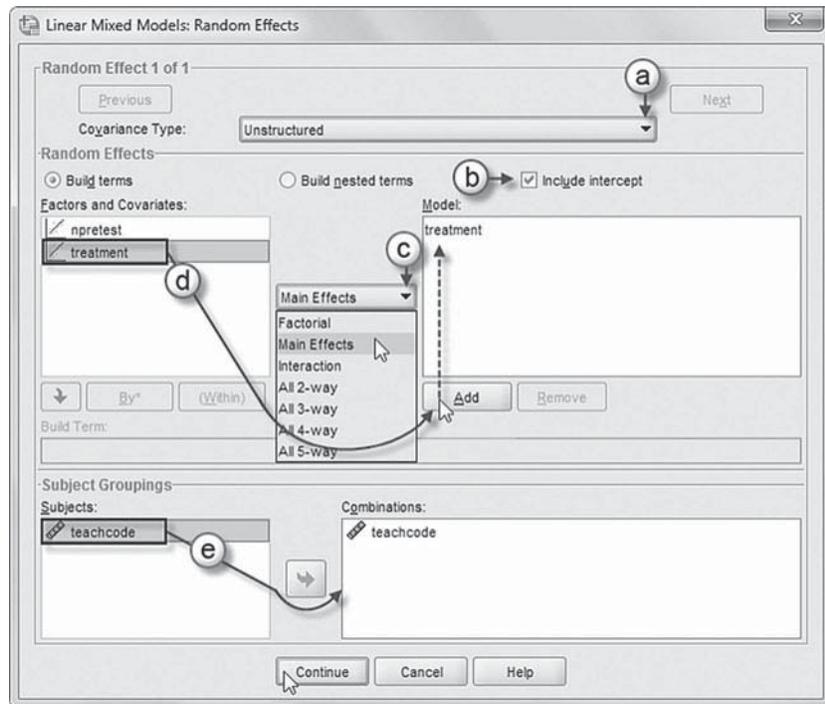


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

We will now add random effects to this model.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box

- 5a. Within the *Linear Mixed Models: Random Effects* box, change the covariance type by clicking on the pull-down menu and selecting *Unstructured*.
- b. Click to select: *Include intercept*.
- c. Change *Factorial Effects* by clicking on the pull-down menu and selecting *Main Effects*.
- d. Click to select *treatment* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
- e. Click *teachcode* from the *Subjects* box, and then click the right-arrow button to add the variable to the *Combinations* box.

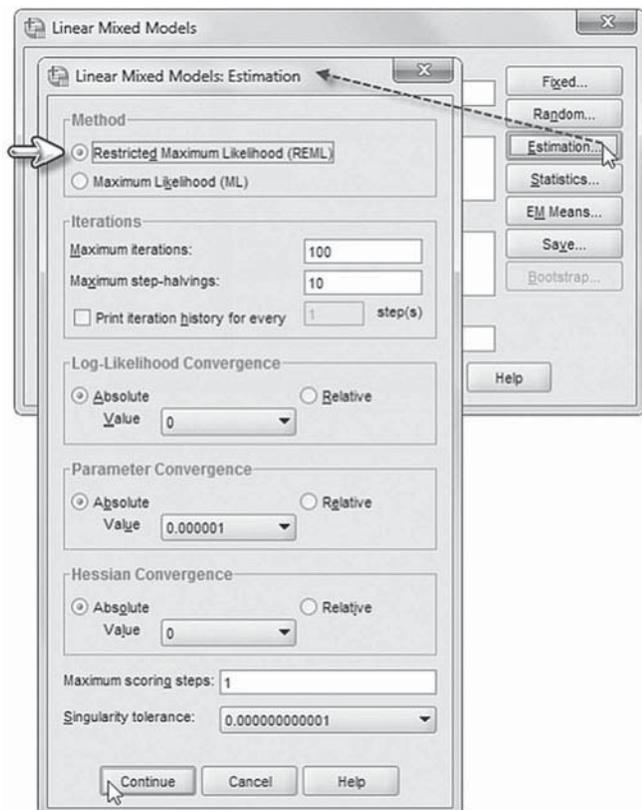


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

6. The *Linear Mixed Models: Estimation* dialog box displays two estimation method choices: ML or REML.

In this chapter, we will use the default setting of REML to estimate the models.

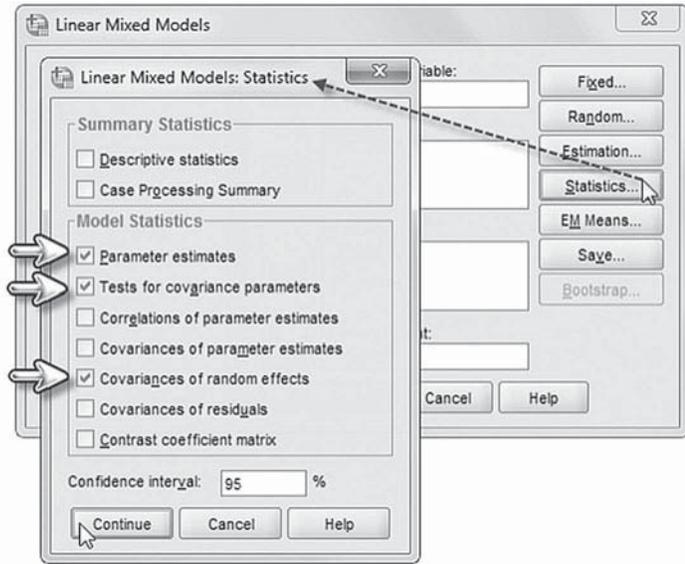
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



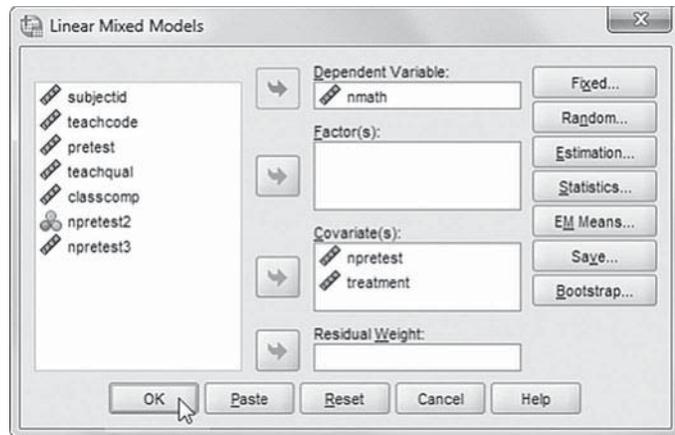
7. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



8. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 2.1**

The fixed-effect estimates in Table 6.16 suggest that the mean for students in the control group (holding other variables at 0) was 671.818. Higher pretest scores tend to be related to higher scores (0.465,  $p < .01$ ). Students in the treatment group also have significantly higher scores than students in the control group ( $\gamma = 4.710, p < .05$ ).

**TABLE 6.16** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	671.818	2.126	76.299	316.041	.000	667.584	676.051
npretest	0.465	0.161	4,540.936	2.889	.004	0.150	0.781
treatment	4.710	2.382	397.711	1.978	.049	0.028	9.392

<sup>a</sup> Dependent variable: nmath.

Table 6.17 presents the summary of the covariance parameters. The table suggests that math achievement varies across teachers in the study, as we might expect. There is some evidence that the treatment also varies across teachers (Wald  $Z = 1.603$ , one-tailed  $p < .06$ ).

The results suggest we might build a model to examine variability in the math achievement intercepts and the treatment effect across teachers in the study.

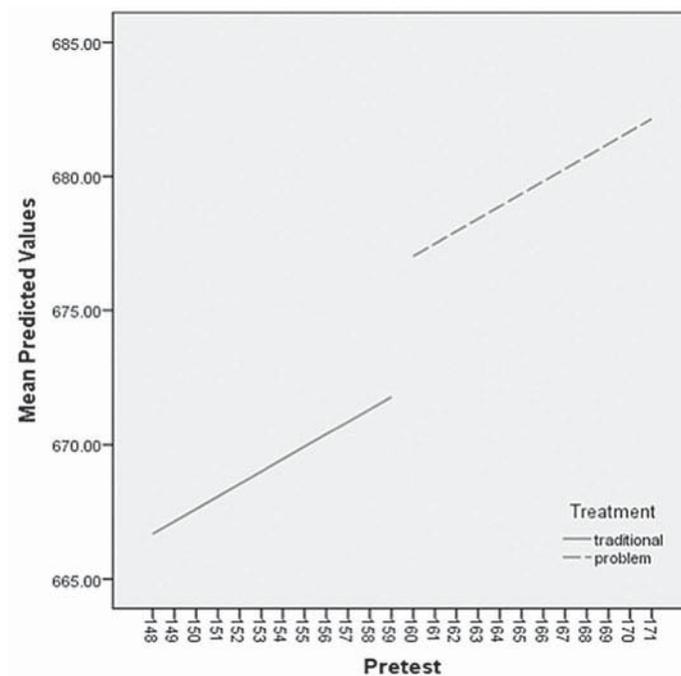
In Figure 6.5, we summarize the predicted scores of individuals based on their pretest and membership in the treatment or control group. The figure illustrates the discontinuity present between the pretest scores (i.e., with 159 describing the highest score in the control group and 160 the lowest score in the treatment group).

As noted, we transformed the pretest variable ( $npretest$ ) so that it is centered on 159 (i.e., the highest score in the control group). This suggests that individuals in the treatment group with the lowest scores on the pretest (i.e., 160) will have predicted scores of about 5.18 points higher (i.e.,  $4.71 + 0.47 = 5.18$ ) than their immediate peers in the control group, as summarized in Table 6.16.

**TABLE 6.17** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		1,339.071	28.210	47.469	.000	1,284.907	1,395.519
Intercept + treatment	UN (1, 1)	148.064	36.588	4.047	.000	91.224	240.320
[subject = teachcode]	UN (2, 1)	6.180	17.257	0.358	.720	-27.643	40.002
	UN (2, 2)	25.430	15.859	1.603	.109	7.490	86.336

<sup>a</sup> Dependent variable: *nmath*.



**FIGURE 6.5** Regression discontinuity design illustrating treatment effect.

## Adding Explanatory Variables at Level 2

We might add two explanatory predictors at Level 2 (i.e., a classroom composition variable and a teacher quality variable). For the intercept model, we propose that the student composition (*classcomp*) in each class affects overall classroom achievement. We also propose that teacher quality (*teachqual*) influences student progress in each class.

$$\beta_{0j} = \gamma_{00} + \gamma_{01}teachqual_j + \gamma_{02}classcomp_j + u_{0j} \quad (6.31)$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}teachqual_j + \gamma_{22}classcomp_j + u_{2j} \quad (6.32)$$

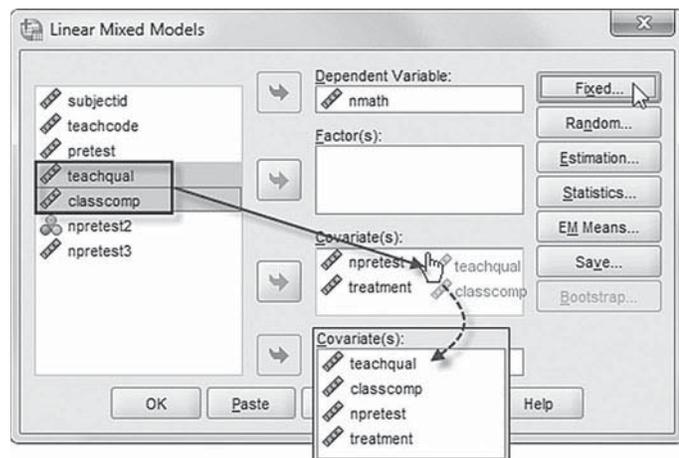
When we substitute these into the Level 1 equation (Eq. 6.27), we will have a combined equation with 11 parameters to estimate (i.e., seven fixed effects, three random effects, and one Level 1 residual).

$$Y_{ij} = \gamma_{00} + \gamma_{01}teachqual_j + \gamma_{02}classcomp_j + \gamma_{10}npretest_{ij} + \gamma_{20}treatment_{ij} + \gamma_{21}teachqual_j * treatment_{ij} + \gamma_{22}classcomp_j * treatment_{ij} + u_{2j}treatment_{ij} + u_{0j} + \varepsilon_{ij} \quad (6.33)$$

### Defining Model 2.2 with IBM SPSS Menu Commands

Continue using the *ch6RD-1data.sav* data. Settings default to those used for Model 2.1.

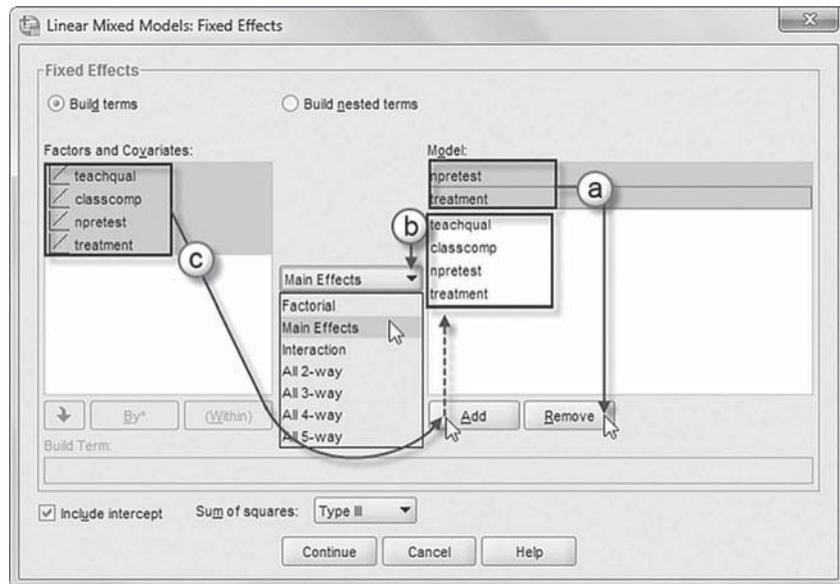
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from Model 2.1. The variable *teachcode* is within the *Subject* box. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.
3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.



We will introduce two additional predictors to be used in the model (*teachqual* and *classcomp*). First, click to select *teachqual* and *classcomp*, and then “drag” the variables to the *Covariate(s)* box above *npretest*. The sequence of the variables is the following: *teachqual*, *classcomp*, *npretest*, and *treatment*.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading the output tables, we will first remove the two variables (*npretest* and *treatment*) by clicking to select the variables and then clicking the REMOVE button.

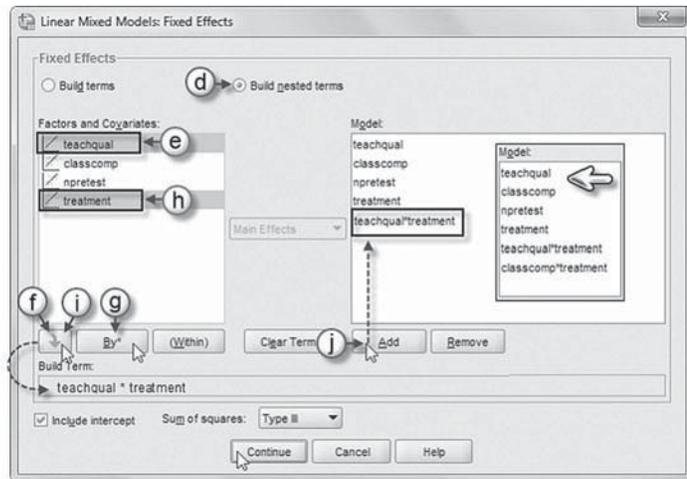


- b. Confirm that the factorial setting is *Main Effects* before adding the predictor variables to the model.
- c. Now click to select all the variables from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box.

Two cross-level interactions (or nested terms) will be created and added to the model:  $classcomp * treatment$  and  $teachqual * treatment$ . These interactions will tell us whether (a) teacher quality (*teachqual*) influences the strength of the treatment effect in each class and (b) student composition (*classcomp*) in each class influences the treatment effect.

#### Add First Interaction to Model 2.2: $teachqual * treatment$

- d. Click to select *Build nested terms*.
- e. Click to select the variable *teachqual* from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves *teachqual* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button, which will insert the computation command symbol:  $teachqual*$ .
- h. Click to select *treatment* from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move *treatment* into the *Build Term* box and complete the interaction term:  $teachqual * treatment$ .
- j. Click the ADD button to transfer the interaction into the *Model* box.

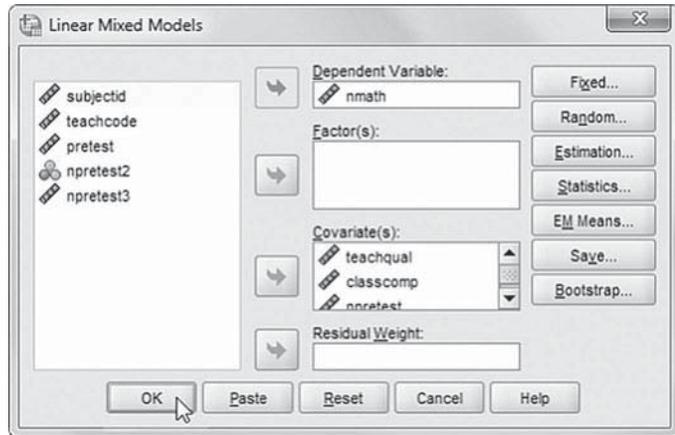


#### Add Second Interaction to Model 2.2: $classcomp * treatment$

Repeat steps 4d to 4j using *classcomp* and *treatment* for the interaction. The final model is shown in the insert.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

- Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 2.2**

We provide the fixed effects in Table 6.18. The results suggest that teacher quality affects achievement levels ( $\gamma_{01} = 3.418, p < .05$ ) and, as we might expect, classroom composition does affect achievement levels ( $\gamma_{02} = -15.644, p < .001$ ). With respect to the treatment, however, neither teacher quality nor classroom composition affects the strength of the treatment effect. In terms of our research goals, we can interpret these results in positive terms by suggesting that the problem-based treatment appeared to enhance student learning, regardless of the specific classroom composition or teacher quality.

**Investigating a Change Due to Policy Implementation**

In this final example, we examine whether a policy, or some other type of intervention, has an impact on its intended targets. One design that is useful in examining trends before and after a policy is introduced is a time series design. The essence of a time series design is the presence of periodic measurements and the introduction of an experimental change into this time series of measurements. The introduction of the treatment is expected to produce a discontinuity in the series of measurements at some hypothesized interval after its introduction. Consider an example concerning the introduction of a policy to increase the academic standards for freshman student athletes and institutions’ responses to recruiting freshman student athletes before and after the policy was introduced (e.g., see Heck & Takahashi, 2006, a more extended treatment of this approach). In this case, we examine whether the introduction of Proposition 48 during the

**TABLE 6.18** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	671.918	1.679	85.023	400.185	.000	668.579	675.256
teachqual	3.418	1.476	62.980	2.316	.024	0.468	6.368
classcomp	-15.644	2.419	45.706	-6.466	.000	-20.514	-10.773
npretest	0.462	0.161	4,544.787	2.865	.004	0.146	0.777
treatment	4.912	2.406	360.340	2.042	.042	0.180	9.644
teachqual*treatment	0.542	1.521	68.353	0.356	.723	-2.493	3.578
classcomp*treatment	-0.756	2.362	43.662	-0.320	.751	-5.517	4.006

<sup>a</sup> Dependent variable: nmath.

mid-1980s had any impact on institutional behavior regarding freshman student athletes. Proposition 48 was implemented to increase institutions' graduation rates by upgrading their academic standards for admitting student athletes.

A thorough test of the policy's impact on changing institutions' recruiting behavior would require the analyst to compare the behavioral trend before and after the policy was introduced. A piecewise growth model—where the growth trajectories are split into two or more trends—can be used to compare growth rates during two different prepolicy and policy implementation periods. In our example, we study freshman student athletes admitted to Division 1A football programs during a 9-year period. More specifically, 3 years of data were collected before the introduction of the policy, and 6 years of data were collected after the policy's implementation. In a time series design, evidence of a treatment's effect is indicated by a discontinuity in the measurements recorded in the time series (Campbell & Stanley, 1966). In this example, the time series design may be diagrammed as follows:

$$0_10_20_3X0_40_50_60_70_80_9,$$

where the three 0s preceding the  $X$  represent a yearly trend in freshman athletes admitted before the introduction of the policy. The design is a sound quasi-experimental design, provided certain threats to internal validity can be successfully argued away (Campbell & Stanley, 1966). Basically, the problem of internal validity reduces to the question of whether plausible competing hypotheses offer likely alternative explanations for any shift in the time series other than the introduction of the policy (Campbell & Stanley, 1966).

The major threats to the internal validity of the single-group time series design are instrumentation, testing, and history. Instrumentation and testing can be argued away more easily in this case because the data were collected utilizing the same variables and no repeated testing on individuals was done, as might be the case if the data were collected from individuals who received a treatment of some type. The determination of change is considered solely on the fluctuations in the institutional data prior to policy implementation and the years following its implementation. Threats due to history, however, could be a potential problem. Rival explanations could include changes in the institutional norms within the set of schools (that may or may not correspond to the policy's introduction) or perhaps cyclical events. The observational series can be arranged to hold these types of cycles relatively constant (e.g., the data are collected at the same time and over a relatively long period of time). This lessens the possibility that some other corresponding extraneous event would produce the expected trends other than the implementation of the policy (or other type of treatment). To deal with history as a rival explanation, however, it is important for the researcher to specify in advance the expected relationship between the introduction of the treatment and the manifestation of an effect (Campbell & Stanley, 1966)—that is, how soon the effect would be seen. Importantly, as the time between implementation and resultant effects increases, the effects of extraneous events become more plausible.

By raising institutions' academic standards to increase graduation rates (i.e., the policy's intended effect), it is likely that the policy also resulted in a reduction of freshman athletes being admitted to programs, especially in some types of institutions. If this were true, over the course of data collection, first, we should note a discontinuity of measurements (i.e., the decline of freshman being admitted) after the policy was introduced. Second, it is likely that a shrinking pool of student athletes might affect the football recruiting practices of some schools more than others. We might hypothesize that schools with greater prestige (defined as schools with outstanding on-field performance and frequent bowl appearances) would be less affected in their efforts to recruit freshman athletes than would less prestigious schools. We might also test whether the policy affected public and private institutions differently in terms of recruiting.

We can formulate a piecewise growth model to test these hypotheses against the data. In our formulation, we first focus on comparing the growth trend before the policy was implemented versus the growth trend after it was implemented. We will assume that the effects of the policy will be seen immediately after the policy was introduced (since compliance was immediate). It is possible, however, to investigate lagged (or delayed) effects. The second part of the analysis involves examining how covariates such as institutional prestige and institutional type (i.e., public or private) might affect the key initial status and growth coefficients before and after the policy was introduced. Piecewise growth models can be used to represent different phases of development or change. One way to do this is to examine whether there is a “discontinuity,” or change, in the time series at the point where the policy is introduced ( $X$ ) or at some specified point afterward. In this example, it is likely that the growth (slope) parameter of freshman admits in Division I football programs would be negatively affected by the introduction of Proposition 48. Therefore, we can develop separate growth slopes before and after the policy was introduced. Covariates can be added to the model to help refine the examination of the contrasting trends.

### The Data

The data in this example consist of outcome data on the number of freshman student athletes admitted to 105 Division 1A football programs over a 9-year period (i.e., 3 years before the introduction of the policy and 6 years after its implementation). Data were also collected on program prestige (i.e., program success over a 5-year period of time) and institutional type (public or private). The variables are presented in Table 6.19.

**TABLE 6.19** Data Definition of *ch6RD-2data.sav* ( $N = 105$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>schid</i>	Individual	School identifier (105 schools).	Integer	Ordinal
<i>private</i>	School	Dichotomous variable identifying institutions as private or public.	0 = Not Private 1 = Private	Scale
<i>prestige</i>	School	Predictor variable measuring program success.	2.00 to 4.551	Scale
<i>Index1</i>	Individual	Variable represents the nine repeated measures of graduation status.	(1, 2, 3, 4, 5, 6, 7, 8, 9)	Nominal
<i>freshadmit</i>	Individual	Dependent variable representing the number of freshman student athletes admitted to football programs.	1 to 38	Scale
<i>implement</i>	Individual	Independent variable representing the prepolicy and policy periods.	0 = No Policy 1 = Policy	Scale
<i>time</i>	Individual	Preliminary variable representing the pre-policy and policy time periods (0,1,2,0,1,2,3,4,5).	(0, 1, 2, 3, 4, 5)	Nominal
<i>implement0</i>	Individual	Recoded <i>Index1</i> variable to a time-related variable representing the prepolicy period's yearly growth (0, 1, 2, 2, 2, 2, 2, 2).	(0, 1, 2, 2, 2, 2, 2, 2)	Nominal
<i>implement1</i>	Individual	Recoded <i>Index1</i> variable to a time-related variable representing the policy period's yearly growth (0, 0, 0, 1, 2, 3, 4, 5, 6).	(0, 0, 0, 1, 2, 3, 4, 5, 6)	Nominal

<sup>a</sup> Individual = Level 1; school = Level 2.

### Model 3.1: Establishing the Prepolicy and Policy Trends

Model 3.1 consists of one intercept and two growth trends. The data were coded such that the intercept represents initial status (i.e., Year 1). The first growth parameter then describes the *change per year* over the 3 years before the policy was implemented. The second growth trend represents change taking place after the policy was implementation (i.e., Year 4). Once again, we can open the *Repeated* dialog box using the *Index1* variable, which represents the nine repeated measures of graduation status. We then can recode this variable into two time-related variables, which we named *implement0* (representing the prepolicy period) and *implement1* (representing the policy implementation period). Readers may wish to examine the data set to see how the two time-related variables are coded. *Implement0* is coded to indicate yearly growth during the prepolicy period (0, 1, 2, 2, 2, 2, 2, 2, 2). The first measurement is coded 0, so it will be interpreted as the intercept, or the average number of freshman student athletes admitted to Division 1A football programs at the beginning of the study. The 2s from Year 4 to the end of the study indicate that the prepolicy period ends after Year 3. The policy implementation variable is coded to indicate no growth during the prepolicy period (0, 0, 0, 1, 2, 3, 4, 5, 6) and then captures the yearly change beginning at Year 4. Interested readers can consult Raudenbush and Bryk (2002) or Orsuwan and Heck (2009) for further information about coding piecewise growth models.

From Equation 6.1, at Level 1, for institution  $i$  at time  $t$  we have the following model:

$$Y_{it} = \pi_{0i} + \pi_{1i} \text{implement}0_{it} + \pi_{2i} \text{implement}1_{it} + \varepsilon_{it}, \quad (6.34)$$

where  $\pi_{0i}$  is the initial status intercept in terms of entering freshman student athletes,  $\pi_{1i}$  is the yearly rate of change before the policy was implemented (*implement0*),  $\pi_{2i}$  is the yearly rate of change after the policy was implemented (*implement1*), and  $\varepsilon_{it}$  represents errors in estimating each institution's growth trajectory, which are assumed to be normally distributed with a mean of 0 and some variance. We note that it is possible to add a second intercept for the policy implementation trend, which requires defining a multivariate model (see Chapter 7).

The Level 2 (between institutions) model is defined as

$$\begin{aligned} \pi_{0i} &= \beta_{00} + u_{0i}, \\ \pi_{1i} &= \beta_{10}, \\ \pi_{2i} &= \beta_{20}, \end{aligned} \quad (6.35)$$

where  $\beta_{10}$  and  $\beta_{20}$  are the prepolicy implementation and policy implementation intercepts and  $u_{0i}$  is a random initial status intercept. In preliminary analyses, we found that the policy slopes did not vary across institutions, so we fixed those parameters ( $u_{1i}$ ,  $u_{2i}$ ) between institutions in Equation 6.35. Through substitution, the combined model is then

$$Y_{it} = \beta_{00i} + \beta_{10i} \text{implement}0_{it} + \beta_{20i} \text{implement}1_{it} + u_{0i} + \varepsilon_{it}. \quad (6.36)$$

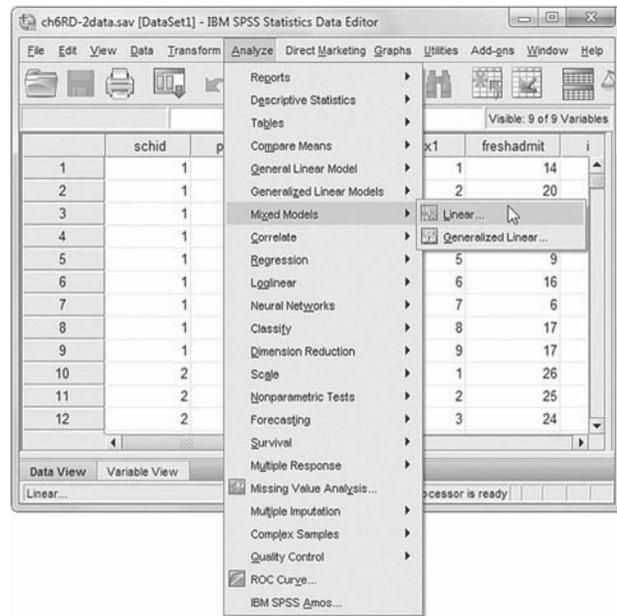
We found in preliminary investigation that a diagonal covariance structure fit the data well at Level1, and there is only a single random intercept at Level 2.

### Defining Model 3.1 with IBM SPSS Menu Commands

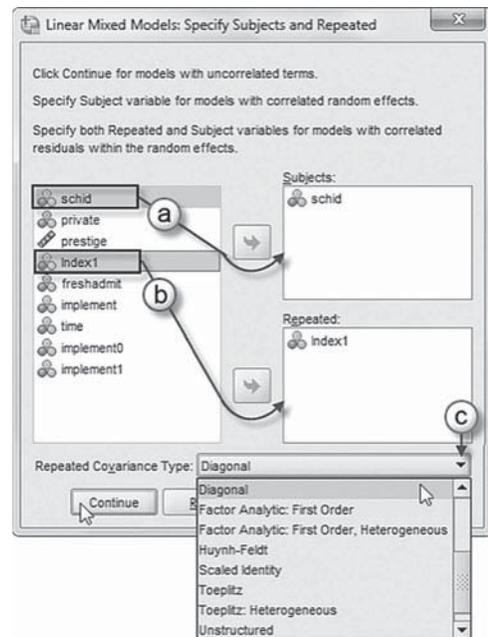
Launch the IBM SPSS program application, and select the *ch6RD-2data.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



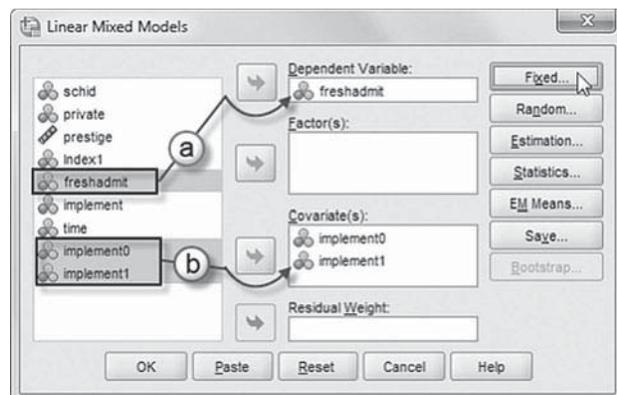
- 2a. Within the *Linear Mixed Models: Specify Subjects and Repeated* dialog box, click to select the *schid* variable from the left column, and then click the arrow button to transfer the variable into the *Subjects* dialog box.
- b. The *Repeated* box allows specifying variables that identify repeated observations. For this model, *Index1* identifies repeated observations over nine time periods. Click to select *Index1*, and then click the right-arrow button to move the variable into the *Repeated* box.
- c. The *Repeated Covariance Type* specifies a model's covariance structure. For this model, we will use the *Diagonal* covariance matrix. Click the pull-down menu to select the *Diagonal* covariance matrix as the *Repeated Covariance Type*.



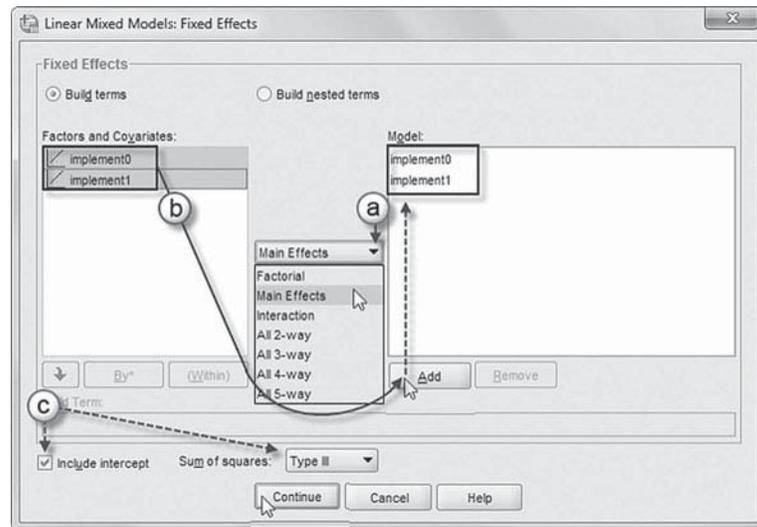
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

- 3a. In the *Linear Mixed Models* dialog box, click to select the *freshadmit* variable from the left column listing, and then click the right-arrow button to move it into the *Dependent Variable* box.
- b. Click to select *implement0* and *implement1*, and then click the right-arrow button (or “drag them”) to move them into the *Covariate(s)* box.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
- b. Click to select *implement0* and *implement1* from the *Factors and Covariates* box, and then click the **ADD** button to move the variables into the *Model* box.
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



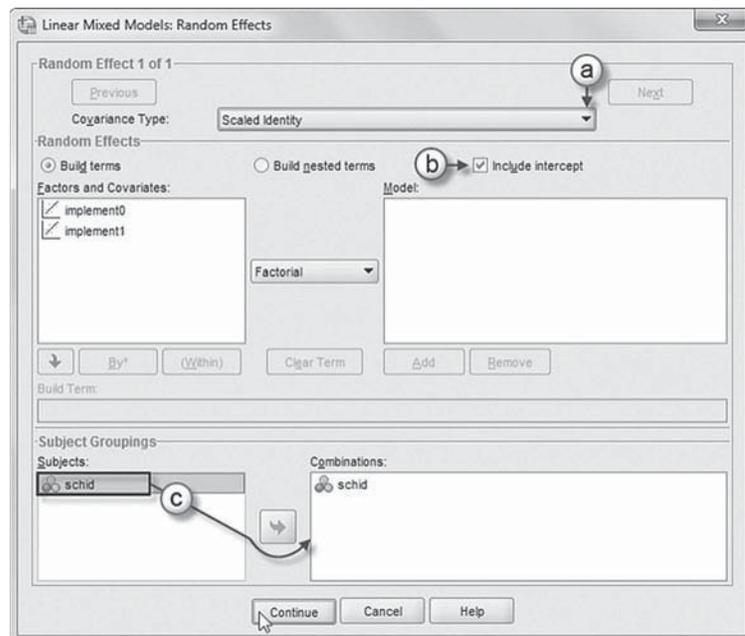
Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

We will now add random effects to this model.

Click the **RANDOM** button to access the *Linear Mixed Models: Random Effects* dialog box.

- 5a. Within the *Linear Mixed Models: Random Effects* box, change the covariance type by clicking on the pull-down menu and selecting *Scaled Identity*.
- b. Click to select *Include intercept*.
- c. Click *schid* from the *Subjects* box, and then click the right-arrow button to add the variable to the *Combinations* box.

Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

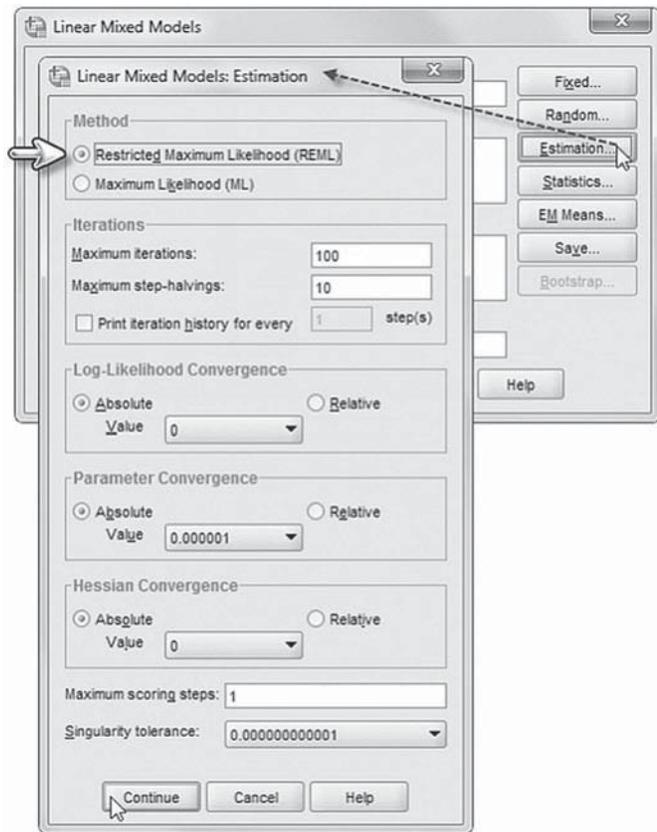


6. In the *Linear Mixed Models* dialog box, click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box.

The *Linear Mixed Models: Estimation* dialog box displays two estimation method choices: ML or REML.

In this chapter, we will use the default setting of REML to estimate the models. Readers should keep in mind that if they compare models with regression slopes and covariance parameters, ML estimation is preferred (Hox, 2010).

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

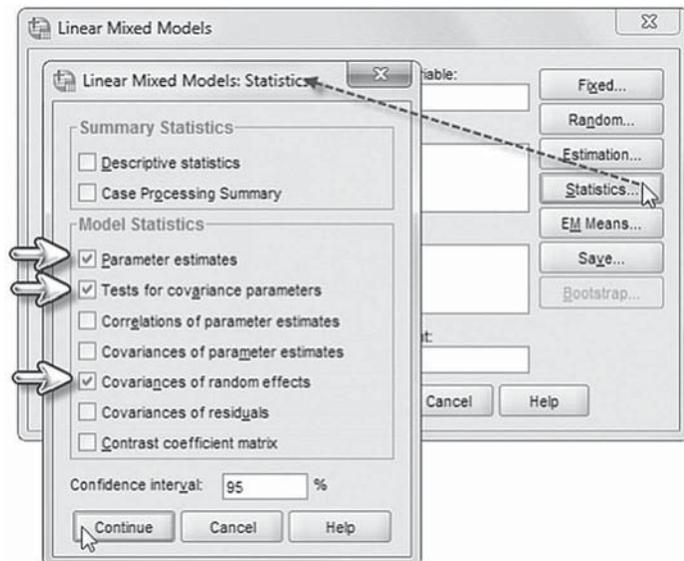


7. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

8. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 3.1**

This proposed model suggests 13 parameters to estimate (three fixed effects, one random effect, and nine diagonal variances at Level 1). This can be confirmed in Table 6.20, which summarizes the model dimensions.

We present the fixed effects in Table 6.21 on page 292. We can see that the first slope (before the policy was introduced) is negative but not declining significantly ( $\beta_{10} = -0.441, p > .05$ ). The slope after the introduction of the policy, however, does decline significantly ( $\beta_{20} = -0.585, p < .001$ ).

We present the variance components in Table 6.22. We can see that there is significant variance in initial status intercepts across institutions (Wald  $Z = 5.317, p < .001$ ). The variances for the repeated measures are considerably different, providing some evidence for why in our preliminary models the diagonal structure fit the data better than an autoregressive or compound symmetry covariance structure.

**TABLE 6.20 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	implement0	1		1		
	implement1	1		1		
Random Effects	Intercept	1	Identity	1	schid	
Repeated Effects	Index1	9	Diagonal	9	schid	105
Total		13		13		

<sup>a</sup> Dependent variable: freshadmit.

**TABLE 6.21 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	22.206	0.444	176.312	49.978	.000	21.329	23.083
implement0	-0.441	0.251	224.738	-1.755	.081	-0.935	0.054
implement1	-0.585	0.070	357.231	-8.391	.000	-0.722	-0.448

<sup>a</sup> Dependent variable: freshadmit.

TABLE 6.22 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [Index1 = 1]	18.209	2.729	6.671	.000	13.573	24.427
	Var: [Index1 = 2]	23.806	3.487	6.827	.000	17.865	31.722
	Var: [Index1 = 3]	19.046	2.846	6.692	.000	14.210	25.527
	Var: [Index1 = 4]	24.754	3.644	6.792	.000	18.549	33.034
	Var: [Index1 = 5]	20.872	3.084	6.768	.000	15.624	27.882
	Var: [Index1 = 6]	15.408	2.341	6.583	.000	11.441	20.752
	Var: [Index1 = 7]	12.117	1.888	6.417	.000	8.928	16.445
	Var: [Index1 = 8]	12.287	1.909	6.436	.000	9.061	16.661
	Var: [Index1 = 9]	9.960	1.599	6.228	.000	7.271	13.644
Intercept [subject = schid]	Variance	5.230	0.984	5.317	.000	3.617	7.561

<sup>a</sup> Dependent variable: freshadmit.

### Final Model with Covariates Added

Our final model adds school type ( $\beta_{01}$ ), defined as private (coded 1) versus (public = 0), and institutional prestige ( $\beta_{02}$ ) at Level 2 for the intercept model:

$$\pi_0 = \beta_{00} + \beta_{01}private_i + \beta_{02}prestige_i + u_{0i}. \quad (6.37)$$

The nonrandomly varying slope models will be the following:

$$\begin{aligned} \pi_{1i} &= \beta_{10} + \beta_{11}private_i + \beta_{12}prestige_i + u_{1i}, \\ \pi_{2i} &= \beta_{20} + \beta_{21}private_i + \beta_{22}prestige_i + u_{2i}. \end{aligned} \quad (6.38)$$

TABLE 6.23 Model Dimension<sup>a</sup>

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	private	1		1		
	prestige	1		1		
	implement0	1		1		
	implement1	1		1		
	private * implement0	1		1		
	prestige * implement0	1		1		
	private * implement1	1		1		
	prestige * implement1	1		1		
Random Effects	Intercept	1	Identity	1	schid	
Repeated Effects	Index1	9	Diagonal	9	schid	105
Total		19		19		

<sup>a</sup> Dependent variable: freshadmit.

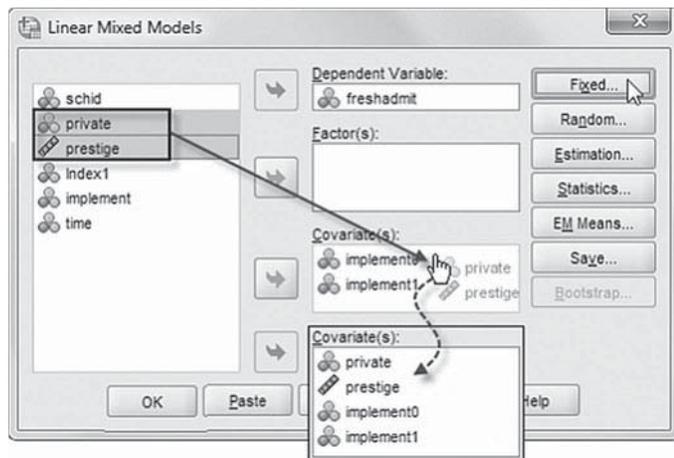
The Level 1 model remains the same as Equation 6.34. Though substitution of Equations 6.37 and 6.38 into Equation 6.34, we obtain the combined model (we leave this last step to readers). As the previous equations suggest, this model will add six more parameters to be estimated (i.e., two effects on the intercept, two effects on the prepolicy trend, and two effects on the policy implementation trend), which can be confirmed in the model dimension table (Table 6.23).

### Defining Model 3.2 with IBM SPSS Menu Commands

Continue using the *ch6RD-2data.sav* data. Settings default to those used for Model 3.1.

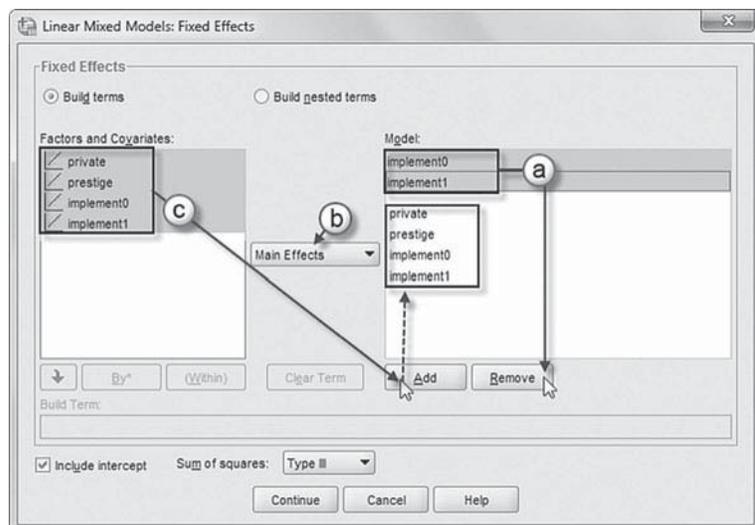
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from Model 3.1. The variables *schid* and *Index1* are in the *Subject* and *Repeated* box, respectively. The chosen covariance type is *Diagonal*. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.
3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

We will introduce two additional predictors to be used in the model (*private* and *prestige*). First, click to select *private* and *prestige*, and then “drag” the variables to the *Covariate(s)* box above *implement0*. The sequence of the variables is the following: *private*, *prestige*, *implement0*, and *implement1*.



Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To make reading the output tables easier, we will rearrange the sequence order of the variables by first removing the two variables (*implement0* and *implement1*). Click both variables to select them, and then click the REMOVE button.
- b. Confirm that the factorial setting is *Main Effects* before adding the predictor variables to the model.
- c. Now click to select all the variables from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box.



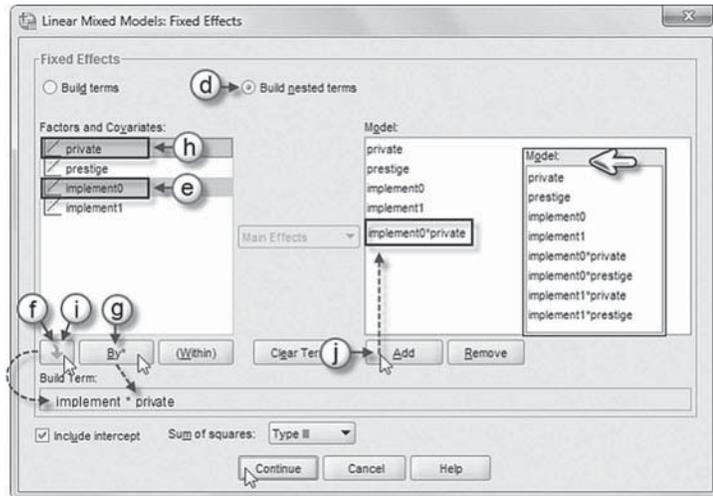
Four cross-level interactions (or nested terms) will be created and added to the model:

*implement0\*private*, *implement0\*prestige*, *implement1\*private*, and *implement1\*prestige*.

### Add First Interaction to Model 3.2:

*implement0\*private*

- d. Click to select *Build nested terms*.
- e. Click to select the variable *implement0* from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves *implement0* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button. This will insert the computation command symbol: *implement0\**.
- h. Click to select *private* from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move *private* into the *Build Term* box and complete the interaction term: *implement0\*private*.
- j. Click the ADD button to transfer the interaction into the *Model* box.



### Add Second Interaction to Model 3.2: *implement0\*prestige*

Repeat steps 4d to 4j using *implement0* and *prestige* for the interaction.

### Add Third Interaction to Model 3.2: *implement1\*private*

Repeat steps 4d to 4j using *implement1* and *private* for the interaction.

### Add Fourth Interaction to Model 3.2: *implement1\*prestige*

Repeat steps 4d to 4j using *implement1* and *prestige* for the interaction.

The completed model is shown in the insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

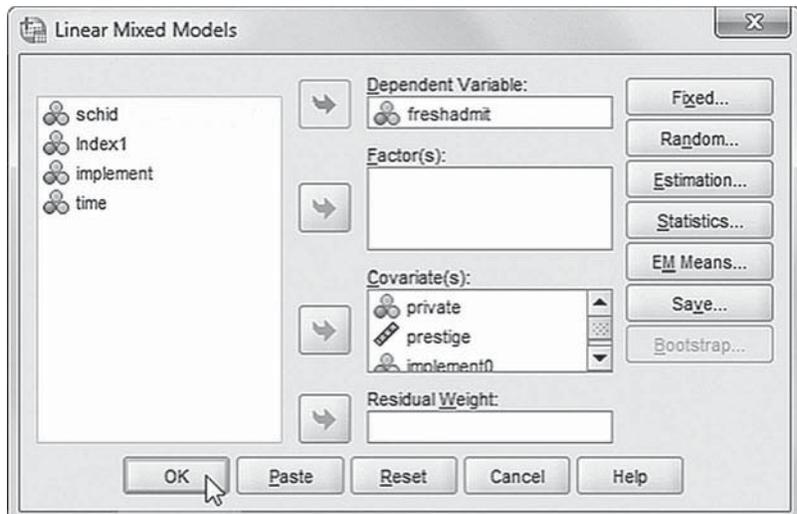


TABLE 6.24 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	22.147	0.485	171.064	45.660	.000	21.190	23.105
private	-0.426	1.138	171.064	-0.374	.709	-2.672	1.821
prestige	0.904	0.346	171.064	2.611	.010	0.221	1.588
implement0	-0.447	0.278	224.030	-1.607	.109	-0.995	0.101
implement1	-0.651	0.077	355.077	-8.434	.000	-0.802	-0.499
private * implement0	0.189	0.652	224.030	0.290	.772	-1.096	1.475
prestige * implement0	-0.203	0.199	224.030	-1.025	.307	-0.595	0.188
private * implement1	0.374	0.181	355.077	2.064	.040	0.018	0.730
prestige * implement1	-0.008	0.055	355.077	-0.144	.886	-0.116	0.100

<sup>a</sup> Dependent variable: freshadmit.

### Interpreting the Output From Model 3.2

We next provide the fixed-effects results in Table 6.24. The intercept model suggests that initially schools with higher prestige got more freshman athletes ( $\beta_{02} = 0.904, p = .01$ ). School type, however, did not influence freshman athletes admitted initially. Moreover, neither predictor affected the institutional growth trend before the policy was implemented ( $p > .05$ ). After implementation, however, private schools got increased numbers of students over each yearly interval ( $\beta_{21} = 0.374, p < .05$ ). In contrast, higher prestige schools did not appear to receive more freshmen after implementation ( $\beta_{22} = -0.008, p > .05$ ). We could, of course, also add other variables to the model, but our analysis seems to suggest the policy did result in fewer freshman student athletes being entered to Division 1A programs ( $\beta_{20} = -0.651, p < .001$ ), after controlling for other variables that might also affect the admission of freshman student athletes. The variance components are similar to previous models, so we do not provide them here.

## Summary

Longitudinal analysis represents a rapidly growing application of multilevel-modeling techniques. Because they provide stronger ways for dealing with causal relationships between variables than cross-sectional analyses, they should continue to draw the increased attention of researchers. In this chapter, we introduced a basic multilevel model considering change at several levels of a data hierarchy. The approach is very flexible for fitting a number of research purposes and designs (e.g., experimental, time series, and nonexperimental).

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# CHAPTER 7

## Multivariate Multilevel Models

In the last chapter, we introduced a three-level modeling framework for examining change in individuals and groups over time. Another variation on the basic three-level modeling framework is a multivariate multilevel model—that is, a model that has more than one dependent variable (defined at Level 1), with individuals at Level 2 and groups at Level 3. The multivariate multilevel model follows directly from the traditional single-level multivariate analysis of variance (MANOVA) model. One of the advantages of the multilevel formulation is that subjects with partial data on outcomes can be included in the analysis, which is a limitation of MANOVA (Hox, 2010). A second advantage is that the multivariate approach facilitates the development of more flexible models with multiple response variables, which can have different sets of explanatory variables (Wright, 1998). A third advantage is that the multivariate provides simultaneous estimation of the outcomes and adjustment for correlations between them. Moreover, it facilitates the use different covariance structures at multiple levels along with a choice in methods to estimate them (Wright, 1998).

There are a number of different ways to specify and set up multilevel models with multivariate outcomes. In this chapter, we provide three examples of two- and three-level models with multivariate outcomes. The first example is where several survey items are combined to define one or more latent constructs at the lowest level of the model. This type of latent variable formulation allows the incorporation of measurement error, as well as possible missing data on items, in the analysis of differences in the constructs within and between groups. The second is where there are two or more observed outcomes, for example, where we might examine students' reading, math, and language test scores simultaneously, rather than modeling each outcome separately. This formulation is useful in adjusting parameters for the expected correlation between students' performance on each test. The third example is where we are interested in examining individual development in two or more domains simultaneously, rather than examining each growth trajectory separately.

We hope that our basic introduction to these types of models will encourage readers to think about situations in their own research where they might examine several outcomes simultaneously. Multilevel models with multivariate outcomes can be constructed much like the univariate growth models presented in Chapters 5 and 6. We do caution, however, that multivariate formulations can be more complicated to specify and to interpret than their univariate counterparts. They may also require more computer memory space and, therefore, be more challenging to estimate. Readers can also consult Raudenbush and Bryk (2002) or Hox (2010) for other examples of multivariate multilevel models.

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### Multilevel Latent-Outcome Model

Our first example focuses on using a set of observed indicators (e.g., survey items) to measure an underlying construct (e.g., job satisfaction or motivation). We propose a model consisting of two latent constructs measured with three survey items each (with items measured on five-point

scales). Individuals were asked several questions about their *job satisfaction* (i.e., my work is challenging and varied; I feel my work is valued; I feel like I am a member of a team) and the evaluation of their *work performance* (i.e., my work is regularly assessed; I am provided with feedback regarding my progress; I am evaluated according to clearly known evaluation standards). We assume the two constructs are positively correlated at the individual and organizational levels. We will next build a three-level model to examine variability in employee perceptions within and between organizations. We note in passing that this same model could also be defined as a two-level structural equation model (i.e., individuals nested in organizations).

## The Data

The data consist of 650 employees in 105 organizations who were asked several questions about their work lives. At Level 1, there are six survey items nested within 650 individuals. The basic means of specifying multivariate models in MIXED is to define the multiple response (Y) variables vertically in the same manner that the growth models in Chapters 5 and 6 were defined. In this case, when the items are stacked vertically within individuals, we have 3,900 data lines. We use the *Repeated* dialog box to define the measurement model at Level 1. We can use an *index* variable to define the set of items measuring the constructs and a dichotomous indicator (*assessjob*) to define each construct (i.e., with *assessjob* = 0 referring to job satisfaction and *assessjob* = 1 referring to job performance). In our example, each construct is measured by three items, but it is easy to vary the number of items that would define each factor. At Level 2, we obtain background

**TABLE 7.1** Data Definition of *ch7worklifeorg.sav* (N = 650)

Variable	Level <sup>a</sup>	Description	Values	Measurement <sup>3</sup>
<i>id</i>	Individual	Employee identifier (650 employees).	Integer	Ordinal
<i>orgcode</i>	Organization	Organization identifier (150 organizations).	Integer	Ordinal
<i>Rid</i>	Individual	A within-group level identifier representing recoded employee identifiers ( <i>id</i> ) ranking with the organizational identifier ( <i>orgcode</i> ).	(1, 2, 3, . . . , 12)	Ordinal
<i>Index1</i>	Within Individual	Identifier variable resulting from indexing the <i>id</i> individual identifier (1 to 650) to create a new identifier to define each of the six items.	(1, 2, 3, 4, 5, 6)	Nominal
<i>work</i>	Within Individual	Dependent variable measuring work life (scale).	(1, 2, 3, 4, 5)	Nominal
<i>assessjob</i>	Within Individual	Dichotomous indicator variable measuring job assessment.	0 = Job Satisfaction 1 = Job Performance	Nominal
<i>female</i>	Individual	Demographic predictor variable representing gender.	0 = Male 1 = Female	Nominal
<i>stability</i>	Individual	Dichotomous variable representing years employed.	0 = < 5 Years 1 = ≥ 5 Years	Nominal
<i>gmorgprod</i>	Organization	Predictor interval variable (grand-mean centered) measuring organizational productivity.	-0.54 to 1.30	Scale
<i>gmresources</i>	Organization	Predictor interval variable grand-mean centered) measuring resource allocation.	-3.17 to 2.18	Scale

<sup>a</sup> Within Individual = repeated measures, Level 1; Individual = Level 2; Organization = Level 3.

<sup>3</sup> Measurement icon settings displayed in subsequent model screenshots may differ from Tables 7.1, 7.17, and 7.24, but will not affect the output

**TABLE 7.2** Descriptive Statistics

	N	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
W1varied	650	3.96	1.058	-0.892	0.096	0.128	0.191
W2value	650	4.21	0.847	-1.099	0.096	1.339	0.191
W3team	650	3.56	1.096	-0.524	0.096	-0.396	0.191
P1assess	650	3.94	1.028	-0.680	0.096	-0.401	0.191
P2progress	650	3.76	0.917	-0.422	0.096	-0.114	0.191
P3evstand	650	3.78	1.137	-0.722	0.096	-0.248	0.191
Valid N (Listwise)	650						

data on the 650 individuals in the study. We include two predictors—gender (coded male = 0, and female = 1) and stability (coded 0 = less than 5 years with company, and 1 = 5 years or more). At Level 3, we also include two organizational predictors (resource allocation and organizational productivity). A complete description of the data used in the example is provided in Table 7.1.

### Research Questions

The first research question concerns whether individuals' perceptions differ according to their backgrounds and experience working in their organizations. We might ask: Are individuals' perceptions of their job satisfaction and job performance differentiated by gender and their length of time working in their workplace? The second research question concerns whether the slope describing the proposed positive relationship between the length of time individuals have been employed in their workplace and their perceptions of the work life constructs specified varies across organizations. This research question focuses on whether the size of a within-group slope varies randomly across organizations; that is, do organizational variables moderate the effect of length of service within the organization on individuals' workplace perceptions?

### Defining the Constructs

We note that there are a number of potential problems to consider when measuring underlying constructs with observed ordinal indicators. One common problem is the situation where individuals are bunched at the top or the bottom of an ordinal scale. Although with ordinal data from five-point scales acceptable solutions can be generated where data do not depart too much from normality (Boomsma, 1987; Rigdon, 1998), we suggest that researchers check the properties of their data in each specific instance (Flora & Curran, 2004). Alternatively, we can also estimate such models using a multilevel ordinal formulation. For dichotomous items, multilevel logistic regression can be used (Hox, 2010). In Table 7.2, we provide a summary of the descriptive statistics on the individual items across the 650 individuals. We can see that the means are relatively high (ranging from 3.56 to 4.21) and, therefore, the items are negatively skewed. Skewness ranges from -1.10 to -0.422 and kurtosis ranges from -0.401 to 1.34. This suggests more responses in the higher categories (i.e., four and five). Although there may be some bias in calculating correlations based on the assumption that the data are interval, our preliminary investigations suggest we should be able to obtain a satisfactory solution.

The goal of the analysis is to examine individual and organizational predictors of employees' beliefs about their work life and evaluation of their productivity. Our first concern is whether there is evidence that the six items in the study do indeed define two separate latent dimensions. To build our case, we first used exploratory factor analysis (i.e., principal axis factoring

**TABLE 7.3 Factor Matrix**

	Factor	
	1	2
W1varied	-0.035	0.768
W2value	-0.003	0.617
W3team	0.241	0.474
P1assess	0.720	0.069
P2progress	0.836	-0.044
P3evstand	0.764	0.018

Extraction method: principal axis factoring.

Rotation method: oblimin with Kaiser normalization.

**TABLE 7.4 Factor Correlation Matrix**

Factor	1	2
1	1.000	0.823
2	0.823	1.000

Extraction method: principal axis factoring with oblimin rotation.

with oblimin rotation) to examine whether the six items define the two proposed constructs. In Table 7.3, we provide the pattern (or weighted loading) matrix. As the table indicates, the three performance assessment items load primarily on Factor 1 (with hypothesized loadings ranging from 0.720 to 0.836, while the three workplace items (W1–W3) load highly on Factor 2 (with hypothesized loadings ranging from 0.474 to 0.768).

We can also note that the correlation between the two factors for the 650 individuals (ignoring clustering) is 0.823 (see Table 7.4). Given this preliminary evidence, we can proceed with developing our model with the two correlated latent factors as outcomes in our proposed study of employees' work lives.

### Organizing the Data Set

In this type of latent variable multilevel formulation, the *Repeated* dialog box in IBM SPSS MIXED can be used to define a measurement model at Level 1 that consists of multiple measures of an outcome  $Y$  nested within individuals, similar to the multiple measurement occasions of  $Y$  that are used to define growth trajectories (Leyland, 2004; Raudenbush & Bryk, 2002). Closer inspection of the data in Figure 7.1 suggests there are six survey items per individual and that individuals' identity (ID) and organizational code must be repeated in the data set for each individual. The grouping variables (*id* and *orgcode*) are used to identify the other predictors as belonging to a particular level of the data hierarchy. The individual responses to the survey items comprising the work life responses (*work*) are nested within individual identification numbers, and recoded employee IDs (*Rid*) are nested within organizations (*orgcode*). The variable *Index1* is used to identify the items used in defining the latent constructs. Similar to our discussion of growth models, this vertical format will accommodate individuals with partial data on the items comprising the constructs as long as at least one measure is included.

After creating the *Index1* variable, we can also use it to define a dummy-coded job assessment (*assessjob*) variable (with 0 = job satisfaction, and 1 = job performance). This variable is used to

	id	orgcode	Rid	Index1	work	assessjob
1	1	1	1	2	4	0
2	1	1	1	3	4	0
3	1	1	1	4	4	1
4	1	1	1	5	4	1
5	1	1	1	6	4	1
6	1	1	1	1	5	0
7	2	1	2	1	2	0
8	2	1	2	3	3	0
9	2	1	2	4	3	1
10	2	1	2	2	4	0
11	2	1	2	5	4	1
12	2	1	2	6	4	1
13	3	1	3	1	2	0
14	3	1	3	3	3	0
15	3	1	3	5	3	1
16	3	1	3	6	3	1
17	3	1	3	2	4	0
18	3	1	3	4	4	1
19	4	1	4	3	2	0
20	4	1	4	1	4	0

FIGURE 7.1 Data matrix for latent variable analysis.

differentiate the two constructs. We recode *Index1* into *assessjob* by specifying 1–3 on *Index1* as 0 to define job satisfaction and 4–6 on *Index1* as 1 to define job performance. Other predictors (i.e., resources and organizational productivity), which are grand-mean centered, are repeated on each of the six lines comprising each student's data since they will be defined at levels above Level 1.

### Specifying the Model

Following Raudenbush and Bryk's (2002) general notation, a general multilevel, multivariate model can be formulated to specify one or more latent constructs. As we have noted previously, we use the *Repeated* dialog box to represent variation among the items defining the constructs within each individual at Level 1. We also note that we must designate individuals as nested in organizations (*orgcode\* Rid*) on the REPEATED command line (if we refer to the syntax statements). We again use the recoded individual identities (*Rid*) to reduce the required time it takes to estimate the model. In this case, we use Level 1 to define the measurement part of the model—that is, to link the observed items to their underlying constructs. The items measuring *Y* are defined as a vector  $(y_{1jk}, y_{2jk}, \dots, y_{ijk})'$  for individual *j* in school *k* measured on item *i* and, as we noted, are stacked vertically in the data set. The general Level 1 model may then be written as

$$Y_{ijk} = \sum_{p=1}^P \pi_{pjk} a_{pjk} + \varepsilon_{ijk}, \quad (7.1)$$

where  $Y_{ijk}$  is the observed score on item  $i$  for individual  $j$  in organization  $k$ ,  $\pi_{pjk}$  is the latent true score for individual  $j$  in organization  $k$  on construct  $p$ , and  $\varepsilon_{ijk}$  is an error associated with individual  $j$ 's response to item  $i$  (assumed to be normally distributed with a mean of 0 and a variance of  $\sigma^2$ ). The key to specifying a multivariate analysis in MIXED is to create an indicator variable ( $a_{pijk}$ ), which can link the multiple observed items to each construct ( $p$ ). The constructs are specified through using a set of dummy variables specified at Level 1, where  $a_{pijk}$  takes on the value of 1 if the indicator  $i$  measures construct  $p$  and 0 otherwise for constructs  $p = 1, \dots, P$ . In this case, the dummy-coded variable we use to define the constructs is *assessjob*.

As Equation 7.1 indicates, in order to incorporate the multiple measures in defining each construct, as well as specifying more than one construct in the model, we must exclude the usual intercept term at the lowest level. In general, the function of the Level 1 model for a multivariate formulation is to aggregate the separate indicators into one or more constructs. In accomplishing this, we can specify a relatively simplified covariance structure (e.g., scaled identity) or more complete covariance structure (diagonal or unstructured) to describe relationships among the six items within individuals (Raudenbush & Bryk, 2002). We caution that when defining constructs in this vertical manner, it is best if the items or subtests are measured on the same scale (e.g., Likert-type scales or scaled scores) and have similar variances (Hox, 2010). Variables can be rescaled as needed prior to the analyses, however, to achieve more similar error variances. Analysts should also keep in mind that ordinal data may not meet the assumptions of continuous normal data.

The Level 2 model describes the distribution of true scores  $\pi_p$  across individuals within organizations, including any predictors that are proposed to explain this variation. The general model can be written as

$$\pi_{pjk} = \beta_{p0k} + r_{pjk}, \quad (7.2)$$

where  $\beta_{p0k}$  is the true score mean of construct  $p$  in organization  $k$ , and  $r_{pjk}$  is the Level 2 random coefficient for individual  $j$  in organization  $k$  on construct  $p$ .

At Level 3, we represent the organizational-level model. The intercept model with no predictors would be

$$\beta_{p0k} = \gamma_{p00} + u_{p0k}, \quad (7.3)$$

where  $\gamma_{p00}$  is the Level 3 intercept for construct  $p$  and  $u_{p0k}$  is the Level 3 random effect capturing variation in organizational means for construct  $p$ . The dimensionality of the covariance matrices of random effects at Levels 2 and 3 depends on the number of random effects at each level. We note that when predictors are entered into the model at Levels 2 or 3 in multivariate formulations with no intercept at Level 1, they are added as interactions with the variable that is used to define the constructs. In this example, the variable is *assessjob*. This will provide a set of estimates for each construct. If we wish to impose equality constraints across the two constructs, we simply enter the predictors into the model as fixed effects. This latter formulation tests the hypothesis that the effects of the predictors are the same in explaining variation in each construct in the model (see Hox, 2010, for further discussion).

### Model 1.1: The Null or "No-Predictors" Model

For the null model, we can examine the variability in the constructs that exists at each level. Because there are two constructs within individuals (Level 1), we can simply define the performance variable with its six component measures ( $i = 1, 2, 3, 4, 5, 6$ ), as specifying the dummy-coded

job assessment variable (*assessjob*). By declaring no intercept at Level 1 (NOINT), we can obtain estimates for each construct separately. Following Equation 7.1, the first set of three items comprises the job satisfaction construct (which we have coded 0), and the second set of three items comprises the evaluation of performance construct (which we have coded 1):

$$Y_{ijk} = \pi_{pjk} \textit{assessjob}_{jk} + \varepsilon_{ijk}, \quad (7.4)$$

where *assessjob* takes on the value of 1 if item *i* measures construct *p* (in this case, evaluation of performance) and 0 when it measures the other construct (job satisfaction);  $\pi_{pjk}$  is the latent true score for person *j* in school *k* on construct *p*, and  $\varepsilon_{ijk}$  is an error term assumed to be normally distributed with a mean of 0 and variance  $\sigma_e^2$  (see Raudenbush & Bryk, 2002, for further discussion).

At Level 1, we will first assume a simple covariance structure by specifying an identity covariance matrix at Level 1. This implies one constant variance for all items comprising the two constructs. If this does not appear to be a reasonable assumption, another possibility might be a diagonal covariance matrix, which would provide a separate variance for each item. The Level 1 portion of the model, however, is generally not the focus of the analysis.

By not specifying an intercept at Level 1 (NOINT) and specifying the *assessjob* dummy-coded variable as randomly varying at Level 2, we can obtain separate intercepts for each construct. In our example, since we have two latent constructs, the Level 2 model will simply represent the grand mean for each construct. We will label the constructs  $\pi_{1jk}$  (job satisfaction) and  $\pi_{2jk}$  (evaluation of performance). The Level 2 (between individuals) model can then be defined as

$$\begin{aligned} \pi_{1jk} &= \beta_{10k} + r_{1jk}, \\ \pi_{2jk} &= \beta_{20k} + r_{2jk}, \end{aligned} \quad (7.5)$$

where  $\beta_{10k}$  and  $\beta_{20k}$  are intercepts and  $r_{1jk}$  and  $r_{2jk}$  are person-specific, random effects. The random effects are assumed to be multivariate normal with means of 0 and contained in a covariance matrix whose dimensions depend on the number of random effects specified (Raudenbush & Bryk, 2002). We will first assume a diagonal covariance at Level 2.

Similarly, the intercepts at Level 3 are the grand mean of the items defining each construct at the organizational level. Between organizations, the model with random intercepts is defined as

$$\begin{aligned} \beta_{10k} &= \gamma_{100} + u_{10k}, \\ \beta_{20k} &= \gamma_{200} + u_{20k}. \end{aligned} \quad (7.6)$$

We will also assume a diagonal matrix of random effects at Level 3. Substituting the Level 2 and Level 3 equations into Equation 7.4, we arrive at the single-equation model, which includes the two organizational-level intercepts and the variance components for each level of the model:

$$Y_{ijk} = \gamma_{100} + \gamma_{200} + u_{10k} + u_{20k} + r_{1jk} + r_{2jk} + \varepsilon_{ijk}. \quad (7.7)$$

This suggests, in its basic format, seven parameters to estimate (i.e., two fixed Level 3 intercepts, four random effects, and one residual variance at Level 1). We can confirm this familiar specification by examining the model dimensions for the null model in Table 7.5. The fixed-effect intercepts are defined along with the random effects at Level 3 ( $u_{0k}$ ) and Level 2 ( $r_{jk}$ ), and the Level 1 residual ( $\varepsilon_{ijk}$ ).

**TABLE 7.5 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	assessjob	2		2		
Random Effects	assessjob	2	Diagonal	2	orgcode	
	assessjob	2	Diagonal	2	Rid * orgcode	
Repeated Effects	Index1	6	Identity	1	orgcode * Rid	650
Total		12		7		

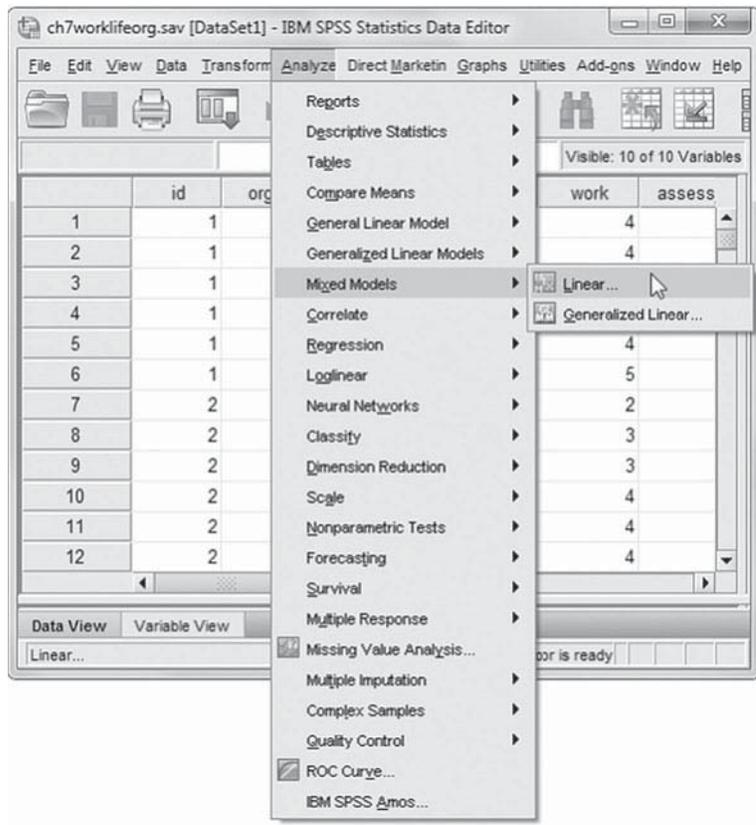
<sup>a</sup> Dependent variable: work.

**Defining the Model 1.1 (Null) with IBM SPSS Menu Commands**

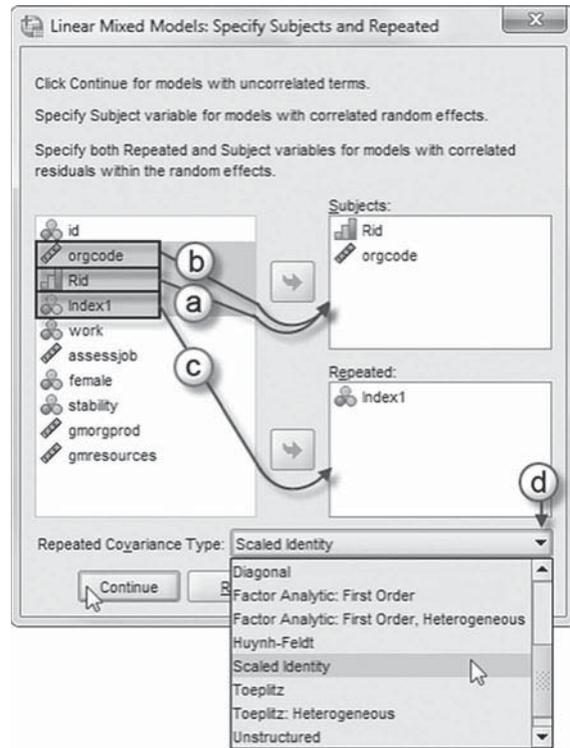
Launch the IBM SPSS program application, and select the *ch7-worklifeorg.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.
- A subject is an observational unit that may be independent of other subjects. For this model, we will designate two subject identifiers for the model (*Rid* and *orgcode*). To facilitate reading the output tables, we will enter the variables in sequence. Click to select *Rid* from the *Variables* column, and then click the right-arrow button to move the variable into the *Subjects* box.
  - Next, click to select *orgcode* from the *Variables* column, and then click the right-arrow button to move the variable into the *Subjects* box.
  - The *Repeated* box allows specifying variables that identify repeated observations. For this model, *Index1* identifies the six items that measure the constructs. Click to select *Index1*, and then click the right-arrow button to move the variable into the *Repeated* box.



The combination of values for *Rid*, *orgcode*, and *Index1* defines a particular employee from a particular organization with data from one to six items (i.e., individuals with partial data will be included in the analysis).

- The *Repeated Covariance Type* specifies a model's covariance structure. For this model, at Level 1 we will use scaled identity. Click the pull-down menu to select the autoregressive covariance matrix, *Scaled Identity*, as the *Repeated Covariance Type*.

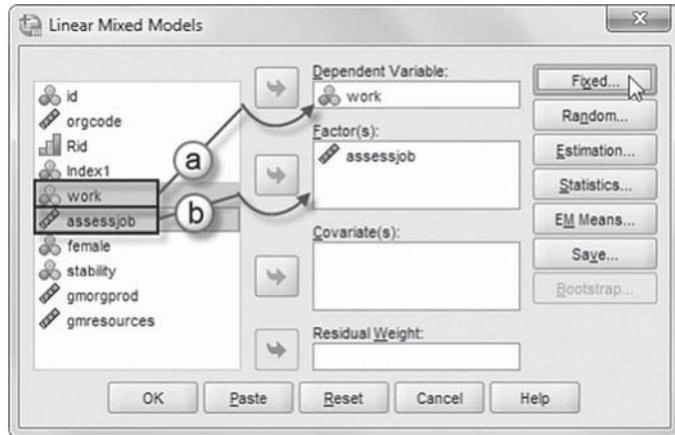
The *Scaled Identity* covariance structure has constant variance and assumes that no covariance exists between any elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, and covariates, as well as access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

a. For this model, we will use *work* as the dependent variable. Click to select the *work* variable from the left column listing. Then click the right-arrow button to transfer *work* into the *Dependent Variable* box.

b. We will create a no-intercept null model that defines the observed measures of *assessjob* (*assessjob0* and *assessjob1*). Click to select the *assessjob* variable from the left column listing. Then click the right-arrow button to transfer *assessjob* into the *Factor(s)* box.

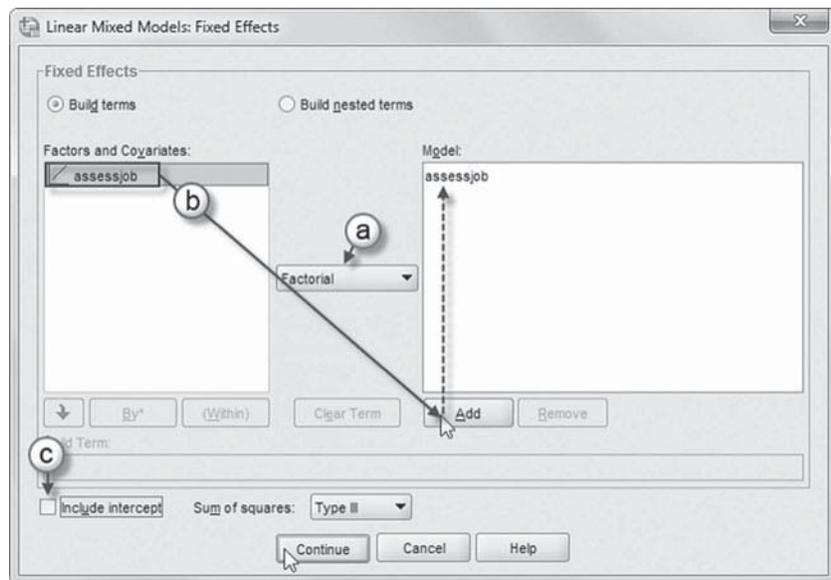


Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, we will retain the default *Factorial* setting, which creates all possible interactions and main effects of the specified variable (IBM Corporation, 2012).

b. Now click to select *assessjob* from the *Factors and Covariates* box. Then click the ADD button to move the variable into the *Model* box.

c. To create the no-intercept model, uncheck the *Include intercept* option.



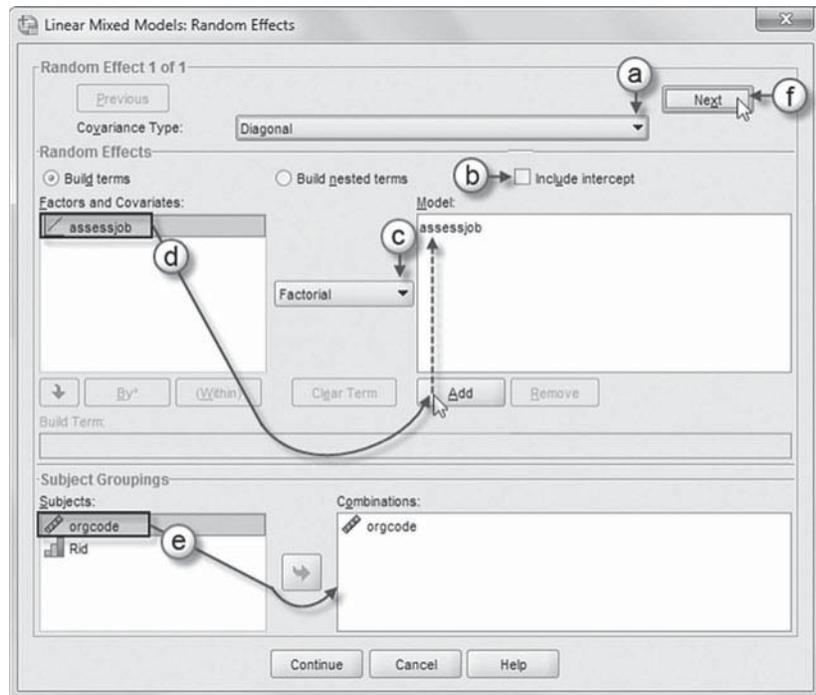
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

We will now add random effects to this model.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen, which is the default when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.

- a. Begin by specifying the covariance structure from the default variance components (VC) to diagonal. To do so, click the pull-down menu and select *Diagonal* (DIAG).

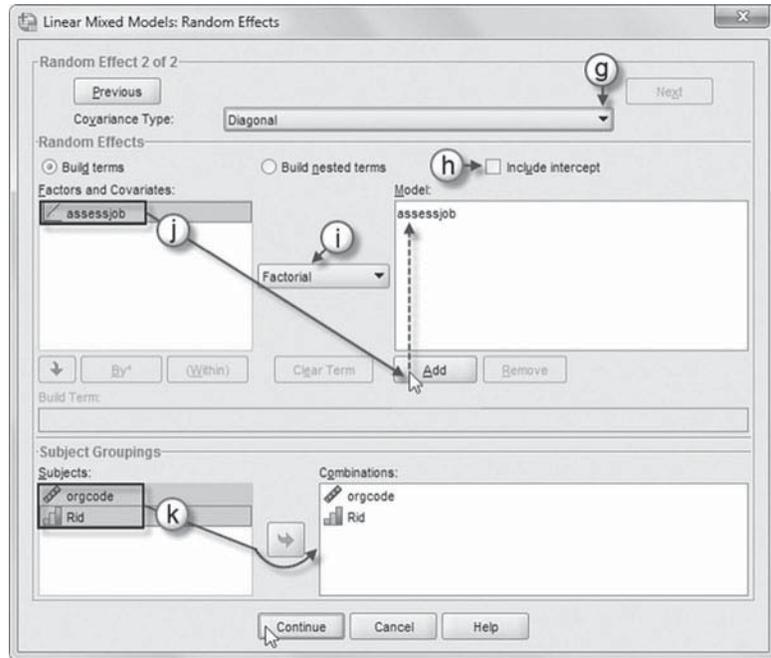


The *Diagonal* covariance type has heterogeneous variances and zero correlation between the elements (IBM Corporation, 2012).

- b. We want the intercept to be excluded from the model, so the model will define the measures of *assessjob*. Therefore, we will retain the current default setting.
- c. We will retain the default *Factorial* setting as this will be a null model (no-predictor, no-intercept), so the setting will have no effect.
- d. Click to select *assessjob*, and then click the ADD button to move the variable into the *Model* box.
- e. The *Subject Groupings* box displays the *orgcode* and *Rid* variables that were specified as subject variables in the *Specify Subjects and Repeated* dialog box show in step 2a. We will specify *orgcode* as the subject for the random-effects Level 1 part of this model. Click to select *orgcode*, and then click the right-arrow button to move the variable into the *Combinations* box.
- f. At the top-right section of the window, click the NEXT button to access the *Random Effect 2 of 2* screen.

The *Random Effect 2 of 2* screen display is similar to the first screen and requires the following changes.

- g. Change the covariance type by clicking on the pull-down menu and selecting *Diagonal*.
- h. We want the intercept to be excluded from the model so that separate constructs comprising *assessjob* will be specified. Therefore, we will retain the current default setting.
- i. Retain the default *Factorial* setting.
- j. Click to select *assessjob*, and then click the **ADD** button to move the variable into the *Model* box.
- k. We will specify *orgcode* and *Rid* as the subjects for the random-effects Level 2 part of this model. Click to select *orgcode* and *Rid*, and then click the right-arrow button to move the variables into the *Combinations* box.

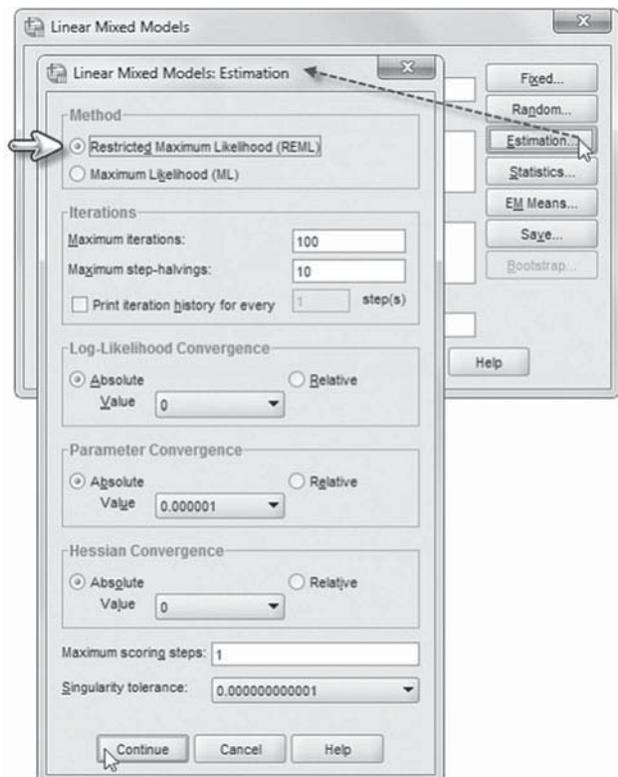


Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

6. Click the **ESTIMATION** button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: maximum likelihood (ML) or restricted maximum likelihood (REML).

In this chapter, we will primarily use the default setting of REML to estimate the models. Where nested models are compared in terms of both variance components and regression components, however, ML estimation should be used (Hox, 2010).

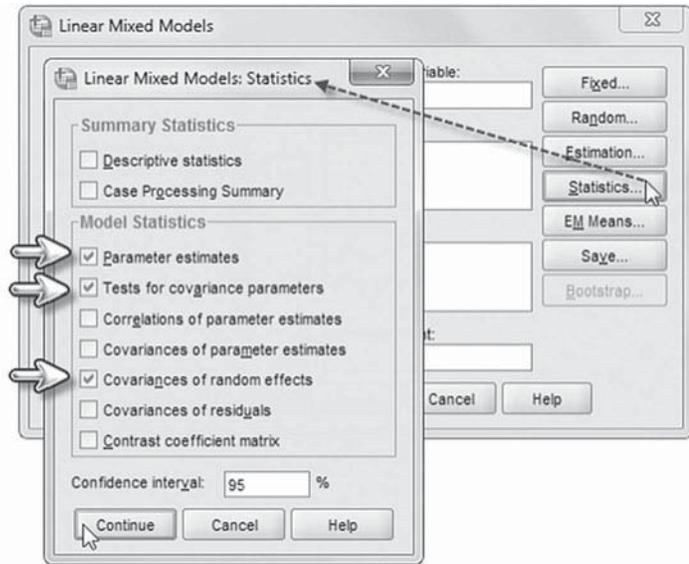
Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.



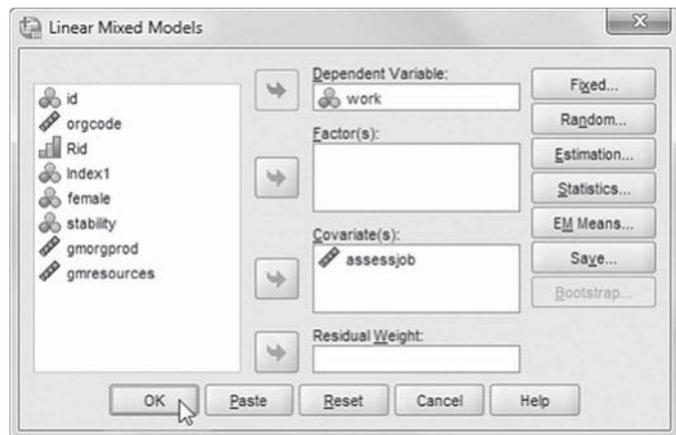
7. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



8. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### **Interpreting the Output From Model 1.1 (Null)**

In this case, we first used a scaled identity (ID) covariance matrix at Level 1 and a diagonal covariance matrix at Levels 2 and 3 since there were two random effects at those levels (i.e., the work life intercept and the evaluation of performance intercept). Table 7.6 suggests that the grand mean for work life was 3.926 (i.e., the mean of the first set of items across the organizations) and 3.845, which represents the grand mean of the second set of items across organizations.

**TABLE 7.6** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[accessjob = 0]	3.926	0.045	105.724	87.347	.000	3.837	4.015
[accessjob = 1]	3.845	0.053	104.133	72.509	.000	3.739	3.950

<sup>a</sup> Dependent variable: work.

**TABLE 7.7** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Variance	0.544	0.015	36.056	.000	0.515	0.574
accessjob [subject = orgcode]	Var: [accessjob = 0]	0.123	0.029	4.238	.000	0.077	0.195
	Var: [accessjob = 1]	0.192	0.041	4.707	.000	0.127	0.291
accessjob [subject = Rid * orgcode]	Var: [accessjob = 0]	0.336	0.032	10.635	.000	0.279	0.404
	Var: [accessjob = 1]	0.413	0.036	11.385	.000	0.347	0.490

<sup>a</sup> Dependent variable: work.

**TABLE 7.8** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Variance	0.544	0.015	36.056	.000	0.515	0.574
accessjob [subject = orgcode]	Var(1)	0.124	0.029	4.249	.000	0.078	0.196
	Var(2)	0.192	0.041	4.707	.000	0.127	0.291
	Corr(2, 1)	0.773	0.070	10.995	.000	0.595	0.879
accessjob [subject = Rid * orgcode]	Var(1)	0.335	0.032	10.637	.000	0.279	0.403
	Var(2)	0.413	0.036	11.385	.000	0.347	0.490
	Corr(2, 1)	0.953	0.034	27.766	.000	0.812	0.989

<sup>a</sup> Dependent variable: work.

The variance components associated with each level are summarized in Table 7.7. The table suggests a single variance at Level 1 (0.544), separate intercept variances for the job satisfaction and evaluation of performance constructs at Level 3 (0.123 and 0.192, respectively), and separate variances for the same two constructs at Level 2 (0.336 and 0.413, respectively).

We might next examine the correlation between each construct at the individual employee level (Level 2) and at the organizational level (Level 3). We can specify the relevant covariance matrices of random effects at each level as unstructured (UN) and designate (R) to obtain a correlation (UNR). When we do this, we will now have nine parameters to estimate since we are adding a correlation each at Level 2 and Level 3. In Table 7.8, we provide the relevant correlations (model syntax is provided in Appendix A). The table suggests that the items are more strongly correlated at the individual level ( $r = 0.953$ ) than at the organizational level ( $r = 0.773$ ).

### Conducting a Likelihood Ratio Test

We can formulate a likelihood ratio test to evaluate the two added correlation parameters. Any model where one or more parameters are fixed to zero can be referred to as a *restricted model* ( $L_0$ ), which can then be compared to an alternative model ( $L_1$ ) where the parameter or parameters are estimated. The change in model likelihoods can then be compared as long as the restricted model is nested in the alternative model (i.e., can be obtained from the unrestricted model by setting parameters to zero). The second (alternative) model with more parameters estimated will always fit at least as well (have a greater log likelihood) as the first model with restricted parameters. The ratio of the two likelihoods ( $L_0/L_1$ ) is referred to as the likelihood ratio. Where there is no difference between models (i.e., the log likelihoods are the same), the likelihood ratio will be 1.0, and the test statistic will be 0. A larger, more positive test result provides evidence against the null hypothesis that there is no difference between models. The likelihood ratio statistic (referred to as  $LR$ ,  $G^2$ , or  $\Lambda$ ) can then be formulated as follows:

$$G^2 = -2 \log \left( \frac{L_0}{L_1} \right) = -2[\log(L_0) - \log(L_1)] = [-2\log(L_0)] - [-2\log(L_1)] \quad (7.8)$$

The difference between two models has a chi-square distribution with degrees of freedom ( $df$ ) of the test equal to the difference in the number of parameters being tested.

We summarize the tests for these two models in Table 7.9. We estimated both models using REML estimation since we were only examining differences between variance components. If regression parameters are also included, we remind readers that ML should be used to conduct the likelihood ratio test between the two competing models (Hox, 2010). The original  $-2LL$  ( $-2 \times \log$  likelihood) for 7 estimated parameters was 10,358.406. The  $-2LL$  with 9 estimated parameters was 9,995.447. The relevant degrees of freedom, then, is 2 for this test (i.e., from 7 to 9 parameters estimated). The required chi-square coefficient for 2 degrees of freedom at  $p = .05$  is 5.99. In this case, the likelihood ratio test easily exceeded 5.99. This suggests that the alternative model with 9 parameters fit the data better than the restricted model with only 7 parameters estimated.

We can also test the assumption that there is a single variance at Level 1 to describe the items comprising the latent constructs. We might also examine whether a diagonal covariance matrix fits better at Level 1. In changing from a scaled identity covariance matrix to a diagonal covariance matrix, we will add 5 estimated parameters (i.e., resulting in 14 parameters to estimate). To conduct a likelihood ratio test, we can use Model 2 as our nested model ( $-2LL = 9,995.447$ , for 9 estimated parameters). The new model (Model 3) has 14 estimated parameters and a  $-2LL$  of 9,871.989. The relevant degrees of freedom is 5 (which at  $p = .05$  requires a coefficient of at least 11.07). We can see the resulting likelihood ratio coefficient describing the difference in deviance between Model 2 and Model 3 easily exceeds the required coefficient. We can therefore accept that a diagonal covariance structure at Level 1 fits the data better than a single variance estimate.

**TABLE 7.9 Likelihood Ratio Tests (REML Estimation)**

Variables	-2LL	Parameters	$G^2$	$df$	$p$
Models					
Model 1 (No Correlations)	10,358.406	7	NA	NA	NA
Model 2 (Correlations)	9,995.447	9	362.959	2	< .001
Model 3 (Diagonal Covariance Matrix)	9,871.989	14	123.458	5	< .001

TABLE 7.10 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [Index1 = 1]	0.587	0.041	14.383	.000	0.512	0.672
	Var: [Index1 = 2]	0.534	0.039	13.829	.000	0.464	0.616
	Var: [Index1 = 3]	0.863	0.056	15.409	.000	0.760	0.980
	Var: [Index1 = 4]	0.420	0.030	13.892	.000	0.365	0.484
	Var: [Index1 = 5]	0.298	0.024	12.243	.000	0.254	0.350
	Var: [Index1 = 6]	0.565	0.037	15.067	.000	0.496	0.643
assessjob [subject = orgcode]	Var(1)	0.123	0.029	4.288	.000	0.078	0.194
	Var(2)	0.181	0.039	4.653	.000	0.119	0.276
	Corr(2, 1)	0.758	0.074	10.249	.000	0.572	0.869
assessjob [subject = Rid * orgcode]	Var(1)	0.285	0.031	9.118	.000	0.230	0.354
	Var(2)	0.442	0.035	12.575	.000	0.379	0.517
	Corr(2, 1)	0.943	0.038	24.989	.000	0.799	0.985

<sup>a</sup> Dependent variable: work.

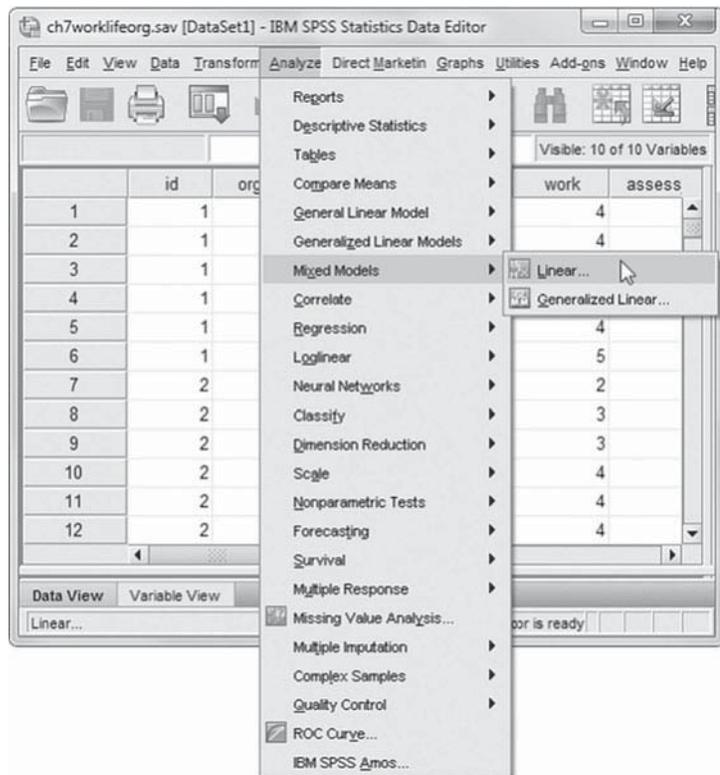
We provide the covariance parameter estimates for this final null model (Model 1.2) in Table 7.10. As noted in Table 7.10, this current model now has 14 estimated parameters. These include the two fixed-effect intercepts and the 12 variance components summarized in the table. We can see, for example, that there is some inconsistency in the items measuring the constructs at Level 1.

### Defining Model 1.2 (Final Null Model) with IBM SPSS Menu Commands

Settings will default to those used in Model 1.1.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

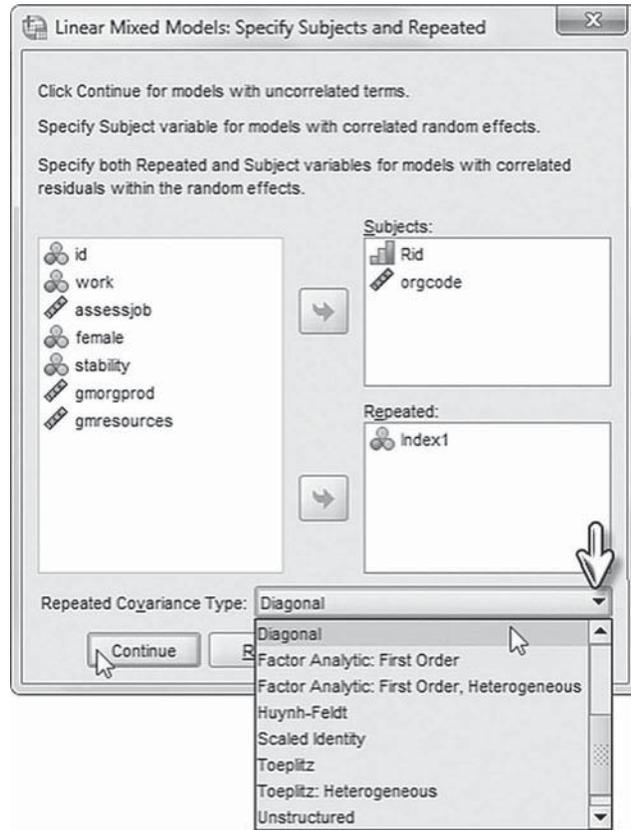


2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

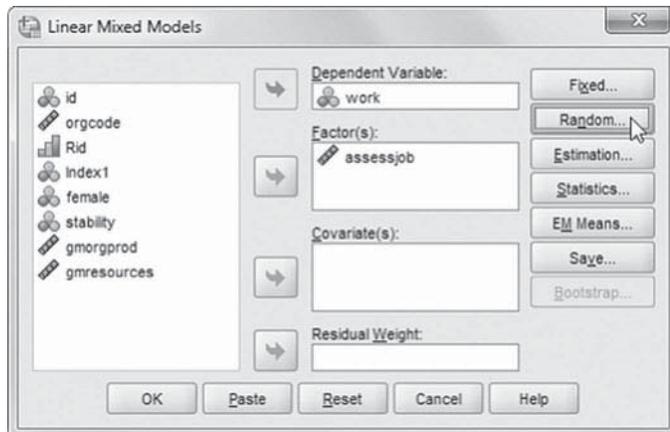
We will change the Level 1 covariance type by clicking the pull-down menu and selecting *Diagonal*.

The *Diagonal* covariance type has heterogeneous variances and zero correlation between the elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. We will change the random-effects covariance type, so from the *Linear Mixed Models* main screen, click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

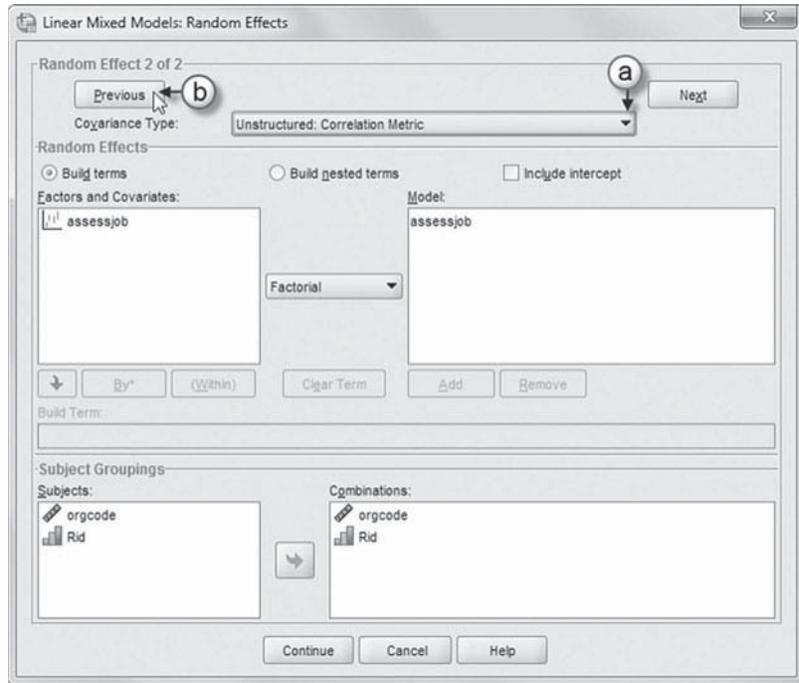


4. The *Random Effect 2 of 2* screen is displayed first as it was the last dialog box used in the prior model.

a. Change the covariance type by clicking the pull-down menu and selecting *Unstructured: Correlation Metric*.

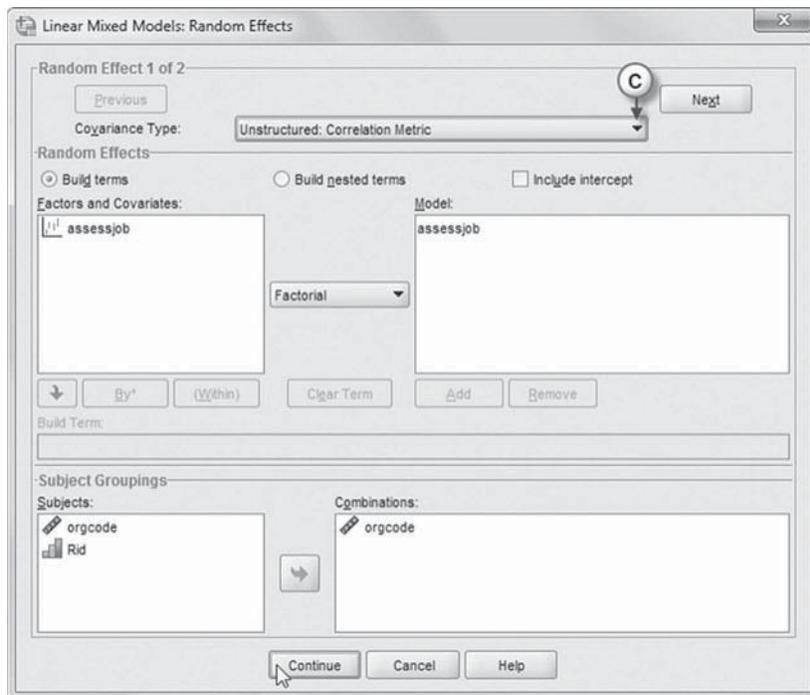
The *Unstructured: Correlation Metric* covariance structure has both heterogeneous variances and correlations (IBM Corporation, 2012).

b. Click the PREVIOUS button to access the *Random Effect 1 of 2* screen.

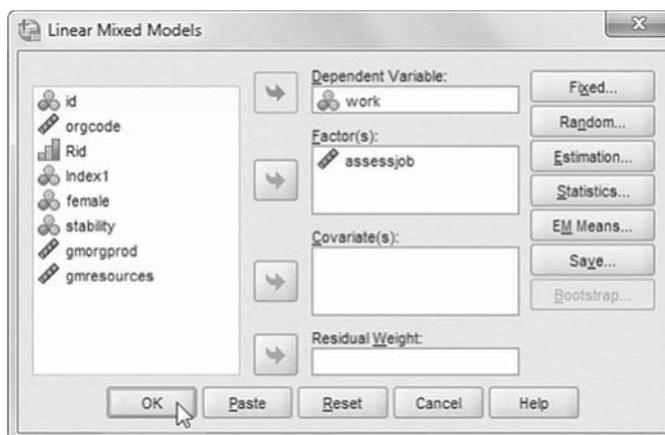


c. From the *Random Effect 1 of 2* screen, change the covariance type by clicking the pull-down menu and selecting *Unstructured: Correlation Metric*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model (Table 7.10).



### Model 1.3: Adding Level 2 Predictors

For the next model, we will add the individual background variables. At Level 2, we propose that employee gender (*female*) may result in different perceptions about work life and evaluation of performance. We will also propose that employees who have been in the organization longer (*stability*) will have more favorable perceptions about work life issues and performance evaluation. We can define the model to estimate perceptions for individual  $j$  in organization  $k$  on the two constructs as follows:

$$\begin{aligned}\pi_{1jk} &= \beta_{10k} + \beta_{11k}female_{jk} + \beta_{12k}stability_{jk} + r_{1jk}, \\ \pi_{2jk} &= \beta_{20k} + \beta_{21k}female_{jk} + \beta_{22k}stability_{jk} + r_{2jk}.\end{aligned}\quad (7.9)$$

Equation 7.9 indicates that individuals' perceptions vary across individuals. The model remains the same at the organizational level (Level 3) as in Equation 7.6, with the intercepts varying at that level also. We will assume the slopes for *female* and *stability* are fixed at Level 3.

For the single-equation (or combined) model, when we add the predictors to the model we multiply them by the dummy indicators:

$$\begin{aligned}Y_{ijk} &= \gamma_{100} + \gamma_{200} + \gamma_{110}assessjob_{jk} * female_{jk} + \gamma_{210}assessjob_{jk} * female_{jk} + \\ &\gamma_{120}assessjob_{jk} * stability_{jk} + \gamma_{220}assessjob_{jk} * stability_{jk} + u_{10k} + u_{20k} + r_{1jk} + r_{2jk} + \varepsilon_{ijk}.\end{aligned}\quad (7.10)$$

This makes a total of 18 parameters to estimate. These include six fixed effects, six random effects combined at Levels 2 and 3 (including covariances), and six Level 1 residual variances. This can be confirmed in Table 7.11.

As we noted previously, multiplying the set of dummy outcome variables (*assessjob*) by each Level 2 (or Level 3) predictor results in a separate regression coefficient for each response variable. If we wish to impose an equality constraint across the response variables (i.e., assume the effect of the predictor is the same on each), we would add the predictor directly to the model instead of multiplying it by the dummy indicators. In our specific case, this would amount to one parameter describing the effect of *gender* and one for the effect of *stability*. One can then test the fit of this “nested” model specification against the model as specified in Equation 7.10.

TABLE 7.11 Model Dimension<sup>a</sup>

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	assessjob	2		2		
	assessjob * stability	2		2		
	assessjob * female	2		2		
Random Effects	assessjob	2	Unstructured Correlations	3	orgcode	
	assessjob	2	Unstructured Correlations	3	Rid * orgcode	
Repeated Effects	Index1	6	Diagonal	6	orgcode * Rid	650
Total		16		18		

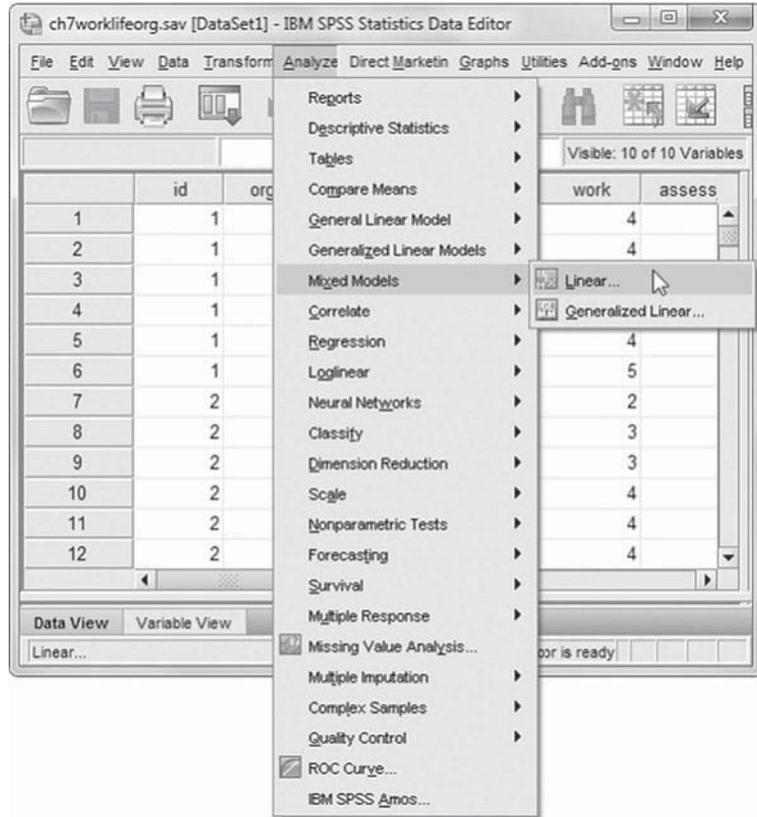
<sup>a</sup> Dependent variable: work.

**Defining Model 1.3 with IBM SPSS Menu Commands**

Settings will default to those used in Model 1.2.

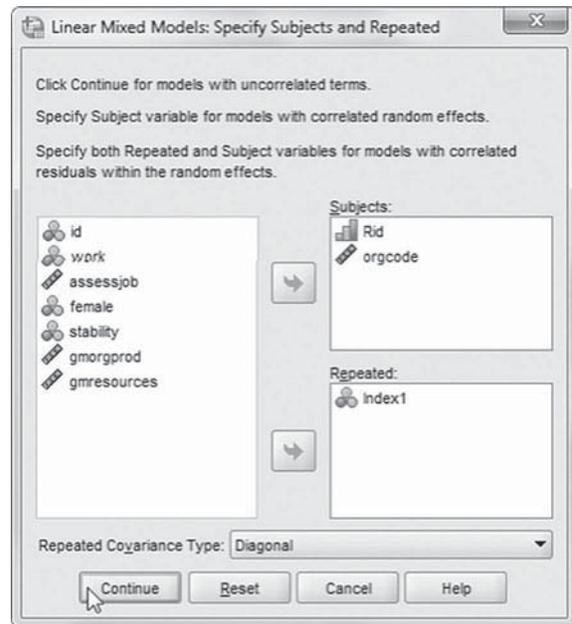
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

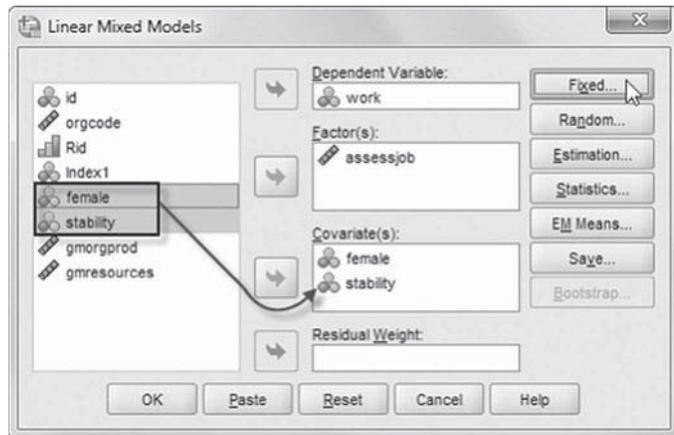
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. We will designate two variables to be used in the model. Locate and click *female* and *stability* from the left column listing, and then click the right-arrow button to move the variable into the *Covariate(s)* box.

We may now proceed to define fixed effects for the variable.

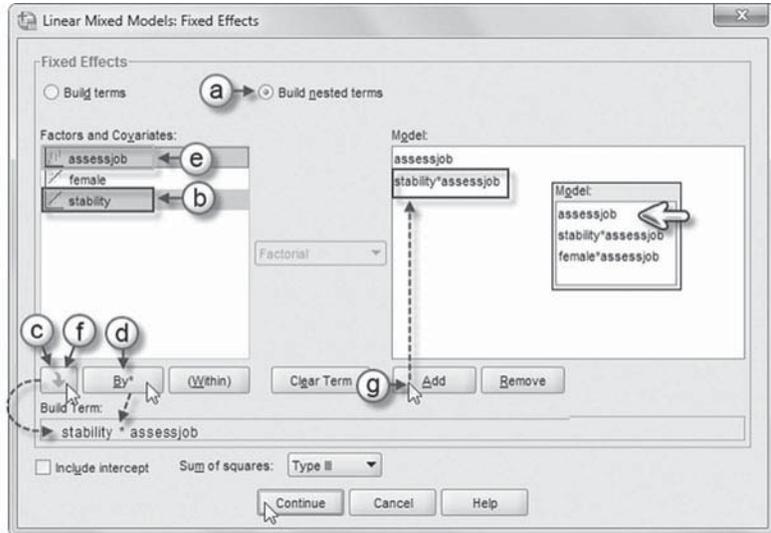
Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



Two cross-level interactions (or nested terms) will be created and added to the model: *stability\*assessjob* and *female\*assessjob*. These interactions will tell us if (a) employees who have been with the organization longer (*stability*) have more favorable perceptions about work life issues (*assessjob*) and (b) employee gender (*female*) may result in different perceptions about work life (*assessjob*).

Add First Interaction to Model 1.3: *stability\*assessjob*

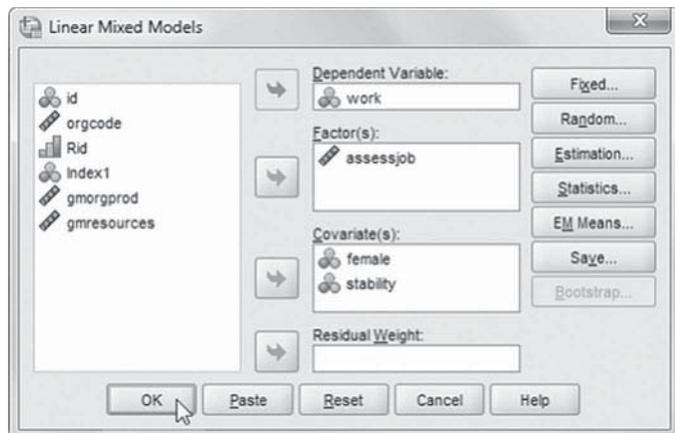
- 4a. Click to select *Build nested terms*.
- b. Now click to select the variable *stability* from the *Factors and Covariates* box.
- c. Click the arrow button below the *Factors and Covariates* box. This moves *stability* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- d. Next, click the BY\* button, which will insert the computation command symbol: *stability\**.
- e. Click to select *assessjob* from the *Factors and Covariates* box.
- f. Click the arrow button below the *Factors and Covariates* box to move *assessjob* into the *Build Term* box and complete the interaction term: *stability\*assessjob*.
- g. Click the ADD button to transfer the interaction into the *Model* box.

Add Second Interaction to Model 1.3: *female\*assessjob*

Repeat steps 4a to 4g using *female* and *assessjob* to create the interaction.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 1.3

The fixed-effect estimates for Model 1.3 are summarized in Table 7.12. After adjusting for the within-organization background controls, the intercepts of the two constructs are 3.939 for job satisfaction ( $assessjob = 0$ ) and 3.762 for evaluation of performance ( $assessjob = 1$ ). The table indicates that gender does not significantly affect either job satisfaction ( $\gamma = -0.031, p > .05$ ) or performance ( $\gamma = 0.005, p > .05$ ) perceptions. Employee stability is positively related to job satisfaction perceptions ( $\gamma = 0.125, p = .05$ ) and perceptions about the assessment of their performance ( $\gamma = 0.171, p < .05$ ).

The variance component output (Table 7.13) suggests that after addition of the set of individual predictors, there is still variance in the constructs to be explained at both the individual and organizational levels.

**TABLE 7.12** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[assessjob = 0]	3.939	0.059	253.776	66.480	.000	3.822	4.055
[assessjob = 1]	3.762	0.067	243.767	56.240	.000	3.631	3.894
[assessjob = 0] * stability	0.125	0.063	609.739	1.968	.050	0.000	0.249
[assessjob = 1] * stability	0.171	0.069	640.637	2.474	.014	0.035	0.307
[assessjob = 0] * female	-0.031	0.058	595.213	-0.532	.595	-0.144	0.083
[assessjob = 1] * female	0.005	0.063	601.425	0.079	.937	-0.118	0.128

<sup>a</sup> Dependent variable: work.

**TABLE 7.13** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [Index1 = 1]	0.588	0.041	14.340	.000	0.513	0.674
	Var: [Index1 = 2]	0.533	0.039	13.782	.000	0.462	0.614
	Var: [Index1 = 3]	0.864	0.056	15.399	.000	0.761	0.981
	Var: [Index1 = 4]	0.418	0.030	13.872	.000	0.363	0.482
	Var: [Index1 = 5]	0.300	0.024	12.260	.000	0.255	0.352
	Var: [Index1 = 6]	0.564	0.037	15.065	.000	0.496	0.643
assessjob [subject = orgcode]	Var(1)	0.114	0.028	4.103	.000	0.071	0.183
	Var(2)	0.170	0.038	4.525	.000	0.110	0.262
	Corr(2, 1)	0.738	0.080	9.185	.000	0.537	0.859
assessjob [subject = Rid * orgcode]	Var(1)	0.287	0.031	9.126	.000	0.232	0.356
	Var(2)	0.442	0.035	12.543	.000	0.378	0.517
	Corr(2, 1)	0.943	0.038	25.038	.000	0.800	0.984

<sup>a</sup> Dependent variable: work.

**Model 1.4: Adding the Organizational Predictors**

Model 1.4 adds two organizational predictors at Level 3. These include a measure describing the level of each organization’s productivity and a measure of its resources allocated to support employees. Both were grand-mean centered. They are each proposed to affect work life perceptions of job satisfaction and evaluation of performance:

$$\begin{aligned} \beta_{10k} &= \gamma_{100} + \gamma_{101}gmorgprod_k + \gamma_{102}gmresource_k + u_{10k}, \\ \beta_{20k} &= \gamma_{200} + \gamma_{201}gmorgprod_k + \gamma_{202}gmresource_k + u_{20k}. \end{aligned} \tag{7.11}$$

This combined model will add 4 more parameters to estimate (i.e., from 18 parameters in the last model to 22 parameters), since we are specifying a separate predictor for each outcome variable at Level 3:

$$\begin{aligned} Y_{ijk} &= \gamma_{100} + \gamma_{200} + \gamma_{101}assessjob_{jk} * gmorgprod_k + \gamma_{201}assessjob_{jk} * gmorgprod_k \\ &+ \gamma_{102}assessjob_{jk} * gmresources_k + \gamma_{202}assessjob_{jk} * gmresources_k + \gamma_{110}assessjob_{jk} * female_{jk} \\ &+ \gamma_{210}assessjob_{jk} * female_{jk} + \gamma_{120}assessjob_{jk} * stability_{jk} + \gamma_{220}assessjob_{jk} * stability_{jk} \\ &+ u_{10k} + u_{20k} + r_{1jk} + r_{2jk} + \epsilon_{ijk}. \end{aligned} \tag{7.12}$$

This can be confirmed in the model dimension table (Table 7.14). We note that we also had to change from an unstructured correlation covariance matrix (UNR) to an unstructured covariance matrix (UN) in order for the model to converge. We estimated this model and the following model with equality constraints with ML estimation (instead of REML) since we are subsequently going to compare the fit of the model in Equation 7.12 against a model with equality constraints added for the fixed-effect regression slopes.

**TABLE 7.14 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	assessjob	2		2		
	assessjob * gmorgprod	2		2		
	assessjob * gmresources	2		2		
	assessjob * stability	2		2		
	assessjob * female	2		2		
Random Effects	assessjob	2	Unstructured	3	Orgcode	
	assessjob	2	Unstructured	3	orgcode * Rid	
Repeated Effects	Index 1	6	Diagonal	6	orgcode * Rid	650
Total		20		22		

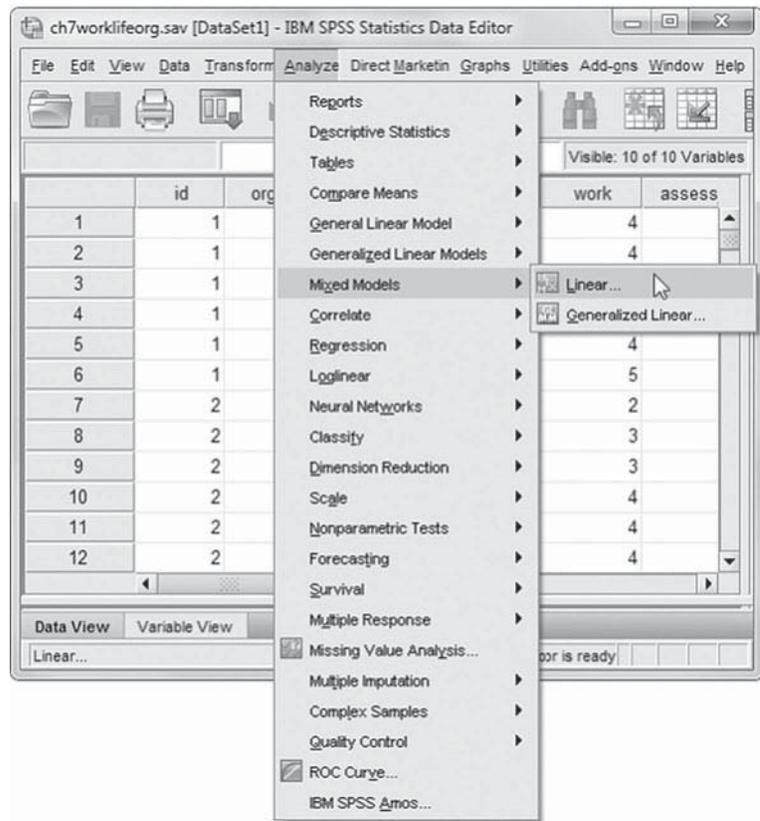
<sup>a</sup> Dependent variable: work.

**Defining Model 1.4 with IBM SPSS Menu Commands**

Settings will default to those used in Model 1.3.

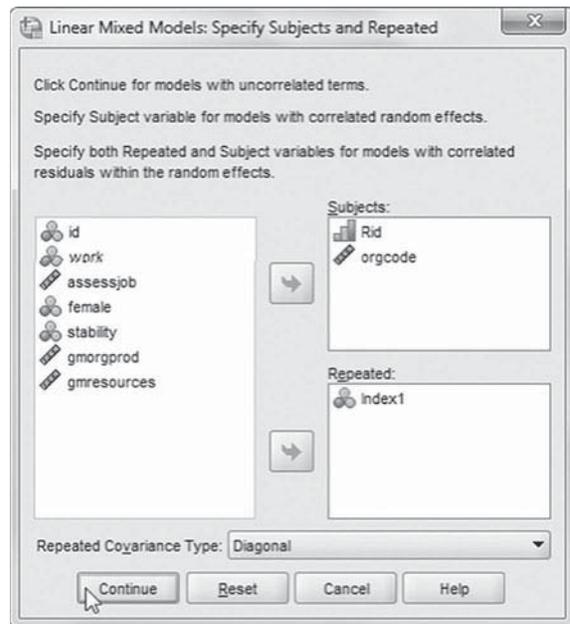
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

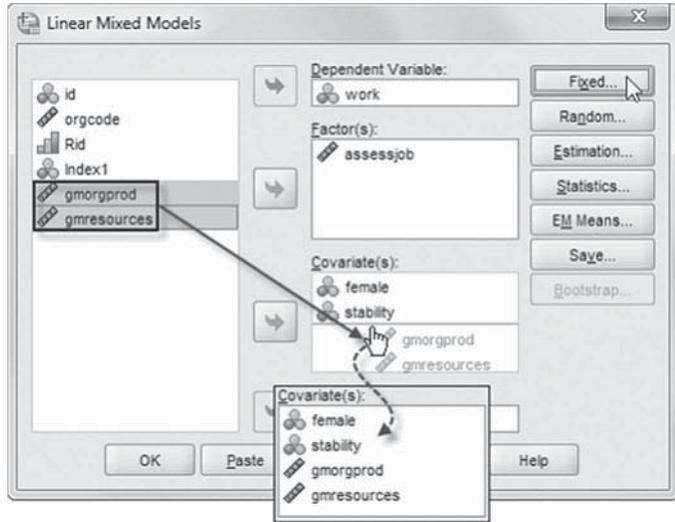


3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

We will introduce two new variables to be used in the model (*gmorgprod* and *gmresources*). First, click to select *gmorgprod* and *gmresources*, and second, “drag” the variables to the *Covariate(s)* box below *stability*.

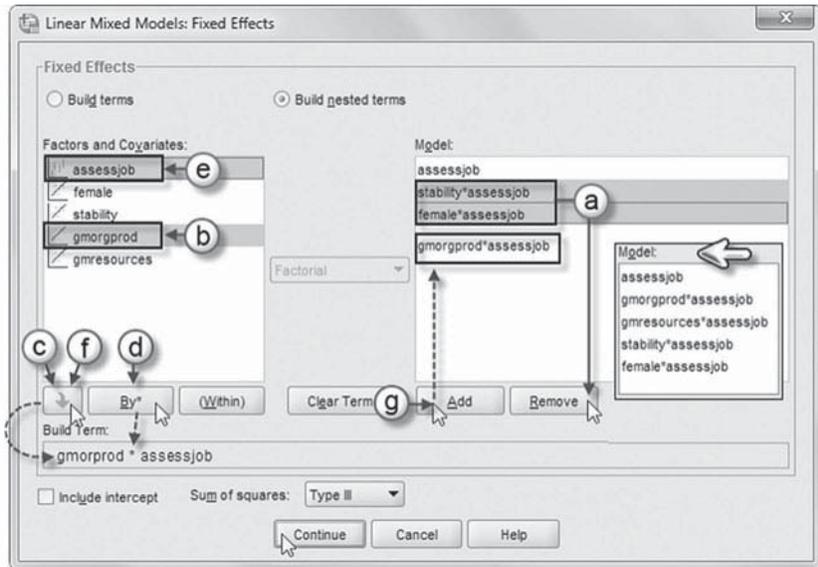
The sequence of the variables is as follows: *female*, *stability*, *gmorgprod*, and *gmresources*.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



Two new cross-level interactions (or nested terms) will be created and added to the model: *gmorgprod\*assessjob* and *gmresources\*assessjob*. These interactions will tell us if (a) each organization’s productivity (*gmorgprod*) supports employees’ perceptions of job satisfaction and performance evaluation (*assessjob*) and (b) each organization’s resource allocation (*gmresources*) supports employees’ job satisfaction and performance evaluation (*assessjob*).

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading of the output tables, we will first remove the two interactions to clear the model and then later add them back into the model. First, click to select *stability\*assessjob* and *female\*assessjob*, and second, click the REMOVE button.



Add First Interaction to Model 1.4: *gmorgprod\*assessjob*

**Note:** *Build nested terms* is the default setting from the prior model.

- b. Click to select the variable *gmorgprod* from the *Factors and Covariates* box.
- c. Then click the arrow button below the *Factors and Covariates* box. This moves *gmorgprod* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- d. Next, click the BY\* button, which will insert the computation command symbol: *gmorgprod\**.
- e. Click to select *assessjob* from the *Factors and Covariates* box.
- f. Click the arrow button below the *Factors and Covariates* box to move *assessjob* into the *Build Term* box and complete the interaction term: *gmorgprod\*assessjob*.
- g. Click the ADD button to transfer the interaction into the *Model* box.

Add Second Interaction to Model 1.4: *gmresources\*assessjob*

Repeat steps 4b to 4g using *gmresources* and *assessjob* to create the interaction.

Add Third Interaction to Model 1.4: *stability\*assessjob*

Repeat steps 4b to 4g using *stability* and *assessjob* to create the interaction.

Add Fourth Interaction to Model 1.4: *female\*assessjob*

Repeat steps 4b to 4g using *female* and *assessjob* to create the interaction.

The completed model is shown in the insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

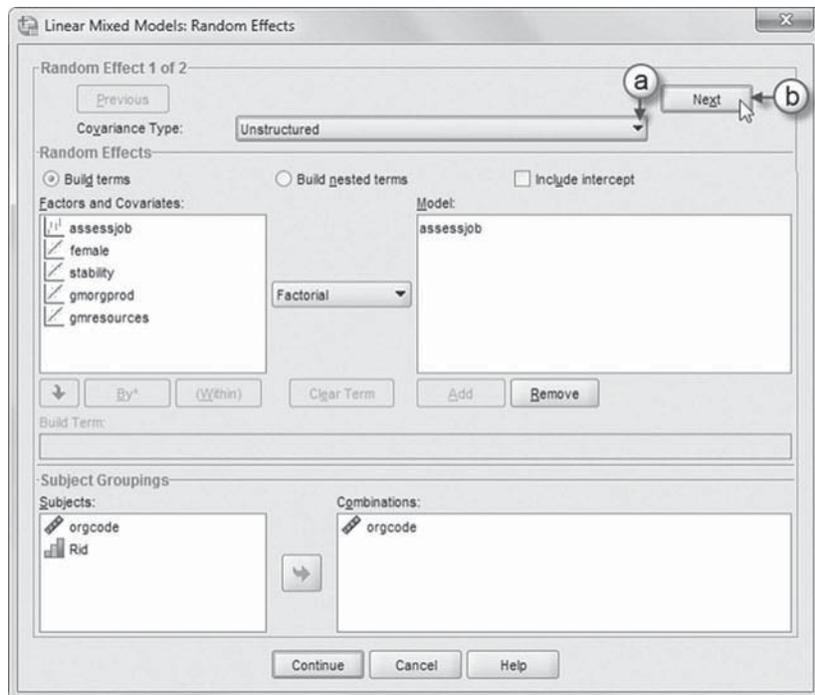
Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Random Effect 1 of 2* screen is displayed first, as it was the last dialog box used in the prior model.

- a. Change the covariance type by clicking the pull-down menu and selecting *Unstructured*.

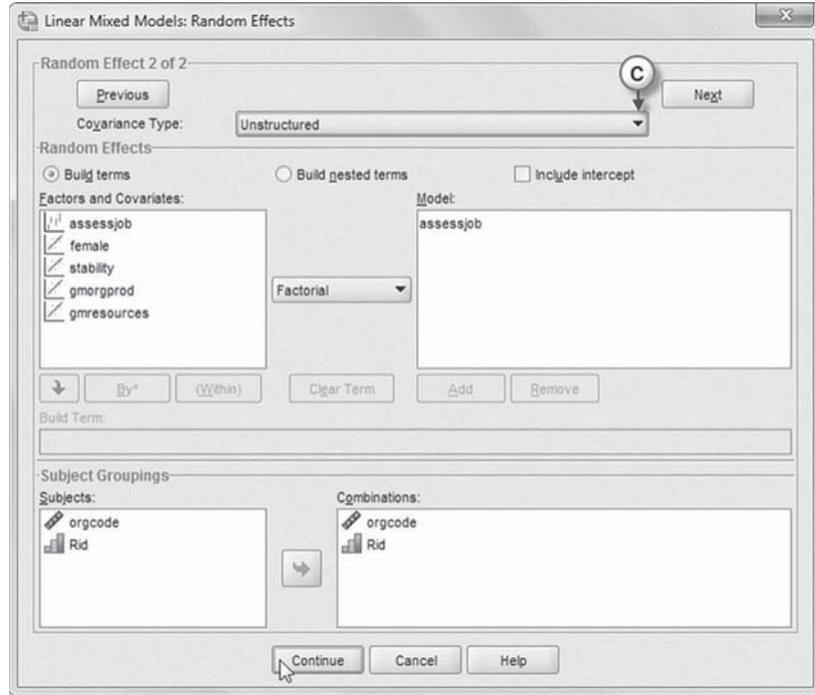
The *Unstructured* covariance structure has a completely general covariance matrix (IBM Corporation, 2012).

- b. Click the NEXT button to access the *Random Effect 2 of 2* screen.



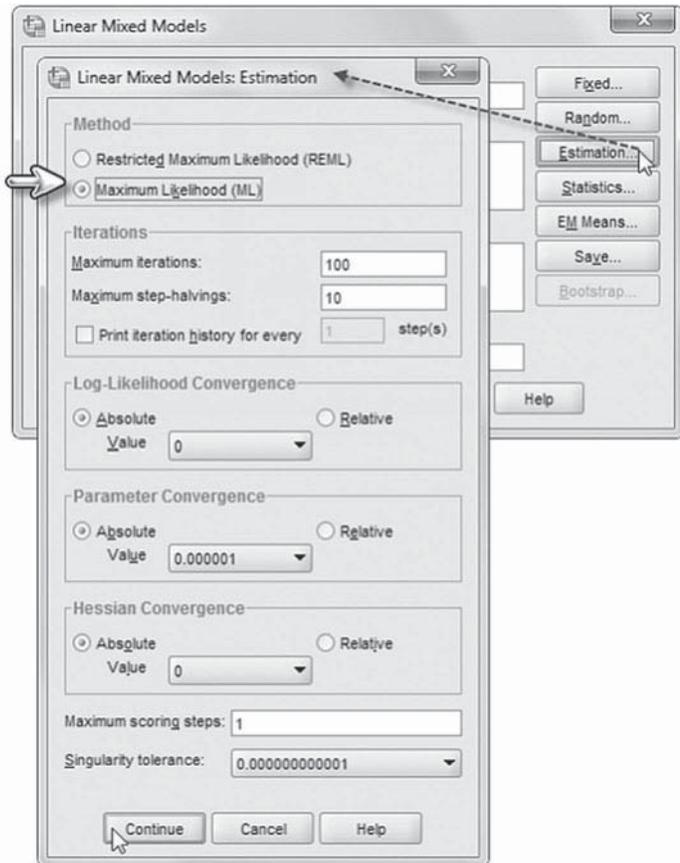
- c. From the *Random Effect 2 of 2* screen, change the covariance type by clicking the pull-down menu and selecting *Unstructured*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

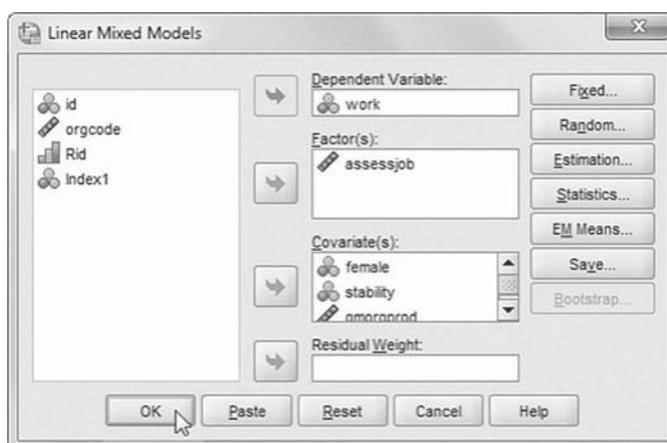


- 6. Click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box. We will change the estimation method by clicking to select ML.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output from Model 1.4

The fixed-effect results are presented in Table 7.15. The table suggests that between individuals (Level 2), gender once again does not affect individual perceptions, but employee stability does. Between organizations (Level 3), resource adequacy affects perceptions of job satisfaction (*assessjob* = 0) but not perceptions about the assessment of individuals' performance (*assessjob* = 1). Finally, organizational performance positively affects both organizational job satisfaction (*assessjob* = 0) and organizational evaluation of employee performance (*assessjob* = 1).

Because the covariance parameters are very similar to the previous model, we do not reproduce them here.

**TABLE 7.15** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[ <i>assessjob</i> = 0]	3.909	0.056	266.183	69.537	.000	3.798	4.019
[ <i>assessjob</i> = 1]	3.738	0.065	250.718	57.681	.000	3.610	3.865
[ <i>assessjob</i> = 0] * gmorgprod	0.344	0.119	106.311	2.878	.005	0.107	0.581
[ <i>assessjob</i> = 1] * gmorgprod	0.383	0.144	106.371	2.666	.009	0.098	0.668
[ <i>assessjob</i> = 0] * gmresources	0.101	0.044	108.941	2.273	.025	0.013	0.188
[ <i>assessjob</i> = 1] * gmresources	0.060	0.053	108.492	1.124	.263	-0.046	0.165
[ <i>assessjob</i> = 0] * stability	0.129	0.062	599.753	2.071	.039	0.007	0.251
[ <i>assessjob</i> = 1] * stability	0.171	0.069	639.944	2.493	.013	0.036	0.306
[ <i>assessjob</i> = 0] * female	-0.038	0.057	604.741	-0.666	.506	-0.151	0.075
[ <i>assessjob</i> = 1] * female	-0.001	0.062	608.417	-0.016	.988	-0.123	0.121

<sup>a</sup> Dependent variable: work.

### Examining Equality Constraints

We can compare the model in Table 7.15 (which has a separate estimate for each predictor on each construct) against a model with equality constraints on the predictors (Hox, 2010). This type of model makes the assumption that the effects of predictors are the same on the constructs. This proposed model has only 18 parameters instead of 22 since 1 parameter is removed for each of the four predictors in the model. We estimated both models with ML to facilitate the comparison of models with different numbers of regression slope coefficients. The model with equality constraints added has a deviance (–2LL) of 9,837.799 and Akaike’s information criterion (AIC) of 9,873.799. The model with 22 parameters in Table 7.15 has a deviance 9,835.713. We can estimate the likelihood ratio test coefficient as 2.086 (which is not significant for 4 *df*). Since the reduced model does not worsen the fit, however, it could be accepted on the grounds that it is more parsimonious (i.e., having only 18 vs. 22 parameters). It also has a smaller AIC.

We present the fixed effects for this model in Table 7.16 (instructions for Model 1.5 shown after the table). Similar to the previous model, it suggests that resources and productivity affect perceptions at the organizational level and stability affects perceptions at the individual level.

**TABLE 7.16** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[assess job = 0]	3.896	0.055	247.746	70.822	.000	3.788	4.005
[assess job = 1]	3.758	0.061	216.609	61.204	.000	3.637	3.879
female	–0.024	0.054	602.279	–0.437	.662	–0.129	0.082
stability	0.143	0.059	614.094	2.426	.016	0.027	0.258
gmorgprod	0.352	0.118	103.555	2.984	.004	0.118	0.586
gmresources	0.091	0.044	105.907	2.077	.040	0.004	0.177

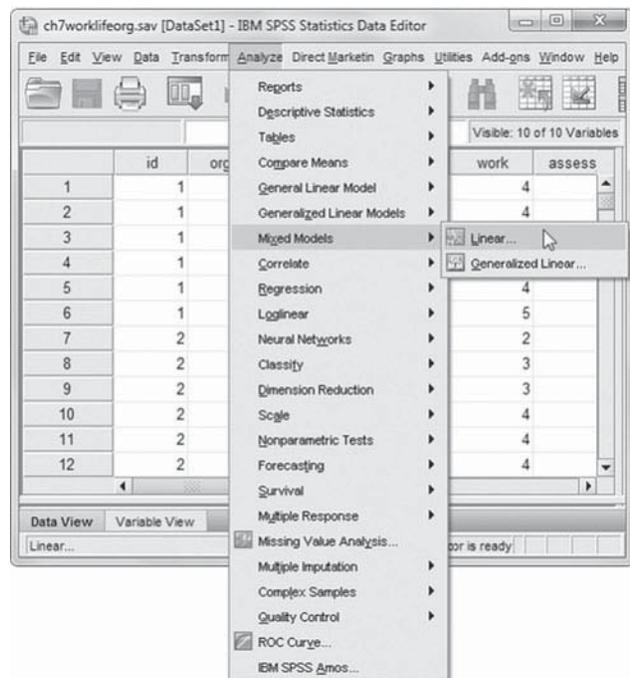
<sup>a</sup> Dependent variable: work.

### Defining Model 1.5 with IBM SPSS Menu Commands

Note: Settings will default to those used in Model 1.4.

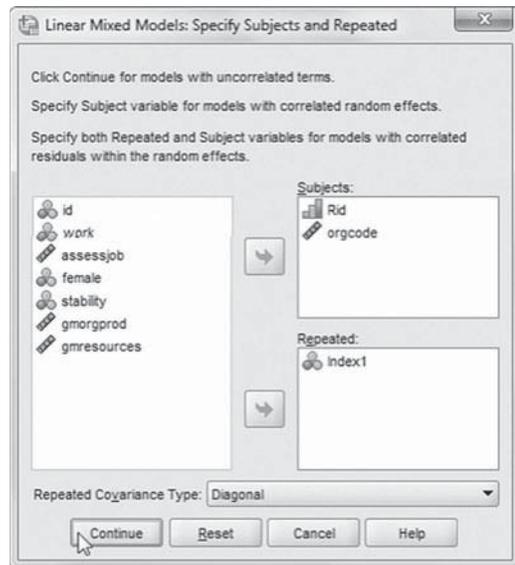
1. Go to the toolbar and select ANA-  
LYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



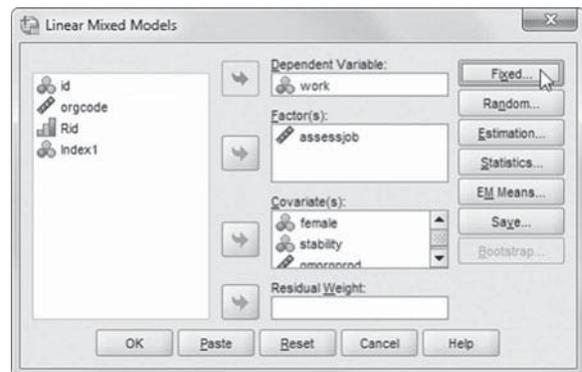
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model. We will retain the default settings.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

We will modify the fixed effects so click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

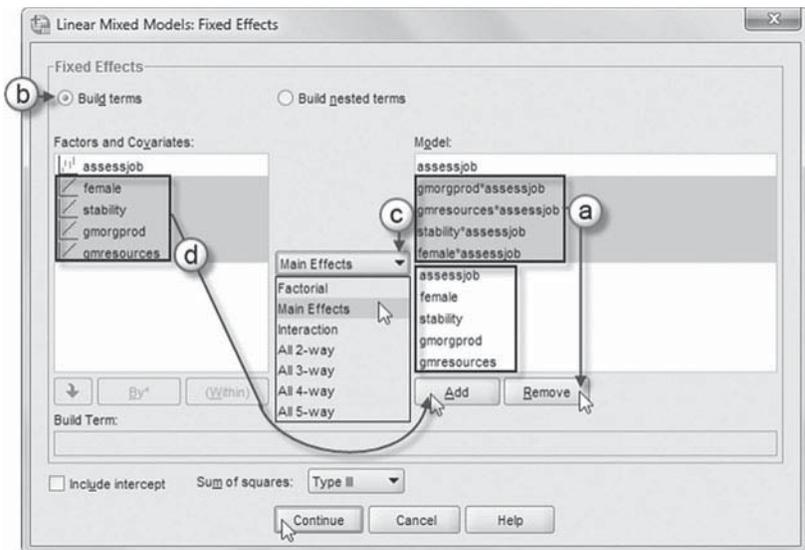


- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. We will first remove the four cross-level interactions from the model by clicking to select them and then clicking the REMOVE button.

- b. Click to select the *Build terms* option, which enables adding nonnested terms to the model.

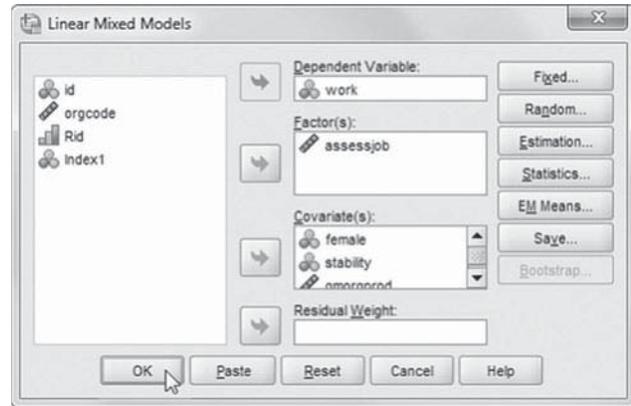
- c. Change the factorial setting by clicking the pull-down menu and selecting *Main Effects*.

- d. Now click to select four variables (*female*, *stability*, *gmorgprod*, and *gmresources*) from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model (Table 7.16).



### Investigating a Random Level 2 Slope

Another of our research goals was to determine whether the effects of particular individuals' background variables (e.g., gender and stability) on perceptions might vary across organizations at Level 3. If a slope parameter does vary across organizations, we can build a model to explain this variation in slopes at Level 3. To illustrate this, we will propose that the slope describing the effect of individuals' stability in the organization on their perceptions of the two constructs varies across organizations (instructions provided in Model 1.6). The stability slopes from Equation 7.9 ( $\beta_{12k}$  and  $\beta_{22k}$ ) can be designated as randomly varying at Level 3 (between organizations) as follows:

$$\begin{aligned}\beta_{12k} &= \gamma_{120} + u_{12k}, \\ \beta_{22k} &= \gamma_{220} + u_{22k},\end{aligned}\quad (7.13)$$

where  $\gamma_{120}$  and  $\gamma_{220}$  are the average slope effects at the organizational level and  $u_{12j}$  and  $u_{22j}$  represent variation in the female-work life and female-performance slopes across organizations. We can summarize this combined model as follows:

$$\begin{aligned}Y_{ijk} &= \gamma_{100} + \gamma_{200} + \gamma_{101}assessjob_{jk} * gmorgprod_k + \gamma_{201}assessjob_{jk} * gmorgprod_k \\ &+ \gamma_{102}assessjob_{jk} * gmresources_k + \gamma_{202}assessjob_{jk} * gmresources_k + \gamma_{110}assessjob_{jk} * female_{jk} \\ &+ \gamma_{210}assessjob_{jk} * female_{jk} + \gamma_{120}assessjob_{jk} * stability_{jk} + \gamma_{220}assessjob_{jk} * stability_{jk} \\ &+ u_{10k} + u_{20k} + u_{12k}assessjob_{jk} * stability_{jk} + u_{22k}assessjob_{jk} * stability_{jk} + r_{1jk} + r_{2jk} + \epsilon_{ijk}.\end{aligned}\quad (7.14)$$

At Level 3, we can specify a UN covariance matrix to capture variability in intercepts, slopes, and the covariance between the intercept and slope. We note that in order to specify each stability slope as randomly varying, we have to create an interaction term ( $assessjob * stability$ ) in the Random command (instructions provided in Model 1.7). This creates a relatively complex covariance matrix of random effects. If instead we wanted to build a random slope on the model with equality constraints, as in Table 7.16, we would not need to specify an interaction term in the Random command. At Level 2, we can also continue to use a UN covariance matrix at the between-individual level.

When we actually specified the model in Table 7.15 as having a random slope, however, we found that the effect of individuals' *stability* on the constructs did not vary significantly across organizations. We also received a warning message that the model did not converge. For the model with equality constraints (Table 7.16), we found we could obtain a solution by specifying

the random-effects covariance matrices to be diagonal at Levels 2 and 3, but, even so, the effect of employee stability on the two constructs did not vary across organizations ( $p = .751$ ). We therefore stopped our model investigation at this point. For interested readers, we provide a comparison of the estimates produced with Mplus (using a two-level latent variable analysis) and the MIXED specification in Table 7.15 in Appendix B. As we might expect, the analyses produce very similar results. We note in passing that for models where there are several latent constructs under consideration, multilevel structural equation modeling may be an approach that is easier to implement. This latter approach requires specialized software that is not available in IBM SPSS, however.

### Defining Models 1.6 and 1.7 with IBM SPSS Menu Commands

Note: IBM SPSS settings will default to those used in Model 1.5.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model. We will retain the default settings.

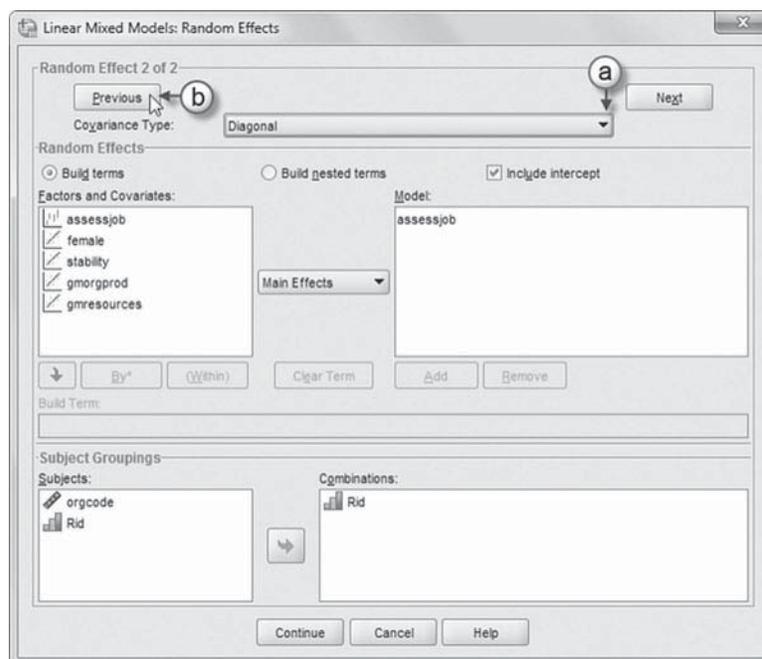
Click the CONTINUE button to display the *Linear Mixed Models* dialog box. We will change the model's random effects, so click the *Random Effects* button to access the random-effects main screen.

### Model 1.6

3. The *Random Effect 2 of 2* screen is displayed first, as it was the last dialog box used in the prior model.
  - a. Change the covariance type by clicking the pull-down menu and selecting *Diagonal*.

The *Diagonal* covariance type has heterogeneous variances and zero correlation between the elements (IBM Corporation, 2012).

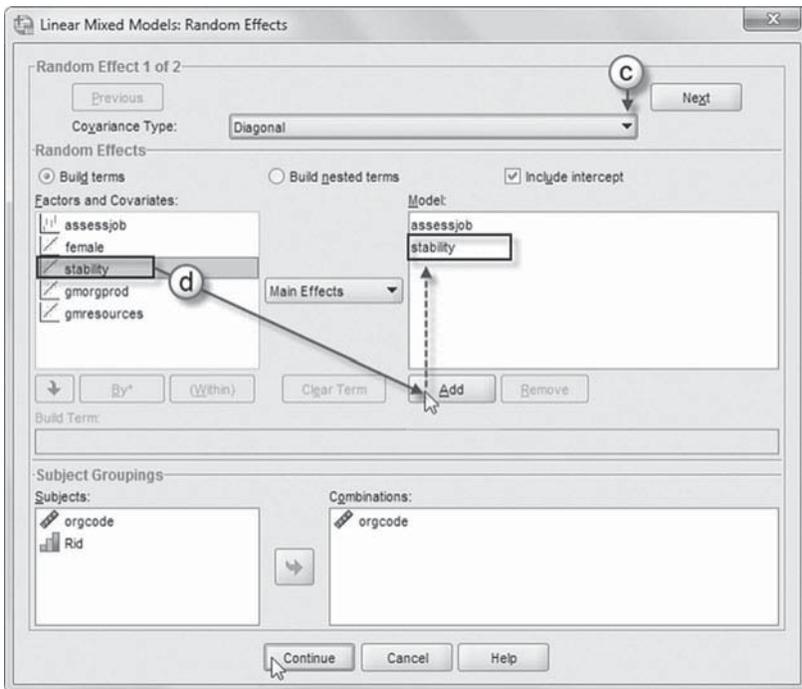
- b. Click the PREVIOUS button to access the *Random Effect 1 of 2* screen.



- c. From the *Random Effect 1 of 2* screen change the covariance type by clicking the pull-down menu and selecting *Diagonal*.
- d. Click to select *stability*, and then click the ADD button to move the variable into the *Model* box.

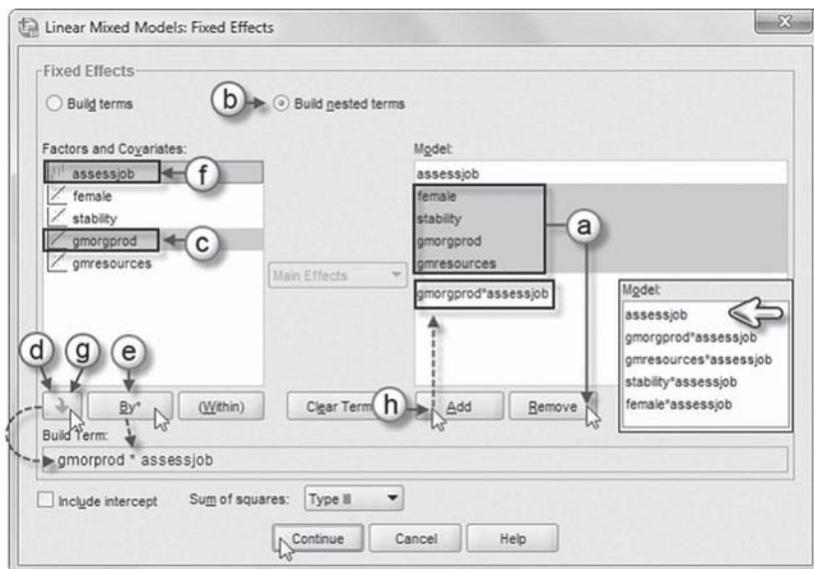
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Click the OK button to run the model.



### Model 1.7

Repeat steps 1 and 2 from Model 1.6, and then click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



### Add First Interaction to Model 1.7: *gmorgprod\*assessjob*

- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. We will modify the model by first removing four variables (*female*, *stability*, *gmorgprod*, and *gmresources*). Click to select *female*, *stability*, *gmorgprod*, and *gmresources*, and then click the REMOVE button.
- b. Click to select *Build nested terms*.
- c. Now click to select the variable *gmorgprod* from the *Factors and Covariates* box.
- d. Click the arrow button below the *Factors and Covariates* box. This moves *gmorgprod* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- e. Next, click the BY\* button, which will insert the computation command symbol: *gmorgprod\**

- f. Click to select *assessjob* from the *Factors and Covariates* box.
- g. Click the arrow button below the *Factors and Covariates* box to move *assessjob* into the *Build Term* box and complete the interaction term: *gmorprod\*assessjob*
- h. Click the ADD button to transfer the interaction into the *Model* box.

Add Second Interaction to Model 1.7: *gmresources\*assessjob*

Repeat steps 4c to 4h using *gmresources* and *assessjob* for the interaction.

Add Third Interaction to Model 1.7: *stability\*assessjob*

Repeat steps 4c to 4h using *stability* and *assessjob* for the interaction.

Add Fourth Interaction to Model 1.7: *female\*assessjob*

Repeat steps 4c to 4h using *female* and *assessjob* for the interaction.

The completed model is shown in the insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

---

## Multivariate Multilevel Model for Correlated Observed Outcomes

In our second example, we use the multivariate multilevel formulation to define separate outcome models that take into consideration the correlation between the multiple outcomes. This formulation is similar to the previous model, with the major difference being there are no underlying constructs to define. In this example, we have three tests summarizing individual achievement in reading, math, and language. The goal is to build a model where the fixed effects are used to control for differences in the means between individual responses and the random effects can be used to model the different variances for the outcomes, as well as the covariances (or correlations) between the outcome measures (Leyland, 2004). One benefit of this specific type of model is in testing the equality of the size of effect of a specific predictor on each of the outcomes.

We note that in this multivariate formulation, the repeated measures (Level 1) specification is again used to collect three subtest measures within individuals; however, we do not include an error term at the lowest level. At Level 2 (between individuals), we specify the individual background variables in the model. These estimates are considered as “fixed” (i.e., not varying) at the individual level. At the school level (Level 3), we can define possible random effects (e.g., variation in the achievement scores across schools and variation in the effects of background variables). We can also consider the effects of school predictors on the randomly varying outcomes.

### The Data

The data set consists of 2,715 students (Level 1,  $N = 8,145$  observations) nested in 353 schools (Table 7.17). Between individuals, we will investigate the effect of *gender* (coded 1 = female, 0 = male) on the three achievement outcomes. Between schools, we investigate the effect of *academic press* (i.e., the same school variable indicating the relative focus on academic outcomes) on the set of correlated subtests.

### Research Questions

In this example, the primary research question concerns whether gender affects students' achievement on each test. In this instance, we wish to control for the likely correlation between student performance on each test. We then can ask whether the size of the gender effect is the same across all three tests. In answering this latter question, we are interested in investigating

**TABLE 7.17 Data Definition of *ch7achievement.sav* (N = 8,670)**

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>schcode</i>	School	School identifier (353 schools).	Integer	Ordinal
<i>id</i>	Individual	Individual student identifier (2,715 students).	Integer	Ordinal
<i>Rid</i>	Individual	Recoded individual student identifiers ( <i>id</i> ) with the school identifier ( <i>schcode</i> ) identifying students within their school groups (1,2,. . .,30).	1,2,. . .,30	Ordinal
<i>female</i>	Individual	Demographic predictor variable representing students gender	0 = Male 1 = Female	Scale
<i>Index1</i>	Within Individual	Identifier variable resulting from representing the three tests comprising achievement within individuals.	(1, 2, 3)	Nominal
<i>achieve</i>	Within Individual	Dependent variable measuring achievement on three achievement outcomes for math and reading.	25.29 to 99.98	Scale
<i>gmses</i>	Individual	Predictor variable (grand-mean centered) measuring the socioeconomic status of students within schools.	-2.05 to 1.52	Scale
<i>gmacademic</i>	School	Predictor variable (grand-mean centered) representing average years of teaching experience of the staff	-2.61 to 339	Scale
<i>gmses_mean</i>	School	Predictor variable (grand-mean centered) representing average years of teaching experience of the staff.	-0.83 to 1.52	Scale
<i>gmacadpress</i>	School	Predictor variable (grand-mean centered) measuring each schools academic outcomes.	-2.51 to 1.22	Scale

<sup>a</sup> Within individual = Level 1; Individual = Level 2; School = Level 3.

hypotheses about the effect of gender on each of the student subtests comprising the multivariate outcome. More specifically, we wish to determine whether the effect of gender is the same across the correlated subtest outcomes or whether it may be related to some subtests but not others. In each case, the hypothesis proposed is that the strength of the effect of the predictor on the three outcomes is the same. For example, for gender, the null hypothesis would be that the three unstandardized betas ( $\beta_1$ ) would be the same:

$$H_0 = \beta_{1Read} = \beta_{1Math} = \beta_{1Language}$$

We will also pose a question about whether the effect of the school's academic press is the same across the three subtests.

### Formulating the Basic Model

Similar to our previous multivariate formulation, we do not specify an intercept in the *Random* dialog box (or syntax subcommand) for the lowest level of the model. We note in passing that we could define the three observed outcomes as a latent achievement variable at Level 1 and examine variation due to individuals at Level 2 and variation due to schools at Level 3. This would be

similar to our previous three-level formulation with measurement model at Level 1. If we were to do this, we should have variables measured on the same scale (e.g., scaled scores) and with similar variance.

As we have suggested previously, the multivariate linear model is commonly formulated as a single-level model specified in matrix notation (e.g., see Leyland, 2004). To define this type of model in MIXED, we stack the single-level model with reading, math, and language subtests in a single variable column ( $Y_{ijk}$ ), where  $i$  refers to the particular subtest ( $i = 1, 2, 3$ ) for individual  $j$  in school  $k$ . As we noted, instead of creating a latent achievement construct at Level 1, in this example we will again use a categorical indicator (*Index1*) to open up the *Repeated* dialog box for the Menu commands. At the lowest level, the categorical indicator is used to identify each response variable. In this case, it is used to generate three design matrix columns corresponding to three intercept terms (Leyland, 2004). Specifying the fixed-effects portion of the model as having no intercept (NOINT) ensures that separate intercepts are obtained for each of the three subtests and prevents MIXED from generating another, unnecessary, intercept column (Leyland, 2004). At this lowest level, we must also designate students as nested in schools [Subject (*schcode*\**Rid*)] on the *Repeated* command line (if we refer to the syntax statements).

We note that this particular specification has the effect of combining Levels 1 and 2 to create a basic two-level (i.e., student and school) multivariate formulation (Leyland, 2004). As we indicated previously, in this formulation there is no modeling of the residual variance ( $\epsilon_{ijk}$ ) at the lowest level of the model. Moreover, because Levels 1 and 2 are combined to serve as the student level, we do not specify a random intercept at the individual level. By declaring *Index1* as the random effect at the highest level (using *schcode* as the subject variable), we can fit a random intercept model for each test between schools.

For ease of presentation, in Equation 7.15, we specify a separate, combined equation for each outcome (using  $R$  for reading,  $M$  for math, and  $L$  for language). We can also add one or more student-level  $X_{jk}$  predictors and school-level  $W_k$  predictors in each combined model. In this case, we will add female as a student-level predictor and academic press (*gmacadpress*) as a school indicator. We could add other predictors, but this should be sufficient to demonstrate this type of model specification adequately. Following Leyland (2004), the fitted two-level models are then as follows:

$$\begin{aligned} Y_{Rjk} &= \gamma_{R00} + \gamma_{R01}gmacadpress_k + \gamma_{R10}female_{jk} + u_{R0k} + r_{Rjk}, \\ Y_{Mjk} &= \gamma_{M00} + \gamma_{M01}gmacadpress_k + \gamma_{M10}female_{jk} + u_{M0k} + r_{Mjk}, \\ Y_{Ljk} &= \gamma_{L00} + \gamma_{L01}gmacadpress_k + \gamma_{L10}female_{jk} + u_{L0k} + r_{Ljk}. \end{aligned} \quad (7.15)$$

This suggests that we have separate intercepts and regression coefficients for the school-level intercepts and within-group and between-group predictors, with  $r$  representing within-school residuals for each equation and  $u$  representing random effects associated with explaining school achievement for each subtest. For this example, we will consider the slope coefficients describing the relationship of gender and each subtest ( $\beta_{1k}$ ) within schools to be fixed between schools ( $\beta_{1k} = \gamma_{10}$ ).

Once again, we note that *Index1* is crossed with the within-school predictors ( $X$ ) and between-school ( $W$ ) predictors in the combined model (e.g., *Index1*\**female*), if we wish to include a separate regression coefficient describing the gender effect on each outcome. This is important in our example since one of our goals is to examine whether the effects of gender are the same or different on each outcome. If we wished to impose equality constraints on the predictor, however, that variable (e.g., female) could be added directly into the model (Hox, 2010).

We will select a UN covariance matrix where the residuals are assumed to be normally distributed ( $N$ ) with 0 means in order to examine the variances and covariances between tests at the school level.

$$\begin{bmatrix} u_{R0k} \\ u_{M0k} \\ u_{L0k} \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{R_u}^2 & \sigma_{RM_u} & \sigma_{RL_u} \\ \sigma_{RM_u} & \sigma_{M_u}^2 & \sigma_{ML_u} \\ \sigma_{RL_u} & \sigma_{ML_u} & \sigma_{L_u}^2 \end{bmatrix} \right) \quad (7.16)$$

We will specify a similar structure at the student level.

$$\begin{bmatrix} r_{R0jk} \\ r_{M0jk} \\ r_{L0rk} \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{R_r}^2 & \sigma_{RM_r} & \sigma_{RL_r} \\ \sigma_{RM_r} & \sigma_{M_r}^2 & \sigma_{ML_r} \\ \sigma_{RL_r} & \sigma_{ML_r} & \sigma_{L_r}^2 \end{bmatrix} \right) \quad (7.17)$$

This fully specified model will have 21 parameters to estimate (i.e., three fixed-effect intercepts, three fixed-effect estimates for female, three fixed-effect estimates for academic press, and 12 covariance parameters).

### Model 2.1: Null Model (No Predictors)

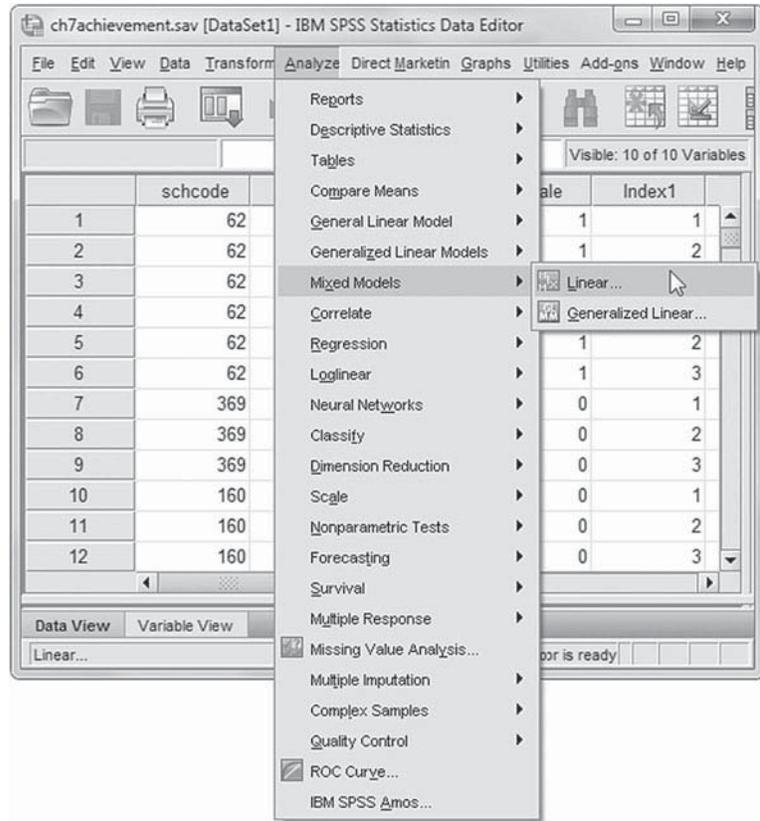
We will first define a simple “no-predictors” model with random intercepts for each response variable at Level 2. This combined model will have 15 parameters to estimate (i.e., three fixed effects and 12 covariance parameters in the two covariance matrices as specified in Eqs. 7.16 and 7.17).

#### Defining Model 2.1 (Null) with IBM SPSS Menu Commands

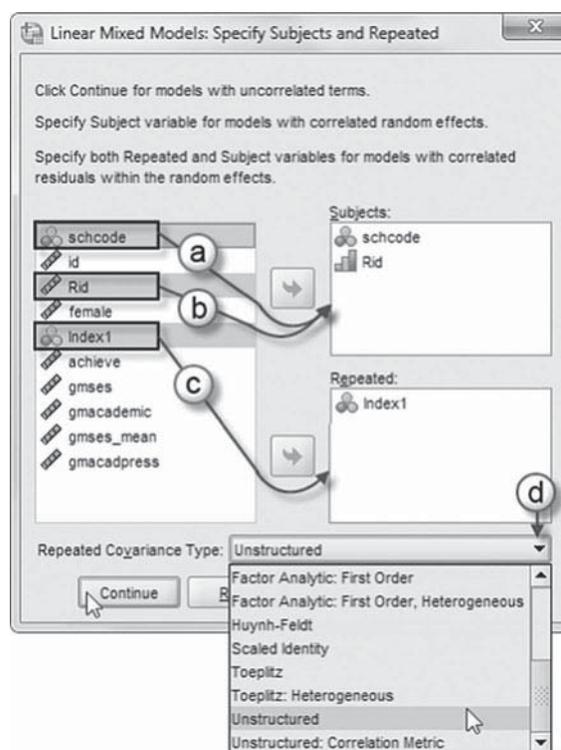
Launch the IBM SPSS program application, and select the *ch7achievement.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.
  - a. A subject is an observational unit that may be independent of other subjects. For this model, we will designate two subject identifiers for the model (*schcode* and *Rid*). Click to select *schcode*, and then click the right-arrow button to move the variable into the *Subjects* box.
  - b. Click to select *Rid*, and then click the right-arrow button to move the variable into the *Subjects* box.
  - c. The *Repeated* box allows specifying variables that identify repeated observations. For this model, *Index1* identifies indexing the *id* individual student identifier to define three test measures each in math and reading. Click to select *Index1*, and then click the right-arrow button to move the variable into the *Repeated* box.



The combination of values for *schcode*, *Rid*, and *Index1* defines a particular student from a particular school group across a single measure of achievement comprising three measures each of reading and math scores.

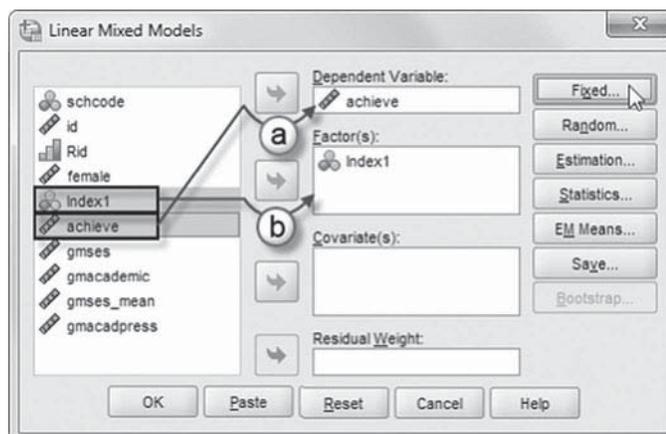
- d. The *Repeated Covariance Type* specifies a model's covariance structure. For this model, we will use an unstructured matrix. Click the pull-down menu to select *Unstructured*.

*Unstructured* is a completely general covariance matrix (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

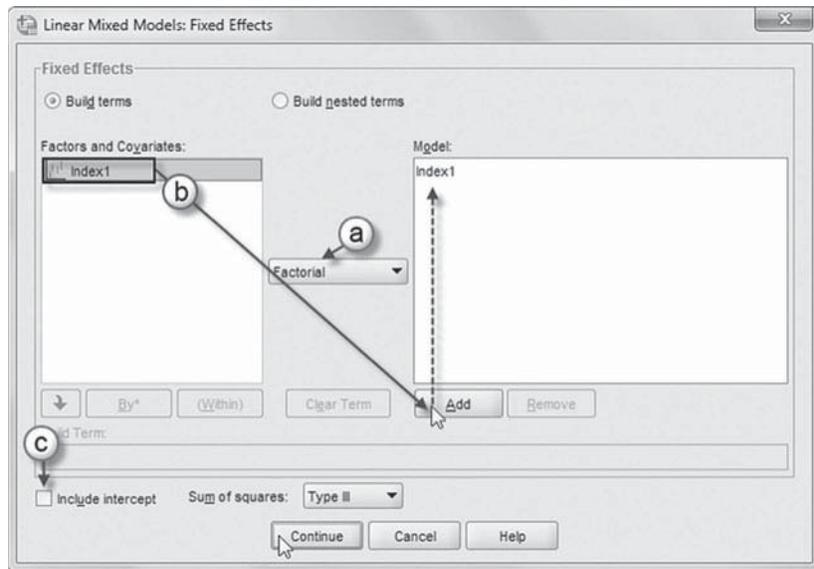
3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, and covariates, as well as access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

- a. For this model, we will use *achieve* as the dependent variable. Click to select the *achieve* variable from the left column listing. Then click the right-arrow button to transfer *achieve* into the *Dependent Variable* box.
- b. We will create a no-intercept null model that uses *Index1*. Click to select *Index1* from the left column listing. Then click the right-arrow button to transfer the variable into the *Factor(s)* box.



Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, we will retain the default *Factorial* setting, which creates all possible interactions and main effects of the specified variable (IBM Corporation, 2012).
- b. Now click to select *Index1* from the *Factors and Covariates* box, and then click the **ADD** button to move the variable into the *Model* box.



- c. To create the no-intercept model, uncheck the *Include intercept* option.

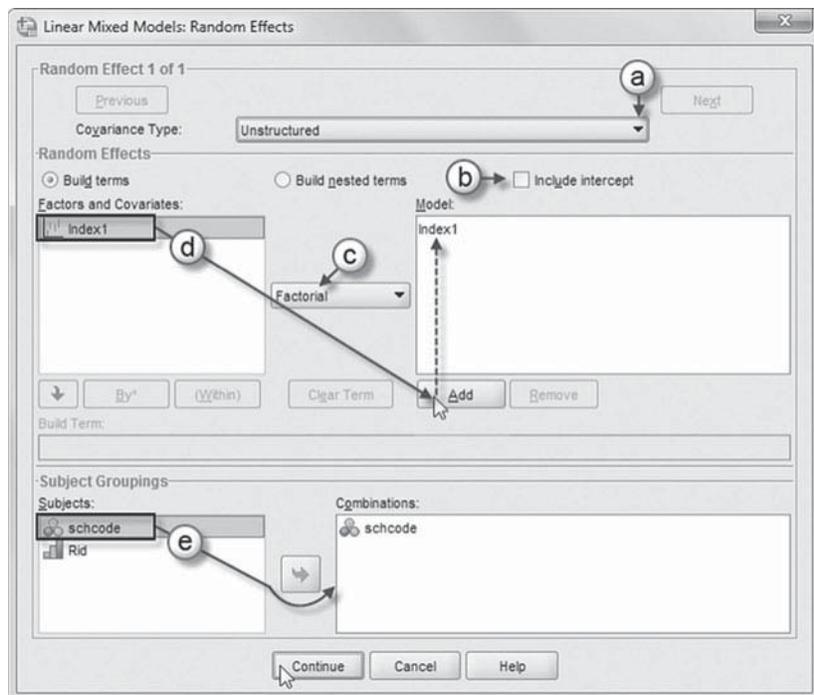
Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

We will now add random effects to this model.

Click the **RANDOM** button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen, which is the default when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.

- a. Begin by specifying the covariance structure from the default variance components (VC) to *Unstructured*. Click the pull-down menu and select *Unstructured* (UN). *Unstructured* is a completely general covariance matrix (IBM Corporation, 2012).



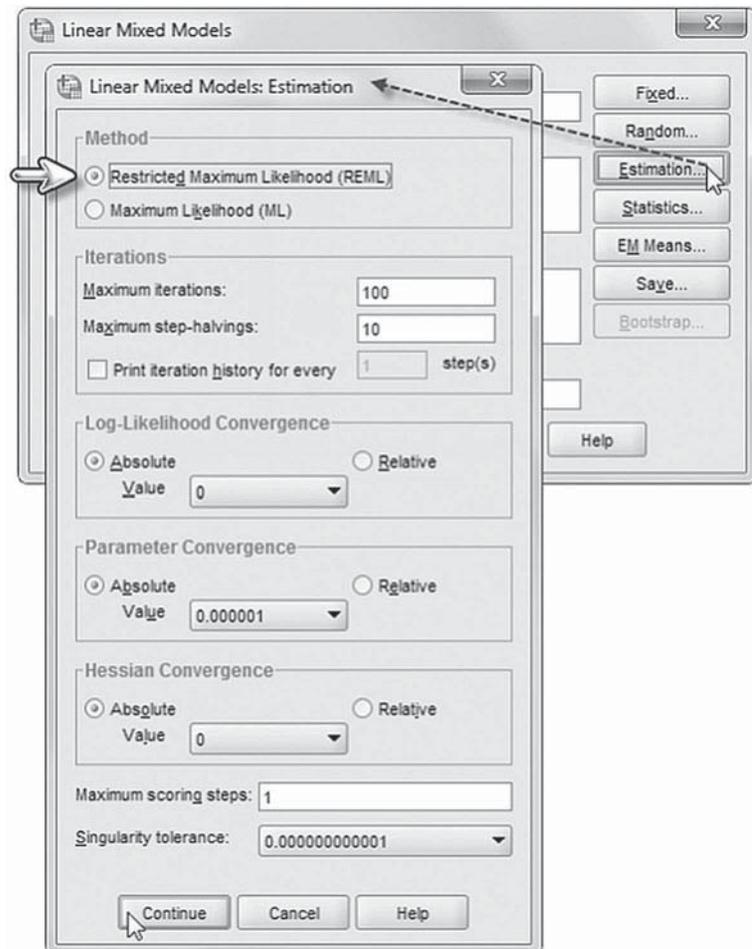
- b. We want the intercept to be excluded from the model, so we will retain the current default setting.
- c. We will also retain the default *Factorial* setting, which creates all possible interactions and main effects of the specified variable (IBM Corporation, 2012).
- d. Click to select *Index1*, and then click the ADD button to move the variable into the *Model* box.
- e. The *Subject Groupings* box displays the *schcode* and *Rid* variables that were specified as subject variables in the *Specify Subjects and Repeated* dialog box show in step 2a. We will specify *schcode* as the subject for the random-effects Level 1 part of this model. Click to select *schcode*, and then click the right-arrow button to move the variable into the *Combinations* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

6. Click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: ML or REML.

We will use the default setting of REML to estimate the models.

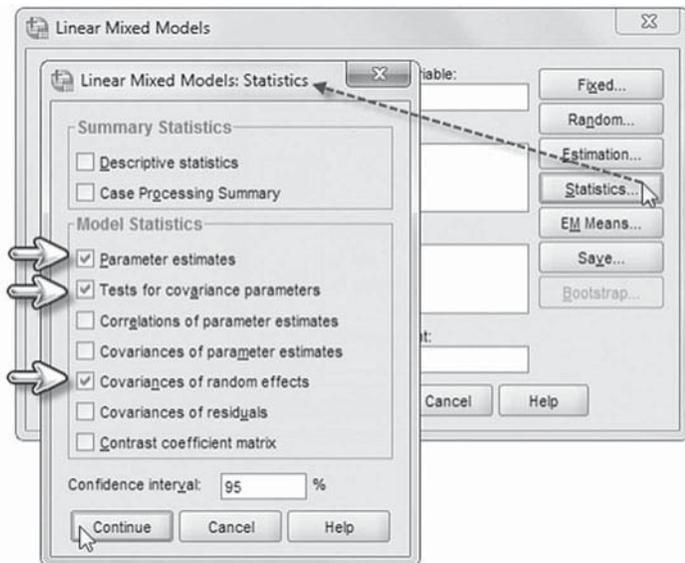
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



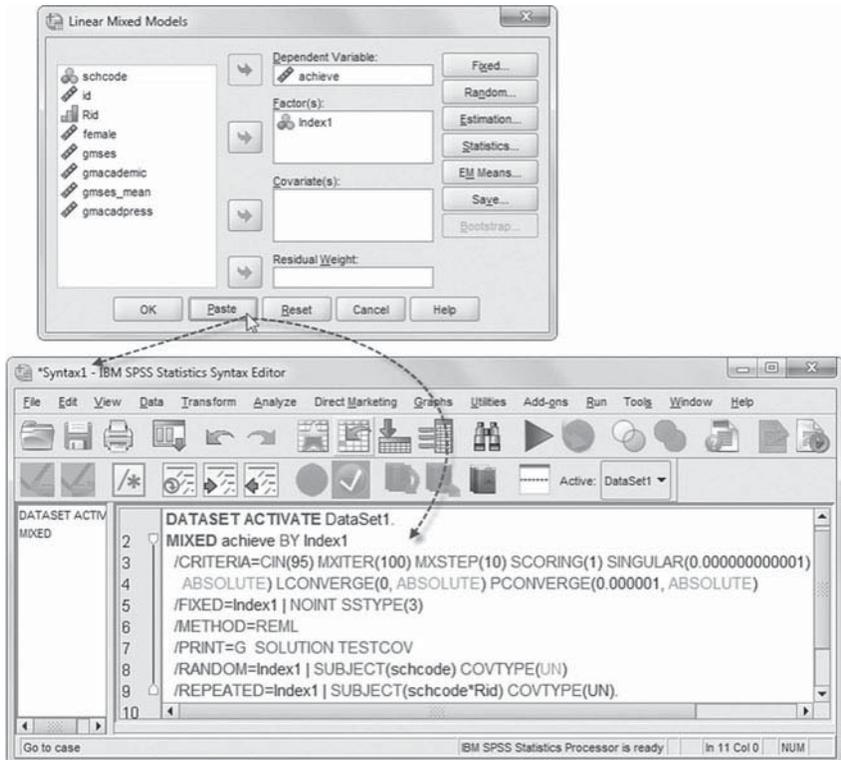
8. We recommend viewing your syntax commands before running the null model to first check if the model has been properly specified.

Click the PASTE button on the *Linear Mixed Models* dialog box, which will open a new window that displays the null model's syntax. Discussion of syntax occurs in the next section, "Examining the Syntax Commands."

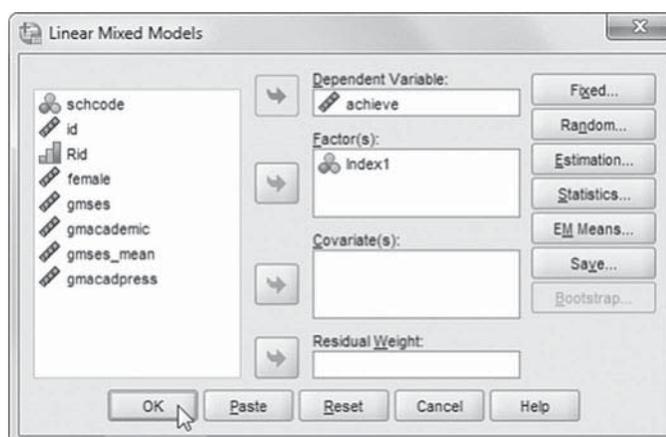
**Note:** Executing the *Paste* command will generate the syntax but also close the *Linear Mixed Models* dialog box. You will then need to repeat several steps in order to reaccess the *Linear Mixed Models* dialog box where you may then be able to run the model:

- Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.
- Within the *Linear Mixed Models: Specify Subjects and Repeated* dialog box, click the CONTINUE button to open the *Linear Mixed Models* dialog box.

Now continue on to the final step (9).



9. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Examining the Syntax Commands

We suggest examining the syntax statements to see whether the model has been set up properly using the *Repeated Covariance Type* menu option. The `/FIXED` command specifies *Index1* as the single predictor and no intercept (NOINT) in the fixed-effects portion of the model. The `/REPEATED` command line shows the variable *Index1* has been defined to represent the subtests and that the proper nesting of students within schools was accomplished correctly (*schcode\*Rid*). Next notice that the `/RANDOM` command includes *Index1* as a random effect, which allows the intercepts for the each test to vary across schools, and the school level (*schcode*) has been properly defined. We note there is no random term at the individual level.

```
MIXED
achieve BY Index1
/CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1)
SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
PCONVERGE(0.000001, ABSOLUTE)
/FIXED = Index1 |NOINT SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM Index1 |SUBJECT(schcode) COVTYPE(UN)
/REPEATED = Index1|SUBJECT(schcode*Rid) COVTYPE(UN) .
```

### Interpreting the Output From Model 2.1

We present the fixed effects for a simple “no-predictors” model first (Table 7.18). The school-level means (corrected for correlations between outcomes) are as follows: reading ( $R = 57.421$ ), math ( $M = 59.238$ ), and language ( $L = 57.816$ ).

**TABLE 7.18** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[Index1 = 1]	57.421	0.259	291.975	221.903	.000	56.912	57.931
[Index1 = 2]	59.238	0.284	304.979	208.703	.000	58.680	59.797
[Index1 = 3]	57.816	0.276	306.376	209.702	.000	57.274	58.359

<sup>a</sup> Dependent variable: achieve.

**TABLE 7.19** Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	UN (1, 1)	89.770	2.591	34.645	.000	84.832	94.995
	UN (2, 1)	55.555	2.082	26.689	.000	51.475	59.635
	UN (2, 2)	80.659	2.339	34.486	.000	76.202	85.375
	UN (3, 1)	54.857	2.163	25.363	.000	50.618	59.096
	UN (3, 2)	59.393	2.137	27.786	.000	55.203	63.582
Index1 [subject = schcode]	UN (3, 3)	91.458	2.637	34.682	.000	86.433	96.775
	UN (1, 1)	9.992	1.755	5.695	.000	7.083	14.097
	UN (2, 1)	11.217	1.746	6.425	.000	7.795	14.639
	UN (2, 2)	15.239	2.128	7.163	.000	11.591	20.035
	UN (3, 1)	9.894	1.644	6.018	.000	6.671	13.116
	UN (3, 2)	12.354	1.865	6.624	.000	8.699	16.010
	UN (3, 3)	12.532	1.966	6.373	.000	9.214	17.044

<sup>a</sup> Dependent variable: achieve.

We also show the specification of the two covariance matrices between and within schools in Table 7.19. As specified, the structure is fairly complex but with all the variances and covariances statistically significant within and between schools.

**Model 2.2: Building a Complete Model (Predictors and Cross-Level Interactions)**

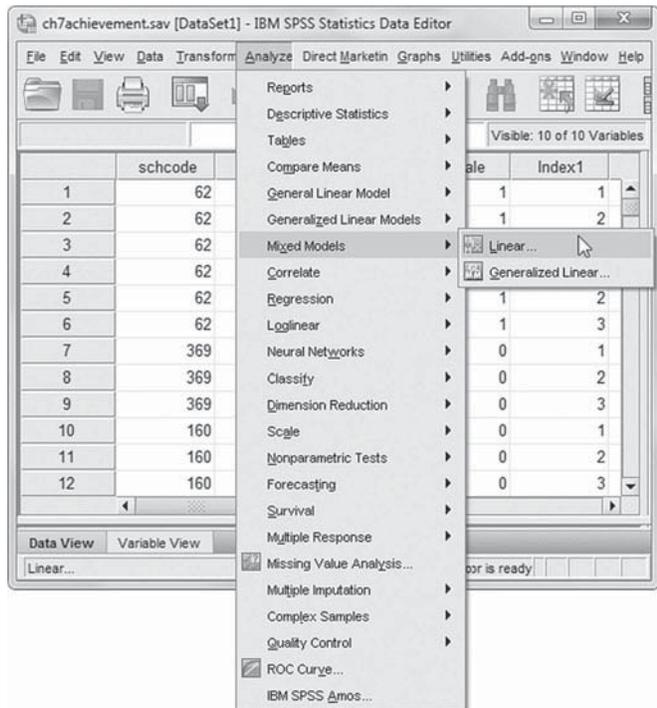
We will next build the model with individual-level and school-level predictors, as specified in Eqs. 7.15–7.17.

**Defining Model 2.2 with IBM SPSS Menu Commands**

Settings will default to those used in Model 2.1.

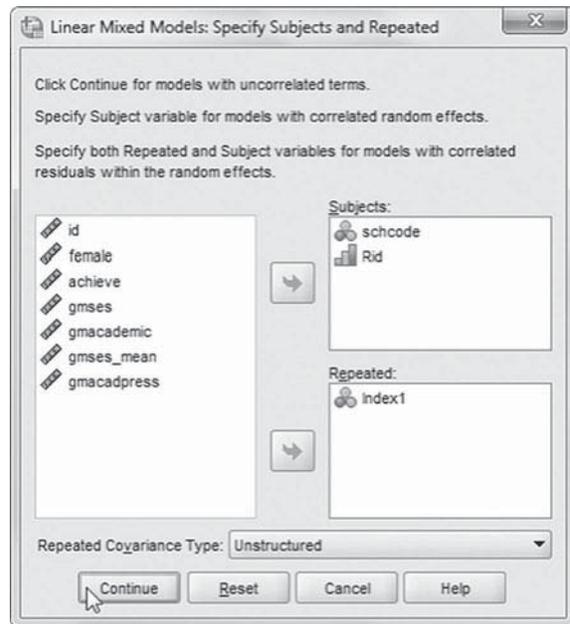
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

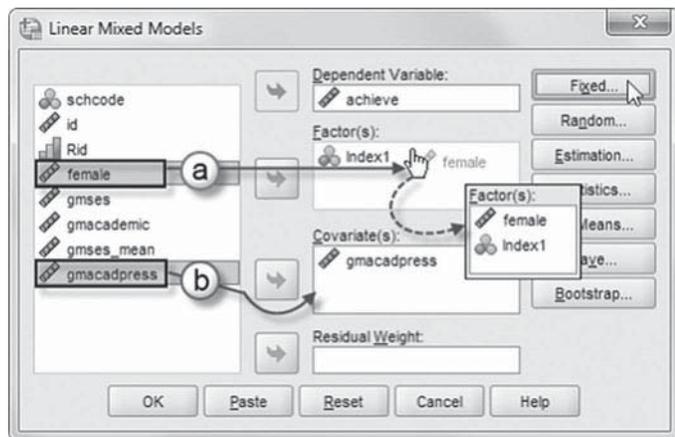
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

We will introduce two additional variables to be used in the model (*female* and *gmacadpress*).

- a. First, click to select *female*, and then “drag” the variable below *Index1* in the *Factor(s)* box.
- b. Next, click to select *gmacadpress*, and then click the arrow button or drag the variable into the *Covariate(s)* box.

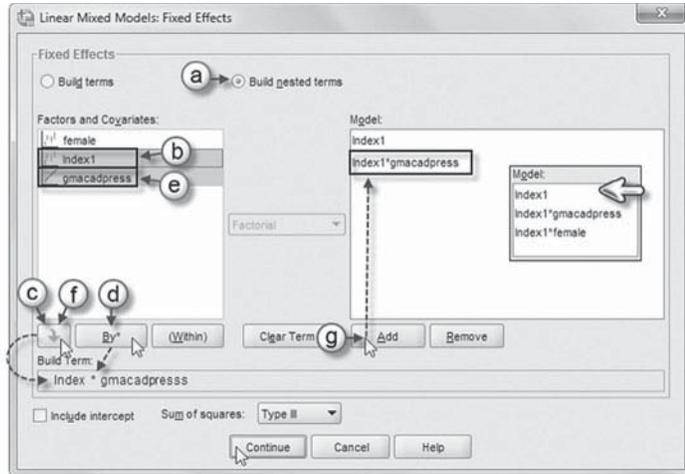


Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

Two cross-level interactions (or nested terms) will be created and added to the model:  $Index1 * gmacadpress$  and  $Index1 * female$ . These interactions will tell us if (a) each school’s academic outcomes affect students’ achievement on each of the subtests (*Index1*) and (b) student achievement on each of the subtests (*Index1*) is affected by gender (*female*).

Add First Interaction to Model 2.2:  $Index1 * gmacadpress$ 

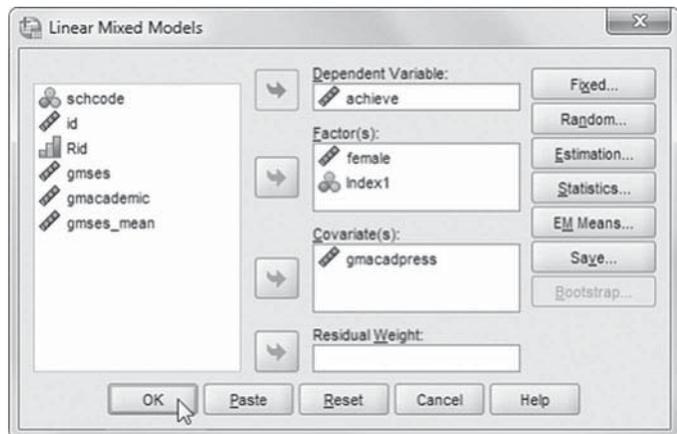
- 4a. Click to select *Build nested terms*.
- b. Click to select the variable *Index1* from the *Factors and Covariates* box.
- c. Then click the arrow button below the *Factors and Covariates* box. This moves *Index1* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- d. Next, click the BY\* button, which will insert the computation command symbol:  $Index1*$ .
- e. Click to select *gmacadpress* from the *Factors and Covariates* box.
- f. Click the arrow button below the *Factors and Covariates* box to move *gmacadpress* into the *Build Term* box and complete the interaction term:  $Index1 * gmacadpress$ .
- g. Click the ADD button to transfer the interaction into the *Model* box.

Add Second Interaction to Model 2.2:  $Index1 * female$ 

Repeat steps 4b to 4g using *Index1* and *female* to create the interaction.

The completed model is shown in the insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

**Interpreting the Output From Model 2.2**

The second model summarized in Table 7.20 presents the adjusted means (i.e., adjusted for gender), which are as follows: reading ( $R = 58.319$ ), math ( $M = 58.466$ ), and language ( $L = 56.109$ ). Because we entered *female* as a factor (categorical variable) in this model (instead of a covariate), we see in the table that there are estimates for *female = 0* (i.e., males), but not for *female = 1* (females). This is because when a categorical variable is used, MIXED declares the last category as the reference group. The output, therefore, will show estimates for the  $k - 1$  categories comprising the variable. If one wishes to change the reference group, it is necessary to recode

TABLE 7.20 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[Index1=1]	58.319	0.306	588.784	190.818	0.000	57.719	58.919
[Index1=2]	58.466	0.320	550.415	182.866	0.000	57.838	59.094
[Index1=3]	56.109	0.322	594.823	174.051	0.000	55.476	56.742
[Index1=1] * gmacadpress	1.590	0.268	289.596	5.929	0.000	1.062	2.118
[Index1=2] * gmacadpress	1.683	0.291	303.129	5.786	0.000	1.111	2.256
[Index1=3] * gmacadpress	1.149	0.289	314.798	3.973	0.000	0.580	1.717
[Index1=1] * [female=0]	-1.964	0.375	2715.088	-5.240	0.000	-2.699	-1.229
[Index1=2] * [female=0]	1.504	0.362	2692.912	4.149	0.000	0.793	2.214
[Index1=3] * [female=0]	3.477	0.378	2706.867	9.197	0.000	2.736	4.218
[Index1=1] * [female=1]	0.000 <sup>b</sup>	0.000	-	-	-	-	-
[Index1=2] * [female=1]	0.000 <sup>b</sup>	0.000	-	-	-	-	-
[Index1=3] * [female=1]	0.000 <sup>b</sup>	0.000	-	-	-	-	-

<sup>a</sup> Dependent variable: achieve.

<sup>b</sup> This parameter is set to 0 because it is redundant.

TABLE 7.21 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[Index1 = 1]	56.356	0.315	649.589	178.770	.000	55.737	56.975
[Index1 = 2]	59.970	0.328	602.740	182.658	.000	59.325	60.615
[Index1 = 3]	59.586	0.332	653.536	179.657	.000	58.935	60.237
[Index1 = 1] * gmacadpress	1.590	0.268	289.596	5.929	.000	1.062	2.118
[Index1 = 2] * gmacadpress	1.683	0.291	303.129	5.786	.000	1.111	2.256
[Index1 = 3] * gmacadpress	1.149	0.289	314.798	3.973	.000	0.580	1.717
[Index1 = 1] * female	1.964	0.375	2,715.088	5.240	.000	1.229	2.699
[Index1 = 2] * female	-1.504	0.362	2,692.912	4.149	.000	-2.214	-0.793
[Index1 = 3] * female	-3.477	0.378	2,706.867	9.197	.000	-4.218	-2.736

<sup>a</sup> Dependent variable: achieve.

the variable. The output suggests that males score significantly lower than females ( $\gamma = -1.964$ ,  $p < .001$ ) in reading ( $Index1 = 1$ , female = 0), but significantly higher than females in math ( $\gamma = 1.504$ ,  $p < .01$ ) and language ( $\gamma = 3.477$ ,  $p < .001$ ). We can also see the redundant effects from the categorical specification used. Turning to the school portion of the model, we can observe that the school's academic press is positively related to students' achievement on each of the subtests (with  $\gamma$  coefficients ranging from 1.149 to 1.683 and  $p < .001$ ).

For comparative purposes, we also provide the output for the same model if we defined female as a covariate instead of as a factor. We provide the syntax for this model in Appendix A. Table 7.21 reveals two noteworthy differences. First, the intercepts will be different. If we take the intercept from Table 7.20 for reading (58.319) and subtract the coefficient for males (1.963), we get a coefficient of 56.356, which represents the reading intercept for males in Table 7.21 (with slight difference due to rounding). Second, the direction of the coefficients regarding

gender is different, reflecting that when female (coded 1) is defined as a covariate, the reference group will now be males (coded 0). The coefficients in the model in Table 7.21 will now refer to females instead of males.

**Testing the Hypotheses**

We can examine whether the effect of gender and academic press are the same across the three subtests. In Table 7.22, we present results from Model 2.2, where female is treated as categorical rather than as a covariate (which produces different coefficients for *Index1* but the same interpretation). The Type III (sum of squares) tests of the fixed effects provide this information. The significant *F* ratios for female and academic press suggest the impact of each predictor is not the same across the subtests. We would, therefore, reject the null hypothesis for each variable.

**Correlations Between Tests at Each Level**

Finally, we turn our attention to the covariance components table (Table 7.23 from Model 2.3). We can specify the UN covariance matrices as unstructured with a correlation metric (UNR) and examine the correlations between the tests in the off-diagonals of the matrix at each level. Within schools, the correlations between the subtests are all moderate and significant (ranging from 0.64 to 0.69 and  $p < .001$ ). Between schools, the correlations between subtests are also significant ( $p < .001$ ) and are considerably stronger than the individual-level correlations (with coefficients ranging from 0.86 to 0.89).

**TABLE 7.22 Model 2.2 (Female Treated as Categorical) Type III Tests of Fixed Effects<sup>a</sup>**

Source	Numerator <i>df</i>	Denominator <i>df</i>	<i>F</i>	Sig.
Index1	3	294.230	19,787.638	.000
Index1 * gmacadpress	3	296.041	14.037	.000
Index1 * female	3	2,703.990	103.927	.000

<sup>a</sup> Dependent variable: achieve.

**TABLE 7.23 Estimates of Covariance Parameters<sup>a</sup>**

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Repeated Measures	Var(1)	88.632	2.558	34.651	.000	83.758	93.790
	Var(2)	80.281	2.327	34.495	.000	75.847	84.975
	Var(3)	88.533	2.550	34.722	.000	83.674	93.674
	Corr(2, 1)	0.666	0.011	58.781	.000	0.644	0.688
	Corr(3, 1)	0.638	0.012	52.958	.000	0.614	0.661
	Corr(3, 2)	0.691	0.011	64.875	.000	0.669	0.711
Index1 [subject = schcode]	Var(1)	8.297	1.598	5.192	.000	5.688	12.103
	Var(2)	12.569	1.902	6.608	.000	9.343	16.909
	Var(3)	11.302	1.815	6.226	.000	8.250	15.484
	Corr(2, 1)	0.894	0.038	23.424	.000	0.789	0.948
	Corr(3, 1)	0.862	0.045	19.014	.000	0.742	0.928
	Corr(3, 2)	0.875	0.034	25.704	.000	0.790	0.928

<sup>a</sup> Dependent variable: achieve.

### Defining Model 2.3 with IBM SPSS Menu Commands

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

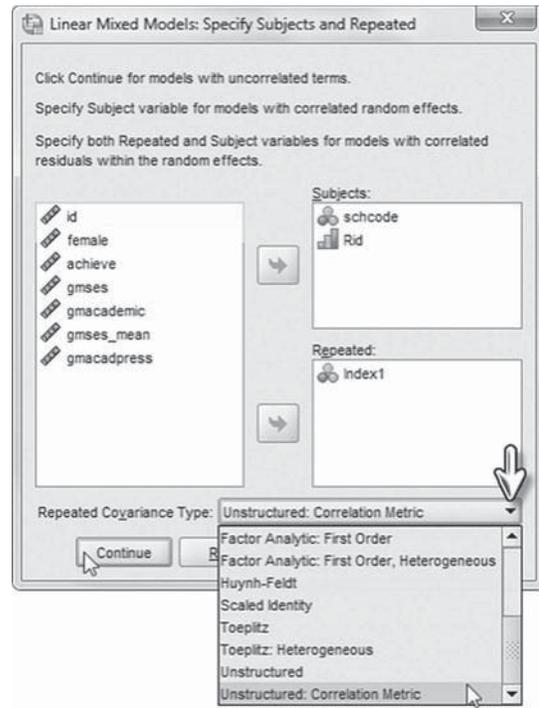
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

We will change the *Repeated Covariance Type* by clicking the pull-down menu and selecting *Unstructured: Correlation Metric* (UNR).

The *Unstructured: Correlation Metric* has heterogeneous variances and correlations (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



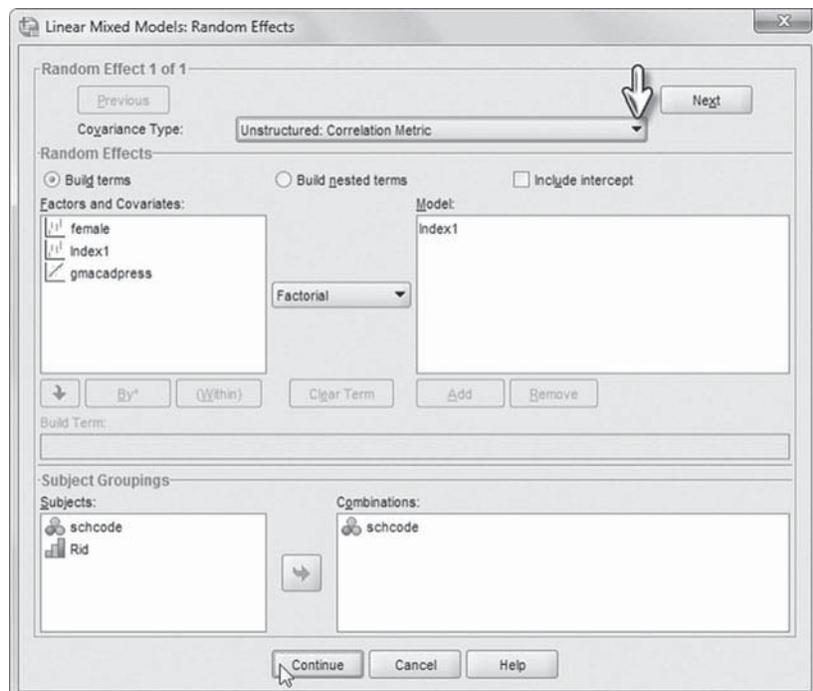
3. The *Linear Mixed Models* dialog box settings default to those used in the prior model.

Click the RANDOM button to access the random-effects main screen.

4. From the *Random Effect 1 of 1* screen, change the covariance type by clicking the pull-down menu and selecting *Unstructured: Correlation Metric*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Click the OK button to run the model.



### Investigating a Random Slope

As in our other models, it is easy to investigate a random gender slope across schools using the previous model formulation. We can specify female as randomly varying by adding it as a random effect at the school level (`/RANDOM Index1 female`). When we do so, we can obtain a covariance estimate for gender with each outcome in the model, as well as a variance component for the gender slope, which is significant ( $p < .05$ ). We might then investigate whether other school predictors (e.g., *gmacadpress* or *gmses\_mean*) explain variation in this slope. We can add such cross-level interactions into the model as three-way interactions (*gmacadpress\*female\*Index1*). If we instead add a school-level predictor as a typical cross-level interaction (*gmacadpress\*female*), this specification becomes a test of an equality constraint across the three outcomes. Tests of fit between the two formulations should be conducted with ML estimation. We leave this investigation to interested readers to conduct (see syntax in Appendix A).

### Defining a Parallel Growth Process

Our third example concerns specifying two growth processes within one model. Similar to our other multivariate models in this chapter, this can be accomplished by stacking the measures of reading and math in one “achievement” column. Once again, we can use an *index* variable to specify the repeated measures and then we can differentiate the reading or math repeated measures by using a dichotomous variable to define each achievement domain. In this case, we will develop an achievement variable consisting of three repeated measures of students’ reading and math scores. We can specify each type of achievement outcome by using a dummy coded variable which we will call *math* (coded 0 = read, 1 = math). This Level 1 specification facilitates building a separate explanatory model for each outcome.

### The Data

For purposes of demonstration, in this example we use a subset of our data (see Table 7.24), as it is relatively easy to run into problems of “insufficient” computer memory because of the complexity of the data structure (i.e., six lines of data for each individual, and 1,836 individuals nested within 30 schools). We also recoded the individual identifiers (*Rid*) within their school groups (1,2,...,n) in order to reduce required computer memory space to estimate the model. We provide the observed means for the reading and math indicators in Table 7.25.

**TABLE 7.24** Data Definition of *ch7PGachievement.sav* ( $N = 1,836$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>id</i>	Individual	Individual student identifier (1,836 students).	Integer	Ordinal
<i>Rid</i>	Individual	Recoded individual student identifiers ( <i>id</i> ) with the school identifier ( <i>schcode</i> ) identifying students within their school groups (1,2,...,30).	(1,2,3,...,30)	Ordinal
<i>schcode</i>	School	School identifier (30 schools).	Integer	Ordinal
<i>female</i>	Individual	Demographic predictor variable representing students gender.	0 = Male 1 = Female	Scale
<i>schcontext</i>	School	Variable measuring school composition.	-1.87 to 1.85	Scale
<i>Index1</i>	Within Individual	Identifier variable resulting from indexing the <i>id</i> individual identifier (1 to 1,836) to create a new identifier to define reading (1-3) and math (4-6).	(1, 2, 3, 4, 5, 6)	Ordinal

(Continued)

TABLE 7.24 (Continued)

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>achieve</i>	Within Individual	Dependent variable measuring achievement on three achievement outcomes for math and reading.	480 to 849	Scale
<i>math</i>	Within Individual	Dichotomous variable specifying student achievement in math or reading.	0 = Read 1 = Math	Nominal
<i>time</i>	Within Individual	Variable representing three linear occasions in time measuring students math achievement.	0 = First Time 1 = Second Time 2 = Third Time	Scale
<i>orthtime</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into an orthogonal linear $-(1, 0, 1)$ sequence.	-1 = First Time 0 = Second Time 1 = Third Time	Scale
<i>orthquadtime</i>	Within Individual	Recoded <i>time</i> variable from three occasions in time (0, 1, 2) into an orthogonal quadratic $(1, -2, 1)$ sequence.	1 = First Time -2 = Second Time 1 = Third Time	Scale

<sup>a</sup> Within Individual = Level 1; Individual = Level 2; School = Level 3.

TABLE 7.25 Observed Reading and Math

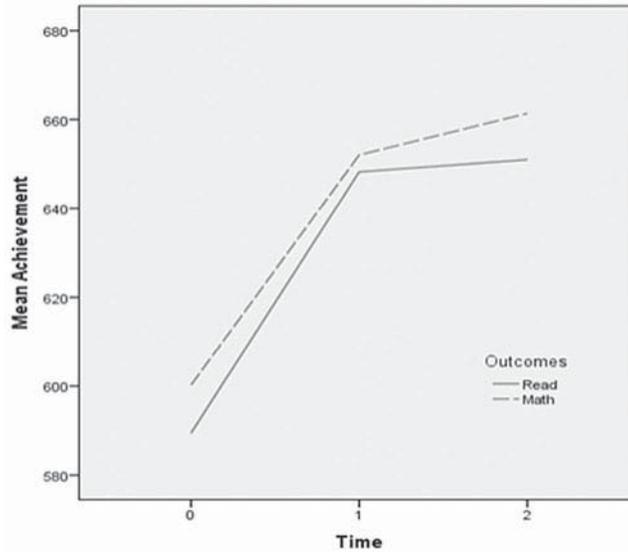
Achieve			
Index	Mean	N	Std. Deviation
Read1	589.44	1,836	34.426
Math1	600.25	1,836	39.638
Read2	648.24	1,836	29.929
Math2	652.06	1,836	30.240
Read3	650.99	1,836	33.720
Math3	661.38	1,836	39.874
Total	633.73	11,016	44.686

## Research Questions

One advantage of specifying a parallel growth model is that researchers can examine the extent to which concurrent individual changes in math and reading are mutually interrelated, as well as the extent to which these changes may be explained by the hypothesized individual- and school-level factors in the model. In particular, the goal was to define a single simultaneous model of parallel student growth in reading and math and to investigate how the specific school factors might moderate student growth. The study addresses one primary research question: Do school measures of teaching effectiveness and school instructional practices moderate student growth trajectories in reading and math?

## Preparing the Data

In specifying this type of stacked outcome, we generally would begin with a horizontal data set with the three repeated measures of reading (*Read1*, *Read2*, and *Read3*) and three repeated measures of math (*Math1*, *Math2*, and *Math3*) specified horizontally. We then use the *Data: Restructure* procedure in IBM SPSS to create a single measure of achievement that is stacked vertically with an *index* variable that specifies the six measures of reading and math, which will now comprise that achievement dependent variable (see Chapter 2). The restructuring procedure



**FIGURE 7.2** Observed math and reading trajectories.

will automatically refer to this new index variable as *Index1*. The first three numbers of the *Index1* variable (1–3) will represent reading, while the last three numbers (4–6) will represent math.

After the data are restructured, we can use the *Index1* variable to create two other necessary variables for defining the parallel growth model. First, we create a dummy variable to define the two outcomes (read, math), which we label *math*. Using the recode command, from the *Index1* variable, we can recode 1–3 to 0 to represent the reading indicators (*math* = 0), and we can recode 4–6 to 1 to represent the math indicators (*math* = 1). We also need to define the time-related polynomials (remembering that we can have  $k-1$  polynomials, or two in this example). In growth modeling, we can also estimate  $k-1$  random effects. Since there are three time points in this example, we can estimate two random effects. This allows us to estimate a random effect for the intercept and a random effect for either the linear component or the quadratic component.

Once again using the *Index1* variable, we can recode 1–3 into a linear *time* variable coded as 0, 1, 2 to define the repeated measures for reading. Then we also recode 4–6 as 0, 1, 2 to define the repeated measures of math. Readers can confirm for themselves the coding of the variables *Index1* (coded 1–6), *math* (coded 0–1), and *time* (coded 0–2) in the data set for this example. We also created some additional variables (i.e., a quadratic time component and orthogonal polynomials) to use as we develop our analyses, but this should be sufficient to get started with the model specification of a parallel growth process.

We illustrate the actual growth trajectories in Figure 7.2. From the trajectories, we can see there is considerable slowing over time (between the second and third observations). This suggests we should add a quadratic component to the model.

### Model 3.1: Specifying the Time Model

To fit our proposed model, we can declare a three-level model, with schools at the highest level [Subject(*schcode*)] and the repeated measures on students combined at Levels 1 and 2 (Leyland, 2004). At the lowest level, we use the *Repeated* dialog box to specify the repeated measures of achievement using the *Index1* variable. This addresses the nesting of the repeated measures within individuals [Subject(*schcode*\**Rid*)]. We can make use of an indicator variable (i.e., in this case, a dummy-coded variable to define the multivariate response structure; Hox, 2010). We can define the indicator variable ( $I_{ijk}$ ) as follows:

$$Y_{ijk} = I_{ijk}Y_{Mjk} + (1 - I_{ijk})Y_{Rjk}, \quad (7.18)$$

where the indicator takes on a value of 1 if  $i$  is the math test and 0 otherwise (Leyland, 2004; Raudenbush & Bryk, 2002) for individual  $j$  in school  $k$ . By using a “no intercept” designation (*NOINT*) in specifying the model’s fixed effects, we ensure that an intercept is estimated for each response variable.

As in the previous example, we do not model variance at the student level in this formulation. More specifically, we do not include a Random subcommand with subjects (*schcode\*Rid*) specified. By specifying the dummy-coded indicator as randomly varying (Random = *math*) and specifying the Subject subcommand *schcode* [Subject(*schcode*)], we can define the two achievement intercepts at the school level. In Equation 7.19, we define a separate model for each outcome, which includes the time-related orthogonal polynomials at the individual level (*orthtime* and *orthquadtime*) and the randomly varying reading and math variance parameters ( $u_{R0k}$  and  $u_{M0k}$ ) at the school level. For individual  $j$  in school  $k$  at time  $t$ , then, we have the following equations for reading and math:

$$\begin{aligned} Y_{Rtkj} &= \gamma_{R00} + \gamma_{R10}orthtime_{tjk} + \gamma_{R20}orthquadtime_{tjk} + u_{R0k}, \\ Y_{Mtkj} &= \gamma_{M00} + \gamma_{M10}orthtime_{tjk} + \gamma_{M20}orthquadtime_{tjk} + u_{M0k}. \end{aligned} \quad (7.19)$$

Because the outcomes are represented as a dummy variable in the multivariate formulation, if we do not multiply the predictors by the dummy outcome (e.g., *math\*orthtime* and *math\*orthquadtime*) in specifying the combined, single-equation model, we will obtain only one set of coefficients, which amounts to imposing equality constraints that the regression coefficients are the same for both outcomes.

At the school level, we assume the intercept residual variances for reading and math are normally distributed with means of 0. The time-related polynomials are fixed at the school level. We will select a UN covariance matrix to examine the variances and covariances between tests at the school level.

$$\begin{bmatrix} u_{R0k} \\ u_{M0k} \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{R_u}^2 & \sigma_{RM_u} \\ \sigma_{RM_u} & \sigma_{M_u}^2 \end{bmatrix} \right) \quad (7.20)$$

Within individuals, we will specify the covariance structure as autoregressive (AR1), with the constraint  $|\rho| \leq 1$  imposed for stationarity. This will include an estimated correlation between repeated measures over time, but result in only two parameters to estimate (the variance  $\sigma^2$  and the  $\rho$  correlation). Of course, we could also investigate other possible covariance structures.

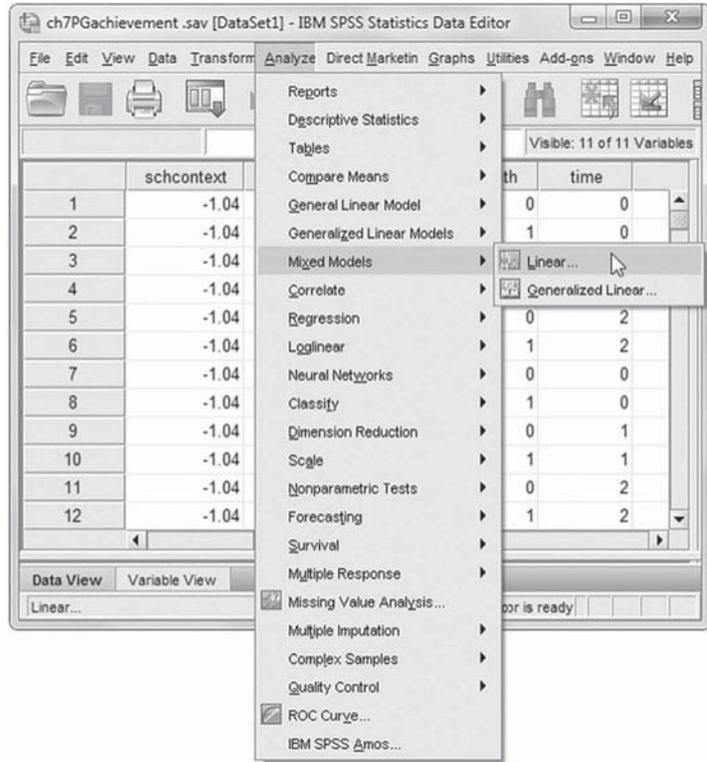
$$\sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \rho^4 & \rho^5 \\ \rho & 1 & \rho & \rho^2 & \rho^3 & \rho^4 \\ \rho^2 & \rho & 1 & \rho & \rho^2 & \rho^3 \\ \rho^3 & \rho^2 & \rho & 1 & \rho & \rho^2 \\ \rho^4 & \rho^3 & \rho^2 & \rho & 1 & \rho \\ \rho^5 & \rho^4 & \rho^3 & \rho^2 & \rho & 1 \end{bmatrix} \quad (7.21)$$

**Defining Model 3.1 with IBM SPSS Menu Commands**

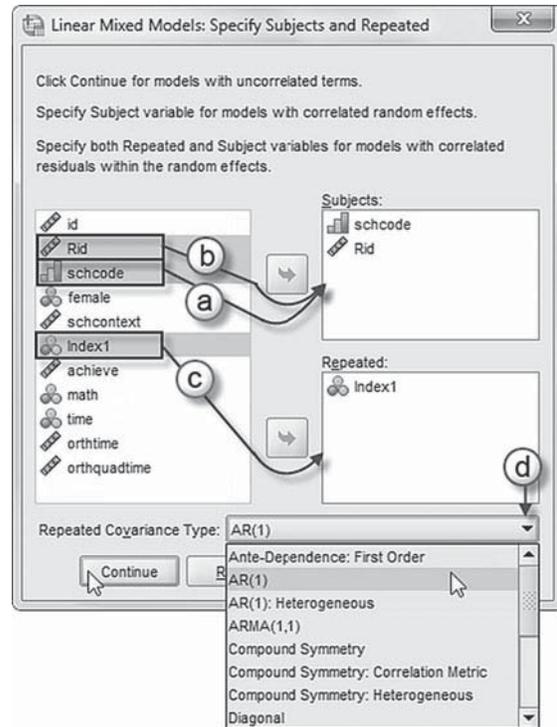
Launch the IBM SPSS program application, and select the *ch7PG-achievement.sav* data file.

1. Go to the toolbar and select **ANALYZE, MIXED MODELS, LINEAR**.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.
  - a. For this model, we will designate two subject identifiers for the model (*schcode* and *Rid*). To facilitate reading of the output tables, we will enter the variables in a particular sequence. First, click to select *schcode*, and then click the right-arrow button to move the variable into the *Subjects* box.
  - b. Next, click to select *Rid*, and then click the right-arrow button to move the variable into the *Subjects* box.
  - c. The *Repeated* box allows specifying variables that identify repeated observations. For this model, *Index1* identifies the repeated measures, which define the math and read achievement. Click to select *Index1*, and then click the right-arrow button to move the variable into the *Repeated* box.



The combination of values for *schcode*, *Rid*, and *Index1* defines a particular student from a particular school across the three math and reading measures.

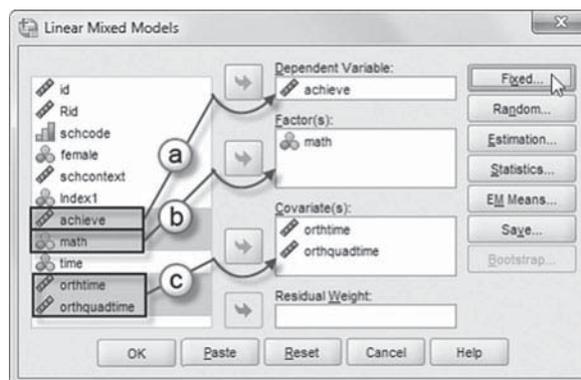
- d. The *Repeated Covariance Type* specifies a model's covariance structure. For this model, we will use the autoregressive covariance matrix,  $AR(1)$ . Click the pull-down menu to select the autoregressive covariance matrix,  $AR(1)$ , as the *Repeated Covariance Type*.

$AR(1)$  is a first-order autoregressive structure with homogenous variances. The correlation between any two elements is equal to  $\rho$  for adjacent elements,  $\rho^2$  for elements that are separated by a third, and so on.  $\rho$  is constrained so that  $-1 < \rho < 1$  (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, and covariates, as well as access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

- a. For this model, we will use *achieve* as the dependent variable. Click to select the *achieve* variable from the left column listing. Then click the right-arrow button to transfer *achieve* into the *Dependent Variable* box.



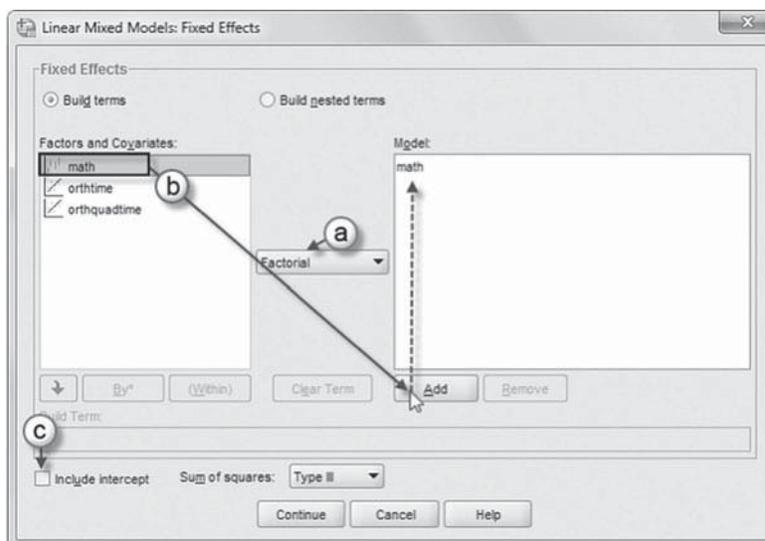
Now we will introduce three additional variables to be used in the model (*math*, *orthtime*, and *orthquadtime*).

- b. First, click to select *math*, and then click the right-arrow button (or “drag” the variable) to move the variable into the *Factor(s)* box.  
c. Next, click to select *orthtime* and *orthquadtime*. Then click the right-arrow button (or “drag” the variables) to move them into the *Covariate(s)* box.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, we will retain the default *Factorial* setting, which creates all possible interactions and main effects of the specified variable (IBM Corporation, 2012).

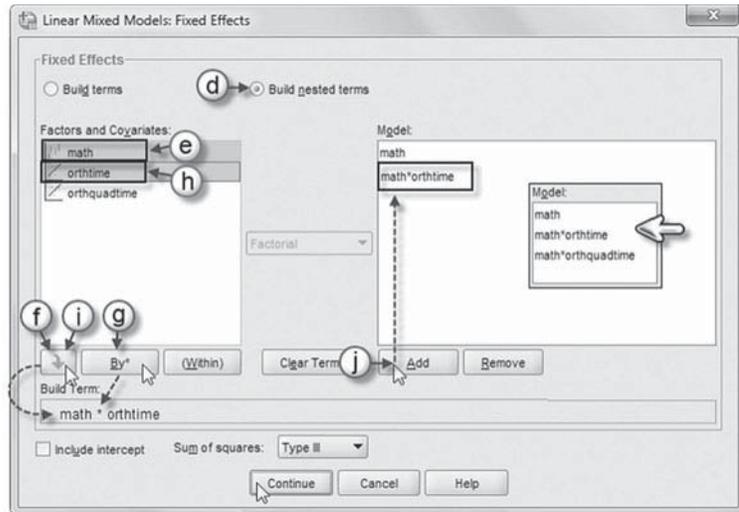
- b. Now click to select *math* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.  
c. To create the no-intercept model, uncheck the *Include intercept* option.



Two interactions (or nested terms) will be created and added to the model:  $math*orthtime$  and  $math*orthquadtime$ . These interactions will tell us if each student's achievement ( $math$ ) trajectories have linear ( $orthtime$ ) and quadratic ( $orthquadtime$ ) components.

### Add First Interaction to Model 3.1: $math*orthtime$

- d. Click to select *Build nested terms*.
- e. Click to select the variable  $math$  from the *Factors and Covariates* box.
- f. Then click the arrow button below the *Factors and Covariates* box. This moves  $math$  into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- g. Next, click the BY\* button, which will insert the computation command symbol:  $math*$ .
- h. Click to select  $orthtime$  from the *Factors and Covariates* box.
- i. Click the arrow button below the *Factors and Covariates* box to move  $orthtime$  into the *Build Term* box and complete the interaction term:  $math*orthtime$ .
- j. Click the ADD button to transfer the interaction into the *Model* box.



### Add Second Interaction to Model 3.1: $math*orthquadtime$

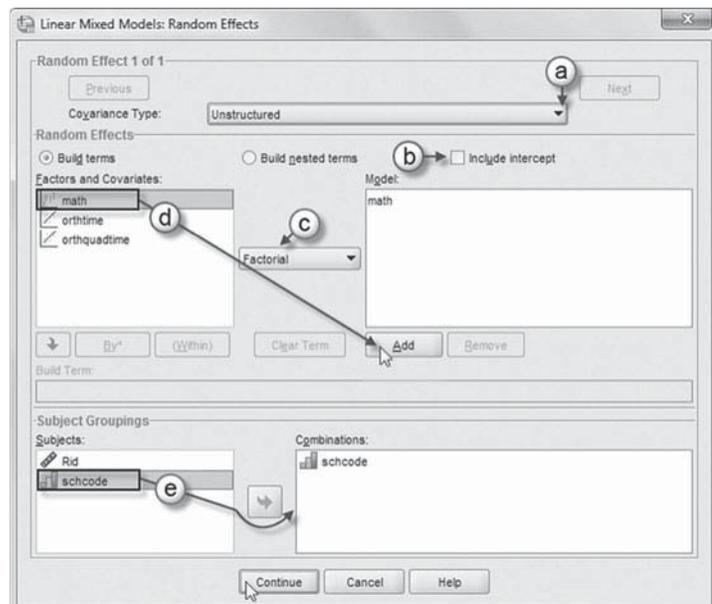
Repeat steps 4e to 4j using  $math$  and  $orthquadtime$  for the interaction.

The completed model is shown in the insert. Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen, which is the default when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.
  - a. Begin by specifying the covariance structure from the default variance components (VC) to *Unstructured*. Click the pull-down menu and select *Unstructured* (UN).

*Unstructured* is a completely general covariance matrix (IBM Corporation, 2012).



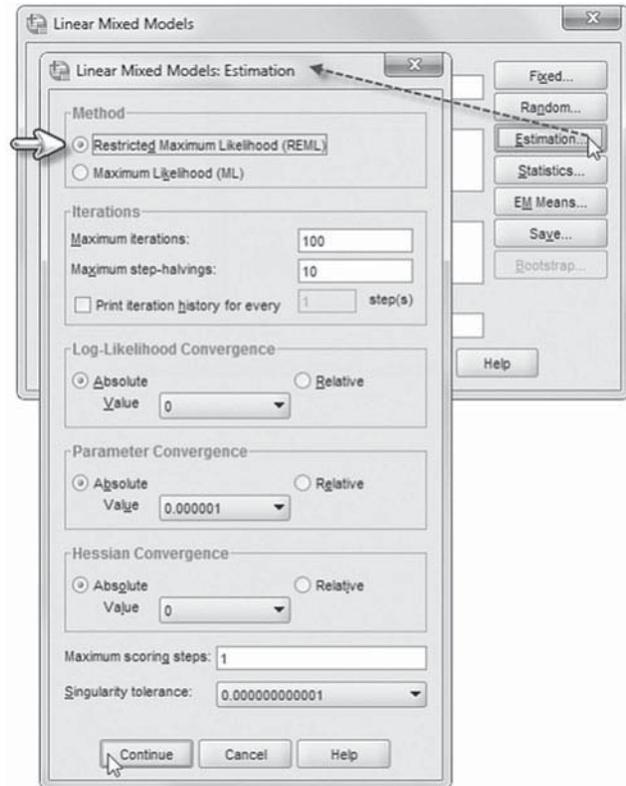
- b. We want the intercept to be excluded from the model, so we will retain the current default setting.
- c. We will also retain the default *Factorial* setting, which creates all possible interactions and main effects of the specified variable (IBM Corporation, 2012).
- d. Click to select *math*, and then click the ADD button to move the variable into the *Model* box.
- e. The *Subject Groupings* box displays the *schcode* and *Rid* variables that were specified as subject variables in the *Specify Subjects and Repeated* dialog box show in step 2a. We will specify *schcode* as the subject for the random-effects Level 1 part of this model. Click to select *schcode*, and then click the right-arrow button to move the variable into the *Combinations* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

6. Click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: ML or REML.

We will use the default setting of REML to estimate the models.

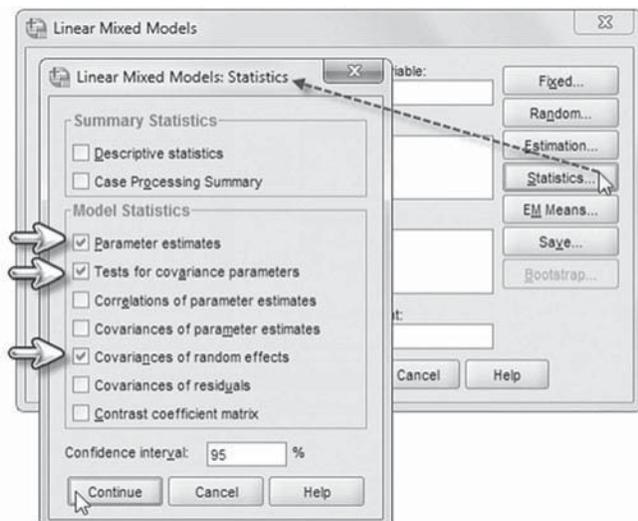
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



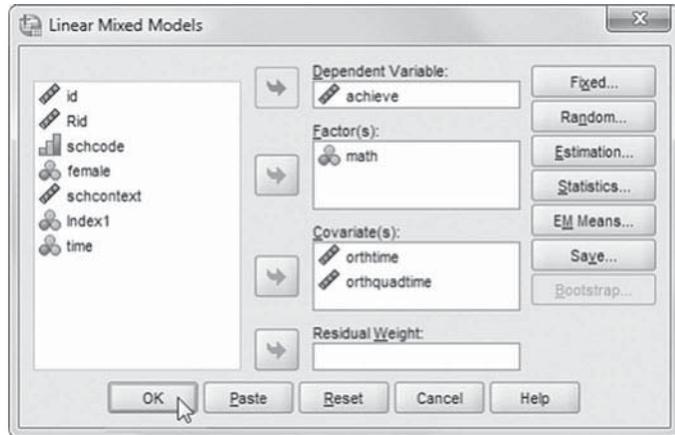
7. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



- Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 3.1**

The model will then have 11 parameters to estimate. This is confirmed in the model dimension table (Table 7.26).

We will specify an unstructured covariance matrix at Level 2 and a first-order autoregressive covariance matrix to simplify the structure at Level 1. Table 7.27 presents the results of this first parallel growth model with orthogonal time-related polynomials. We can see that polynomials are significant for both reading and math, suggesting they should be retained in further analyses.

We provide the variance components for this initial model in Table 7.28.

**TABLE 7.26 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	math	2		2		
	math * orthtime	2		2		
	math * orthquadtime	2		2		
Random Effects	math	2	Unstructured	3	schcode	
Repeated Effects	Index 1	6	First-Order Autoregressive	2	schcode * Rid	1,836
Total		14		11		

<sup>a</sup> Dependent variable: achieve.

**TABLE 7.27 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[math = 0]	635.213	3.016	29.105	210.611	.000	629.046	641.381
[math = 1]	643.254	3.319	28.977	193.810	.000	636.465	650.042
[math = 0] * orthtime	30.770	0.436	10,678.551	70.591	.000	29.916	31.625
[math = 1] * orthtime	30.562	0.436	10,678.551	70.113	.000	29.707	31.416
[math = 0] * orthquadtime	-9.341	0.180	8,582.880	-51.880	.000	-9.694	-8.988
[math = 1] * orthquadtime	-7.080	0.180	8,582.880	-39.323	.000	-7.433	-6.727

<sup>a</sup> Dependent variable: achieve.

TABLE 7.28 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	AR1 Diagonal	1,046.835	23.142	45.234	.000	1,002.445	1,093.190
	AR1 rho	0.760	0.006	130.537	.000	0.748	0.771
math [subject = schcode]	UN (1, 1)	255.352	71.433	3.575	.000	147.579	441.831
	UN (2, 1)	274.344	77.453	3.542	.000	122.539	426.150
	UN (2, 2)	312.753	86.732	3.606	.000	181.613	538.587

<sup>a</sup> Dependent variable: achieve.

### Model 3.2: Adding the Predictors

We can also add one or more student-level  $X_{jk}$  predictors and school-level  $W_k$  predictors. In this case, we will add female as a student-level predictor and school composition (*schcontext*). The combined models for reading and math are summarized in Equation 7.22.

$$\begin{aligned}
 Y_{Rtjk} &= \gamma_{R00} + \gamma_{R01}schcontext_k + \gamma_{R10}orthtime_{tjk} + \gamma_{R20}orthquadtime_{tjk} + \gamma_{R30}female_{jk} \\
 &\quad + \gamma_{R11}schcontext_k^*orthtime_{tjk} + \gamma_{R31}female_{jk}^*orthtime_{tjk} + u_{R0k} \\
 Y_{Mtjk} &= \gamma_{M00} + \gamma_{M01}schcontext_k + \gamma_{M10}orthtime_{tjk} + \gamma_{M20}orthquadtime_{tjk} + \gamma_{M30}female_{jk} \\
 &\quad + \gamma_{M11}schcontext_k^*orthtime_{tjk} + \gamma_{M31}female_{jk}^*orthtime_{tjk} + u_{M0k}
 \end{aligned} \tag{7.22}$$

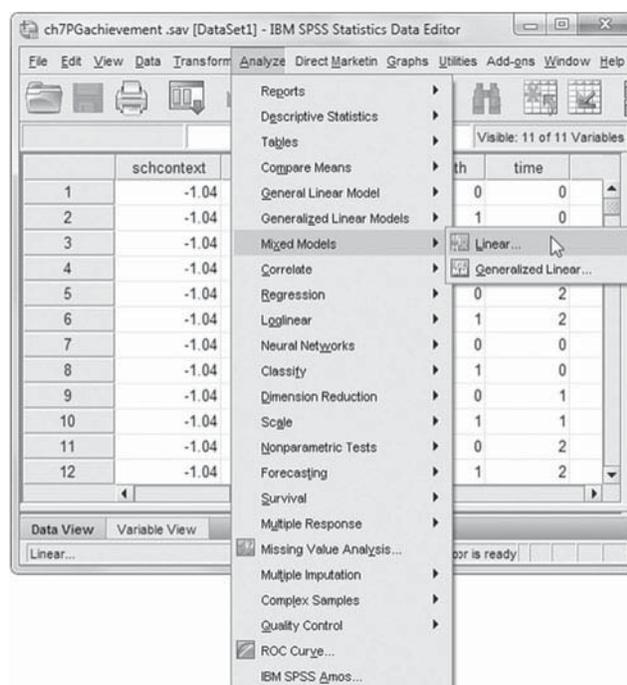
This suggests we have separate intercepts and regression coefficients for the school-level models and within-group and between-group predictors, again with representing random effects associated with explaining school achievement for each subtest. In Equation 7.22, we have added fixed-effect interactions for *female* and school context (*schcontext*) and linear growth (*orthtime*), but we again consider linear and quadratic growth as fixed between schools. We also consider the slope coefficients describing the relationship between female and each subtest to be fixed between schools.

### Defining Model 3.2 with IBM SPSS Menu Commands

**Note:** Settings will default to those used in Model 3.1.

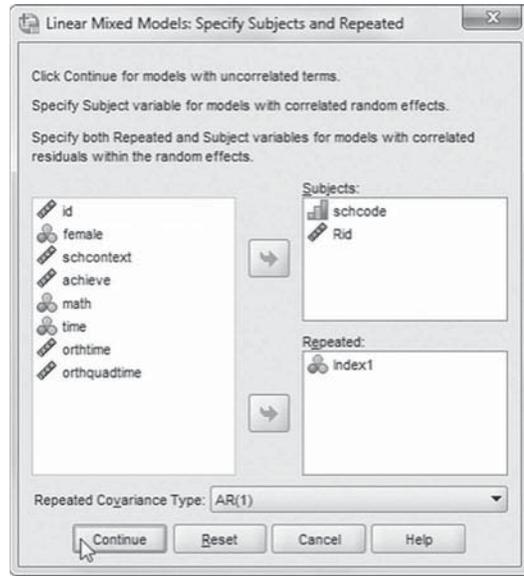
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



- The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

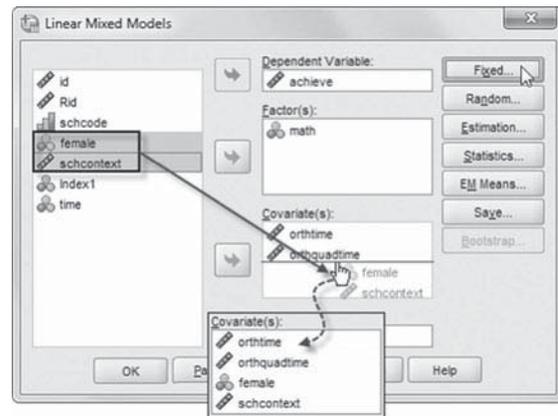
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



- Within the *Linear Mixed Models* main screen we will introduce two additional variables to be used in the model (*female* and *schcontext*).

Click to select *female* and *schcontext*, and then click the right-arrow button (or “drag” the variables) to move them into the *Covariate(s)* box.

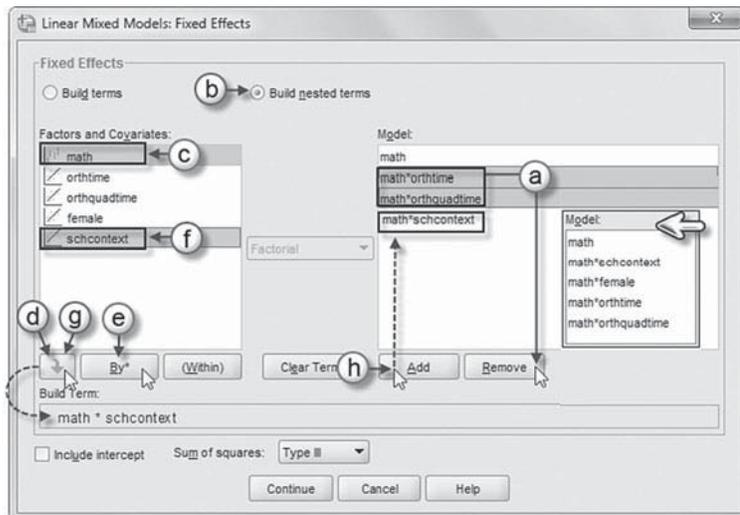
Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



Two new interactions (or nested terms) will be created and added to the model: *math\*schcontext* and *math\*female*. These interactions will tell us if:(a) students’ achievement (*math*) is affected by school composition (*schcontext*) and (b) students’ achievement (*math*) is affected by gender (*female*).

Add First Interaction to Model 3.2: *math\*schcontext*

- The *Linear Mixed Models: Fixed Effects* dialog box displays the default setting from the prior model. To facilitate reading of the output tables, we will rearrange the sequence order of the variables by removing the interactions. Click to select them and then click the REMOVE button.
- Click to select *Build nested terms*.





Add Sixth Interaction to Model 3.2: *female\*math\*orthtime*

Repeat steps 4j to 4q using *female*, *math*, and *orthtime* for the interaction.

The completed model is shown in the insert.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

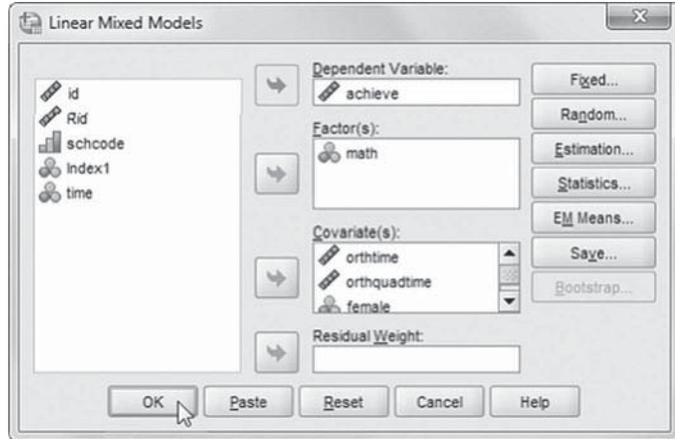


TABLE 7.29 Model Dimension<sup>a</sup>

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	math	2		2		
	math * schcontext	2		2		
	math * female	2		2		
	math * orthtime	2		2		
	math * orthquadtime	2		2		
	math * orthtime * schcontext	2		2		
	math * orthtime * female	2		2		
Random Effects	math	2	Unstructured	3	schcode	
Repeated Effects	Index1	6	First-Order Autoregressive	2	schcode * Rid	1,836
Total		22		19		

<sup>a</sup> Dependent variable: achieve.

**Interpreting the Output From Model 3.2**

Once again, we note that the multivariate outcome *math* is crossed with the within-school predictors (*X*) and between-school (*W*) predictors in the combined model (e.g., *math\*female*), if we wish to have a separate term for each outcome. This is important in our example since one of our goals is to examine whether the effects of gender are the same or different on each outcome. If we wished to impose equality constraints on the predictor, however, that variable (e.g., *female*) could be added directly into the model. Readers will recall that we provided an example of this type of specification in our previous example. The final model as estimated has 19 parameters, which we can confirm in Table 7.29.

We present the fixed effects in Table 7.30. Because of the coding of the orthogonal time variables, the intercepts can be interpreted as the adjusted grand means for reading and math. Both gender and school composition affected levels of reading and math significantly ( $p < .001$ ). We can also observe that gender had a significant effect on growth over time; that is, females experienced greater growth in both subject areas than their male peers ( $p < .001$ ).

TABLE 7.30 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
[math=0]	627.765	1.707	34.232	367.771	0.000	624.297	631.233
[math=1]	636.377	2.086	31.171	305.101	0.000	632.124	640.630
[math=0] * schcontext	-12.006	1.354	29.039	-8.864	0.000	-14.776	-9.236
[math=1] * schcontext	-12.759	1.678	27.933	-7.601	0.000	-16.197	-9.320
[math=0] * female	8.094	1.208	2235.499	6.700	0.000	5.725	10.463
[math=1] * female	6.675	1.209	2237.038	5.523	0.000	4.305	9.045
[math=0] * orthtime	29.674	0.587	10678.166	50.525	0.000	28.523	30.825
[math=1] * orthtime	29.577	0.587	10678.166	50.360	0.000	28.426	30.729
[math=0] * orthquadtime	-9.341	0.180	8583.859	-51.979	0.000	-9.694	-8.989
[math=1] * orthquadtime	-7.080	0.180	8583.859	-39.398	0.000	-7.433	-6.728
[math=0] * orthtime * schcontext	-1.270	0.404	10696.211	-3.140	0.002	-2.062	-0.477
[math=1] * orthtime * schcontext	-0.485	0.404	10696.211	-1.199	0.231	-1.277	0.308
[math=0] * orthtime * female	2.260	0.834	10696.211	2.709	0.007	0.625	3.895
[math=1] * orthtime * female	2.057	0.834	10696.211	2.466	0.014	0.422	3.692

<sup>a</sup> Dependent Variable: achieve.

TABLE 7.31 Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Repeated Measures	AR1 diagonal	1,033.054	22.766	45.377	0.000	989.384	1,078.652
	AR1 rho	0.758	0.006	129.376	0.000	0.746	0.769
math [subject = schcode]	UN (1, 1)	58.311	19.721	2.957	0.003	30.052	113.144
	UN (2, 1)	68.404	23.636	2.894	0.004	22.078	114.730
	UN (2, 2)	98.797	31.117	3.175	0.001	53.291	183.164

<sup>a</sup> Dependent Variable: achieve.

Between schools, there is some evidence in this small school sample that students in schools with more challenging student composition demonstrated significantly less growth in reading over time than their peers in schools at the grand mean of composition ( $\gamma = -1.270$ ,  $p < .01$ ); however, school composition did not affect student growth in math over time ( $p > .05$ ).

Table 7.31 provides the variance component estimates for this model.

Figure 7.3 provides the predicted values of achievement (which we can save within the data set) to illustrate that the model captures the shape of the observed growth trajectories quite well.

### Further Considerations

We note that we can also specify the *orthtime* parameter as randomly varying. Freeing this parameter will add two estimated parameters to estimate. The resulting covariance parameters in Table 7.32 indicate that the random parameter is significant (one-tailed tests,  $p < .05$ ), which

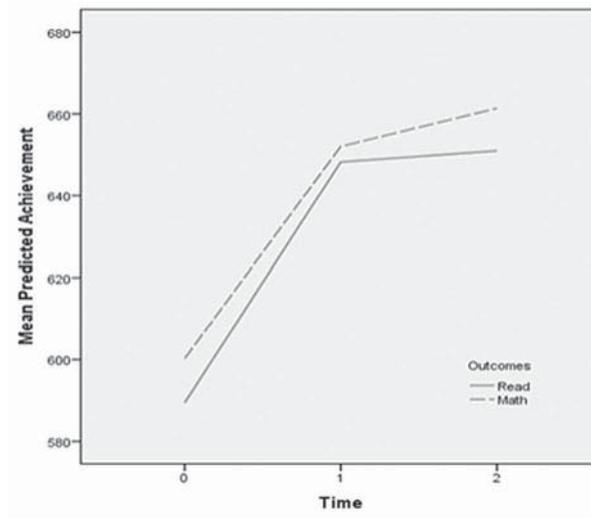


FIGURE 7.3 Predicted achievement values.

TABLE 7.32 Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Repeated Measures	AR1 Diagonal	1,033.674	22.847	45.244	.000	989.851	1,079.437
	AR1 rho	0.759	0.006	129.797	.000	0.747	0.770
math + math * orthtime [subject = schcode]	Var: [math = 0]	41.234	14.873	2.772	.006	20.334	83.615
	Var: [math = 1]	79.831	26.117	3.057	.002	42.043	151.583
	Var: [math = 0] * orthtime	3.287	1.911	1.720	.085	1.052	10.272
	Var: [math = 1] * orthtime	6.726	3.440	1.955	.051	2.468	18.328
	CSH rho	0.770	0.103	7.508	.000	0.483	0.907

<sup>a</sup> Dependent variable: achieve.

suggests that we could continue to build random slope and intercept models to explain this variation across schools. Readers will note we had to change the Level 2 covariance matrix [heterogeneous compound symmetry (CSH)] to simplify model estimation, given the small number of schools in our example (instructions provided at the end of this section). We will stop our investigation at this point.

**Defining Model 3.3 with IBM SPSS Menu Commands**

Settings will default to those used in Model 3.2.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* main screen displays the default settings from the prior model.

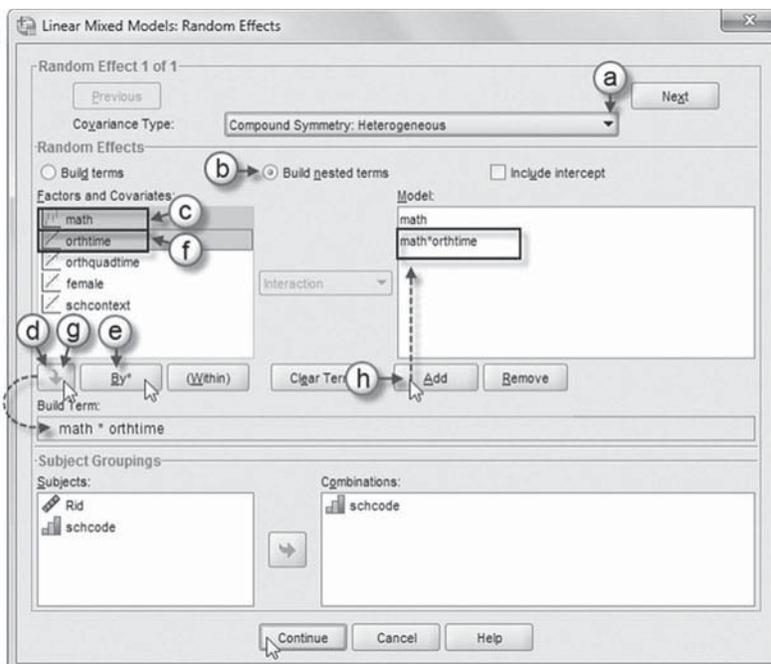
Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen with default settings from the prior model.

- a. Change the covariance structure by clicking the pull-down menu and selecting *Compound Symmetry: Heterogeneous*.

The *Compound Symmetry: Heterogeneous* covariance structure has heterogeneous variances and constant correlations between elements (IBM Corporation, 2012).

- b. Click the *Build nested terms* option.
- c. Click to select the variable *math* from the *Factors and Covariates* box.
- d. Then click the arrow button below the *Factors and Covariates* box. This moves *math* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- e. Next, click the BY\* button, which will insert the computation command symbol: *math\**.
- f. Click to select *orthtime* from the *Factors and Covariates* box.
- g. Click the arrow button below the *Factors and Covariates* box to move *orthtime* into the *Build Term* box and complete the interaction term: *math\*orthtime*.
- h. Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

Click the OK button to generate the model results.

## Summary

In this chapter, we presented three examples using multilevel multivariate outcomes. The composite (or latent construct) formulation is appropriate when one wishes to incorporate multiple measurements in defining constructs. Our first example provided a case where we estimated a construct with a simple error structure within individuals. Since the multilevel (i.e., vertical) formulation does not assume that all outcome data must be available for each individual, it provides another way for incorporating missing data into the analysis (Hox, 2010). We next showed how the repeated measures format at Level 1 can be used to define a multivariate type of model. We

also demonstrated how it is possible to test the equality of regression coefficients across outcome measures in this type of modeling formulation. In our last example, we showed that this type of multilevel model can be generalized to situations where the analyst wishes to examine parallel growth processes. We illustrated what that might look like for examining growth in reading and math simultaneously. This type of growth model can be specified by stacking the reading and math repeated measures in one achievement outcome. As we showed, if each variable has three repeated observations, the *Index1* variable will have six lines. A dummy indicator (i.e., math) can then be defined to specify each outcome. Linear and quadratic time-related variables can be added as needed to define each trajectory and, finally, predictors can then be added within and between groups to explain variation in individuals' growth processes.

## CHAPTER 8

# Cross-Classified Multilevel Models

In the models we have previously presented, the data structures have been purely hierarchical; that is, each individual was in only one Level 2 setting (e.g., a classroom) and only one Level 3 setting (e.g., a school). In this chapter, we present situations where the nesting of subjects within groups is more complicated. Cross-classified data structures represent a special type of multilevel model. In school settings, for example, researchers are often interested in monitoring student progress longitudinally. This has proven difficult, however, since students move regularly (and irregularly) between classrooms and schools. Students may belong to more than one unit (e.g., classrooms) over time at a particular level of the data hierarchy. Another use of a cross-classified model might be where we are interested in studying how students transition from high schools to postsecondary institutions. In this latter case, within a particular state, students from several hundred high schools transition to various 2-year and 4-year institutions. Each student in the study, therefore, would be cross-classified by a high school and postsecondary institution entered after high school. Cross-classified hierarchical models extend the analytic tools available for studying student progress (Hill & Goldstein, 1998). Beretvas (2011), Goldstein (1987), Hox (2010), Raudenbush and Bryk (2002), and Snijders and Bosker (1999) provide general overviews of cross-classified models.

Multiple-membership models extend the cross-classified model to situations where students might be members of more than one unit at the highest level. More specifically, students might attend more than one postsecondary institution in the particular study described previously, for example, depending on whether they entered a 2-year institution and then transferred to a 4-year institution to finish their undergraduate degree. A smaller number may also have entered one 4-year institution and then transferred to another, perhaps if they switched majors. In either case, we might assign Level 1 weights, which describe student membership in one or more postsecondary institutions (Rasbash & Browne, 2001). For interested readers, Beretvas (2011) provides an accessible overview of multiple-membership models.

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### Students Cross-Classified in High Schools and Postsecondary Institutions

Consider the contribution of students' high school and college settings to their undergraduate educational attainment. Students from any particular public high school may attend a number of different college campuses within a state's higher education system. In this first example, we examine whether student background and features of their high school and college entry point influence their college attainment. We will use students' cumulative grade point average (GPA) at the time they earned their highest undergraduate degree (i.e., associate or bachelor's). We could extend this basic model to a multiple-membership model, if we also wanted to consider whether students transferred from one entry campus to another before graduating with a 4-year degree.

## Research Questions

Our first research question might ask simply: Do features of students' high school and college entry campuses affect their educational attainment, after controlling for student background? Second, we ask: Does the slope describing the effect of individual socioeconomic status (SES) on student college attainment vary randomly across either or both Level 2 units? Third, if it does vary randomly across units, are there high school and college variables that explain this variation?

## The Data

In this subset of a larger data set (Table 8.1), we randomly selected 1,767 students within 30 public high schools who entered college at 10 different campuses within a state's higher education system. This type of data is considered a two-level cross-classified data set since we have students at Level 1 cross-classified in high schools and colleges at Level 2.

Using Raudenbush and Bryk's (2002) cross-classified terminology, in our example data matrix summarized in Table 8.2, at Level 2 we will consider high schools as the *row* data set (i.e., the larger number of Level 2 units) and colleges as the *column* data set. In the data matrix, therefore,

**TABLE 8.1** Data Definition of *ch8crossclass1.sav* ( $N = 1,767$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement <sup>4</sup>
<i>schcode</i>	School	School identifier (30 public high schools).	Integer	Ordinal
<i>nschcode</i>	School	Recoded school identifier for high schools.	(1, 2, 3, ..., 30)	Ordinal
<i>campus</i>	School	College campus identifier (10 campuses within a state's higher education system).	(1, 2, 3, ..., 10)	Scale
<i>female</i>	Individual	Demographic predictor variable representing student's gender.	0 = Male 1 = Female	Scale
<i>lowses</i>	Individual	Dichotomous variable representing student socioeconomic status.	0 = Did Not Participate, 1 = Participant in Federal Free/Reduced Lunch Program	Scale
<i>CUM_GPR</i>	Individual	Dependent variable measuring students' cumulative GPA.	0.0 to 4.00	Scale
<i>lowses_mean</i>	School	Predictor variable (grand-mean centered) measuring student socioeconomic status.	0.02 to 0.46	Scale
<i>fouryear</i>	School	Dichotomous variable representing the institutional type students attended.	0 = Not 4-Year 1 = 4-Year	Scale
<i>gmfemale</i>	Individual	Predictor variable (grand-mean centered) measuring student cumulative GPA by gender (female).	-0.51 to 0.49	Scale
<i>gmlowses</i>	Individual	Predictor variable (grand-mean centered) measuring student cumulative GPA by socioeconomic status.	-0.17 to 0.83	Scale
<i>gmLowSES_mean</i>	School	Predictor variable (grand-mean centered) representing an aggregate measure of socioeconomic composition for high school.	-0.15 to 0.29	Scale
<i>gmfouryear</i>	School	Predictor variable (grand-mean centered) measuring student cumulative GPA by institutional type (4-year).	-0.30 to 0.70	Scale

<sup>a</sup> Individual = Level 1; school = Level 2.

<sup>4</sup> Measurement icon settings displayed in subsequent model screenshots may differ from Tables 8.1 and 8.13 but will not affect the output.

**TABLE 8.2** schcode\*campus Cross-Tabulation

Count	Campus										Total	
	1	2	3	4	5	6	7	8	9	10		
schcode	1	2	1	41	9	0	1	1	0	0	3	58
	2	1	1	20	47	0	1	7	0	0	0	77
	3	1	0	16	41	0	0	1	0	0	0	59
	4	2	2	47	43	0	0	4	0	0	0	98
	5	1	1	40	25	0	0	5	0	0	0	72
	6	1	3	17	54	0	0	5	0	2	2	84
	7	2	0	26	25	1	29	2	0	0	1	86
	8	4	2	18	3	0	11	3	0	0	0	41
	9	3	6	27	15	1	68	7	0	0	2	129
	10	5	4	79	24	0	29	9	0	2	2	154
	11	0	0	14	7	0	3	2	0	0	1	27
	12	0	1	9	5	1	1	0	0	0	0	17
	13	1	3	17	8	0	13	1	0	0	2	45
	14	1	0	4	3	0	0	0	0	0	1	9
	15	1	4	35	16	0	58	6	0	0	1	121
	16	2	1	11	9	0	12	0	0	0	1	36
	17	0	2	33	13	0	26	3	0	0	1	78
	18	1	1	37	24	0	3	4	0	7	2	79
	19	2	1	8	2	0	3	1	0	4	0	21
	20	0	2	15	16	0	1	2	0	8	1	45
	21	1	0	17	7	0	1	2	0	3	0	31
	22	72	6	0	0	0	0	3	1	0	0	82
	23	23	3	1	0	0	0	0	1	0	0	28
	24	12	1	1	1	0	0	0	0	0	0	15
	25	9	0	2	0	0	0	0	0	0	0	11
	26	53	0	3	3	1	0	0	0	0	1	61
	27	5	2	0	0	0	0	0	0	0	0	7
	28	10	0	1	0	0	0	0	0	0	0	11
	29	107	8	0	0	0	0	2	0	0	1	118
	30	3	3	6	7	0	2	28	16	0	2	67
Total		325	58	545	407	4	262	98	18	26	24	1,767

each student at Level 1 is cross-classified by a row (high school) and column (college). The data set is considered cross-classified at Level 2 because students attend different high schools and different colleges. Readers will notice that the data in Table 8.2 are not balanced in either rows or columns. For example, in High School 1, 58 students entered the higher education system, primarily enrolling at College Campus 3. Moreover, there were five college campus cells that had small numbers of students (one to three students) and two campuses that did not receive any students from this particular high school. In this type of data configuration, the sample sizes vary considerably, and there are many cells with missing data. Typically, in a cross-classified model, the goal is to generalize from the sample to a population of high schools and college campuses. So the high school and college effects are treated as randomly varying.

The structure of the data allows us to examine the variance components in student attainment that exist *between high schools*, *between colleges*, and *within high school × college* cells. Within each high school–college cell, or the cross-classification, is a unique set of students. For example, 41 students attended High School 1 and College Campus 3. The within-cell model describes the variation among these students on an outcome of interest.

At Level 2, variation in outcomes between cells of students can be attributed to high school effects, college effects, or perhaps a school-by-college interaction. At this level, we can also develop a model to examine high school and college features on students' college attainment. In this case, we will use the mean percentage of low SES students as a high school composition variable that may influence student attainment, and a dummy variable for institutional type (i.e., 2-year or 4-year institution) as a possible college predictor of cumulative GPA. Of course, in this example the data set is a little small ( $N = 10$  colleges), but it will suffice for purposes of our demonstration. We are limited, however, in the number of predictors we could use because of the small size of the college data set.

### Descriptive Statistics

The descriptive statistics at the student level and for the cross-classified Level 2 units are summarized in Tables 8.3, 8.4, and 8.5. We note in Table 8.3 that CUM\_GPR refers to students' cumulative GPA in subsequent analyses.

As a first model, we might examine the variance components in students' cumulative GPA that lie between high schools, between colleges, and within cells (i.e., high school × college

**TABLE 8.3 Descriptive Statistics**

	<i>N</i>	Minimum	Maximum	Mean	Std. Deviation
female	1,767	0.000	1.000	0.512	0.500
lowses	1,767	0.000	1.000	0.169	0.375
CUM_GPR	1,767	0.000	4.000	2.903	0.633
Valid <i>N</i> (Listwise)	1,767				

**TABLE 8.4 Descriptive Statistics**

	<i>N</i>	Minimum	Maximum	Mean	Std. Deviation
lowses_mean	30	0.022	0.462	0.175	0.115
Valid <i>N</i> (Listwise)	30				

**TABLE 8.5 Descriptive Statistics**

	<i>N</i>	Minimum	Maximum	Mean	Std. Deviation
fouryear	10	0	1	0.300	0.483
Valid <i>N</i> (Listwise)	10				

cells). Following Goldstein (1995) and Hox (2010), at Level 1, this unconditional (no predictors) model can be defined as follows:

$$Y_{i(jk)} = \beta_{0(jk)} + \varepsilon_{i(jk)}, \quad (8.1)$$

where  $Y_{i(jk)}$  is the cumulative GPA of student  $i$  within the cross-classification of high school  $j$  and college  $k$ ,  $\beta_{0(jk)}$  is the intercept (overall mean GPA) of students in cell  $jk$  (i.e., students who attended high school  $j$  and college  $k$ ), and  $\varepsilon_{i(jk)}$  is the within-cell residual that represents the deviation of student  $ij$ 's GPA from the cell mean. The residual is assumed to be normally distributed with the mean equal to 0 and some variance ( $\sigma_\varepsilon^2$ ). The Level 1 residual can be further modeled to reflect more complex Level 1 variation if desired (Hill & Goldstein, 1998). The subscripts ( $jk$ ) suggest that we assume that the intercept varies independently across both high schools and colleges that are conceptually at the same level (Hox, 2002).

The Level 2, or “between cells,” model is as follows:

$$\beta_{0(jk)} = \gamma_{00} + u_{0j} + v_{0k} + \delta_{0jk}, \quad (8.2)$$

where  $\gamma_{00}$  is the grand mean GPA for all students,  $u_{0j}$  is the residual random effect for high school  $j$  (i.e., the contribution of high school  $j$  averaged over all colleges),  $v_{0k}$  is the residual random effect for college  $k$  (i.e., the contribution of college  $k$  averaged over all high schools), and  $\delta_{0jk}$  represents a residual random effect for school-by-college cells. This latter component specifies a possible random effect due to being in the two cross-classified units (high school  $j$  and college  $k$ ). As Raudenbush and Bryk (2002) note, however, without sufficiently large within-cell sizes, it is difficult to separate the variance associated with students at Level 1 (i.e., student  $i$  in cell  $jk$ ) and the variance due to this particular Level 2 interaction effect (i.e., high school–college  $jk$ ). Therefore, this potential random effect is typically set to 0 (Beretvas, 2011; Raudenbush & Bryk, 2002). We follow that suggestion for specifying cross-classified models in this chapter.

After substitution (and removing the intra-cell variance component), we arrive at the intercept-only model,

$$Y_{i(jk)} = \gamma_{00} + u_{0j} + v_{0k} + \varepsilon_{i(jk)}, \quad (8.3)$$

where the outcome  $Y_{i(jk)}$  is modeled with an overall intercept  $\gamma_{00}$  and an error term each for high schools and colleges, and  $\varepsilon_{i(jk)}$  is the residual error term for student  $i$  in the cross-classification of high school  $j$  and college  $k$ . The single-equation model in Equation 8.3 implies one fixed effect ( $\gamma_{00}$ ), two random Level 2 effects ( $u_{0j}$ ,  $v_{0k}$ ), and the within-cell (Level 1) residual ( $\varepsilon_{i(jk)}$ ). We note that because of the small data set for our illustration, we will assume a diagonal covariance matrix structure at Level 2 (i.e., no covariance between random effects) in this model and subsequent models.

### Defining Models in IBM SPSS

IBM SPSS is easy to use for investigating a variety of cross-classified models. Compared to the models that we have defined previously with nested hierarchical structures, it is relatively easy to define cross-classified structures. For a two-level cross-classified model, a random effect across

**TABLE 8.6 Mean Student GPA**

	Mean	Std. Error	df	Lower Bound	Upper Bound
Student CUM_GPR	2.717	0.129	10.139	2.429	3.004

**TABLE 8.7 Estimates of Covariance Parameters<sup>a</sup>**

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		0.331	0.011	29.252	.000	0.309	0.354
Intercept	Variance	0.036	0.015	2.494	.013	0.017	0.080
[subject = nschcode]							
Intercept	Variance	0.142	0.073	1.956	.051	0.052	0.387
[subject = campus]							

<sup>a</sup> Dependent variable: CUM\_GPR.

high schools is referred to by using the “random” statements separately (i.e., RANDOM = high school identification code), and a random effect across colleges is referred to in a similar manner (RANDOM = college identification code). This is because the cross-classified part of the model is “independent” (i.e., students can go to different high schools and colleges). The definition of this type of cross-classified data structure is in contrast to the typical structure for a two-level nested model [(RANDOM = college code) and (RANDOM = college code\*high school code)], where the statements would indicate that students are nested in high schools and colleges. This latter data set would only consider students who went to the same high school and college campus.

First, we can consider the GPA intercept and the decomposition of variance across the various cells in the analysis (Table 8.6). The average mean GPA for campuses is 2.72.

Table 8.7 suggests that most variance in student cumulative GPA is due to differences within individuals. As we might expect, college campus attended contributes more to the total variance in GPA than high school campus.

Since within Level 2 the high school and colleges are considered to be independent, we can simply add the estimated variances in the no-predictors model to obtain the total variance. We will estimate three intraunit variance components. The first is the intrarow correlation (i.e., high school), which represents the correlation between outcomes of any two students who attend the same high school but attend different colleges. The intraunit correlation for high schools (*nschcode*) is therefore defined as

$$\rho_{HS} = \sigma_{HS}^2 / (\sigma_{Student}^2 + \sigma_{HS}^2 + \sigma_{College}^2) \tag{8.4}$$

In this case, it can be calculated as  $0.036 / (0.331 + 0.036 + 0.142) = 0.036 / 0.509 = 0.071$ . This suggests that 7.1% of the total variance in students’ cumulative GPA lies between the high schools they attended.

The intraunit correlation for colleges is the correspondence between outcomes of any two students who attend the same college but who attend different high schools:

$$\rho_{College} = \sigma_{College}^2 / (\sigma_{Student}^2 + \sigma_{HS}^2 + \sigma_{College}^2) \quad (8.5)$$

This is estimated as  $0.142 / (0.331 + 0.036 + 0.142) = 0.142 / 0.509 = 0.279$ . This suggests that 27.9% of the variation in students' cumulative GPAs lies between the colleges they attend. The remaining variation represents differences in GPA among students not accounted for by their high schools and colleges (roughly 65% of the variation).

Third, we can examine the correlation between outcomes of students who attended the same high school and college. This intracell component can be summarized as

$$\rho_{HS-College} = (\sigma_{HS}^2 + \sigma_{College}^2) / (\sigma_{Student}^2 + \sigma_{HS}^2 + \sigma_{College}^2) \quad (8.6)$$

In this case, it is estimated as  $(0.036 + 0.142) / (0.331 + 0.036 + 0.142) = 0.178 / 0.509 = 0.350$ . Taken together, the correspondence in outcomes between two students who attend the same high school and college is 0.35. In other words, about 35% of the variance in cumulative GPA can be attributed to attending the same high school and college campus.

### Model 1.1: Adding a Set of Level 1 and Level 2 Predictors

Next, we can add a set of student (Level 1) predictors to the model. In this case, we will add *female* (coded 1) and *lowSES* (code 1 = participation in federal free/reduced lunch program during high school, 0 = else). We grand-mean centered both variables and saved them in the data set. The Level 1 model is now

$$Y_{i(jk)} = \beta_{0(jk)} + \beta_{1(jk)}gmfemale_{i(jk)} + \beta_{2(jk)}gmlowSES_{i(jk)} + \varepsilon_{i(jk)}, \quad (8.7)$$

where  $\beta_{0(jk)}$  is the intercept,  $\beta_{1(jk)}$  and  $\beta_{2(jk)}$  are structural coefficients, and  $\varepsilon_{i(jk)}$  represents the Level 1 residual (mean = 0,  $\sigma^2$ ). The regression slopes for the individual-level variables can be allowed to vary across high schools and/or colleges (as we have shown in past examples).

At Level 2, we might add high school and college predictors that explain students' grade point average. In this case, *gmlowSES\_mean* is an aggregate measure of socioeconomic composition for the high school, and *gmfouryear* describes whether students were enrolled in a 2-year (community college) or 4-year postsecondary institution. The Level 2 intercept model is now

$$\beta_{0(jk)} = \gamma_{00} + \gamma_{01}gmlowSES\_mean_j + \gamma_{02}gmfouryear_k + u_{0j} + v_{0k}. \quad (8.8)$$

The combined model with eight estimated parameters can then be written as follows:

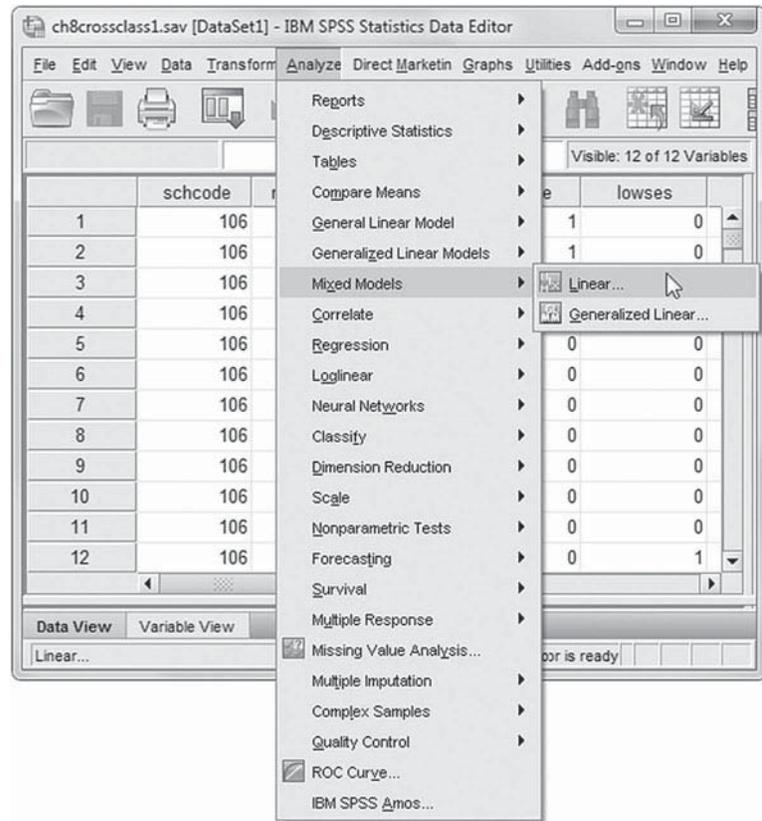
$$Y_{i(jk)} = \gamma_{00(jk)} + \gamma_{01}gmlowSES\_mean_j + \gamma_{02}gmfouryear_k + \gamma_{10}gmfemale_{i(jk)} + \gamma_{20}gmlowSES_{i(jk)} + u_{0j} + v_{0k} + \varepsilon_{i(jk)}. \quad (8.9)$$

### Defining Model 1.1 with IBM SPSS Menu Commands

Launch the IBM SPSS program application, and select the *ch8cross-class1.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

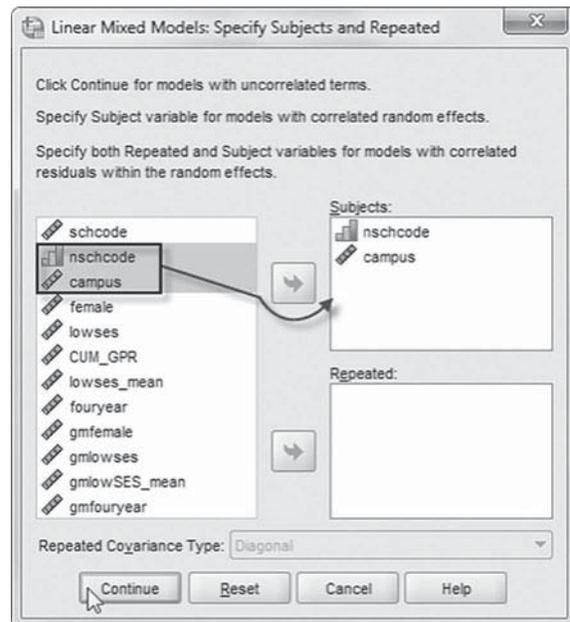
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.

A subject is an observational unit that may be independent of other subjects. For this model, we will designate two subject identifiers for the model (*nschcode*, and *campus*). Click to select *nschcode* and *campus*, and then click the right-arrow button to move the variables into the *Subjects* box.

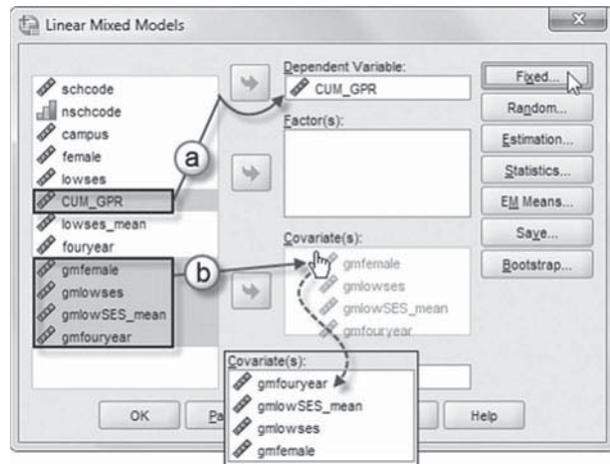
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, and covariates, as well as accessing the dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

a. For this model, we will use *CUM\_GPR* as the dependent variable. Click to select *CUM\_GPR* from the left column listing, and then click the right-arrow button to transfer the variable into the *Dependent Variable* box.

b. We will designate four predictor variables to be used in the model (*gmfemale*, *gmloswses*, *gmloswses\_mean*, and *gmfouryear*). Click to select the four variables, and then “drag” them to *Covariate(s)* box. To facilitate reading of the output tables, we will change the sequence of the variables by dragging individual variables into the following order (see insert): *gmfouryear*, *gmloswses\_mean*, *gmloswses*, and *gmfemale*.



**Note:** An alternate method for arranging the variables in the desired sequence order is to select variables individually and then use the right-arrow button to move each variable into the *Covariate(s)* box.

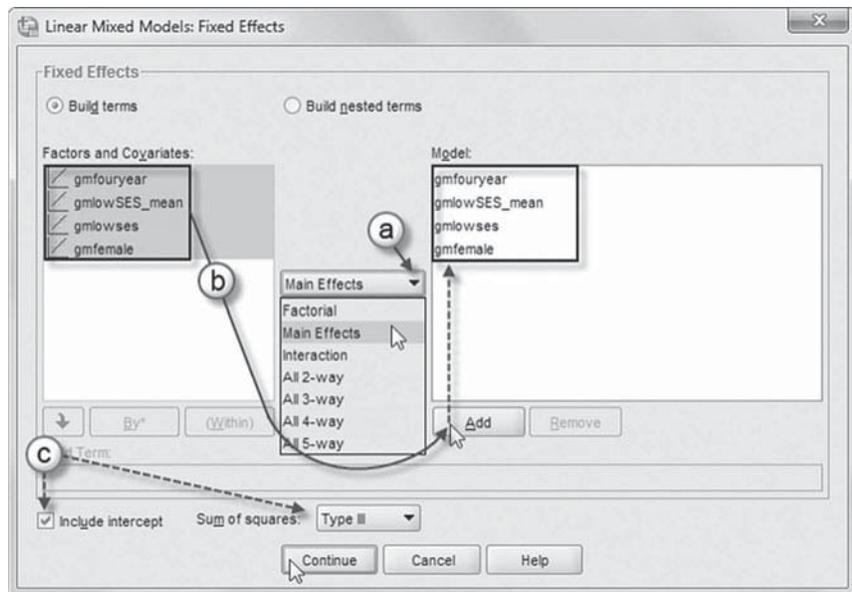
We may now proceed to define fixed effects for the variables.

Click the **FIXED** button to access the *Linear Mixed Models: Fixed Effects* dialog box.

4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.

b. Click to select the four variables from the *Factors and Covariates* box. Then click the **ADD** button to move the variables into the *Model* box.

c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



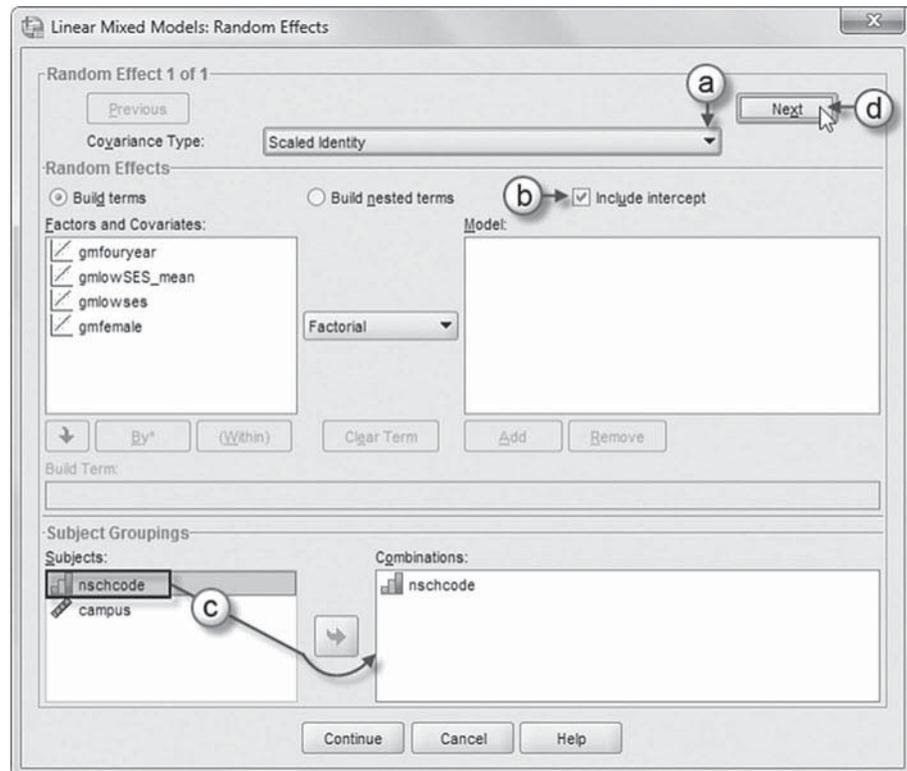
Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.

We will now add random effects to this model.

Click the **RANDOM** button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen, which is the default when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.

- a. Begin by specifying the covariance structure from the default variance components (VC) to scaled identity. Click the pull-down menu and select *Scaled Identity* (ID).



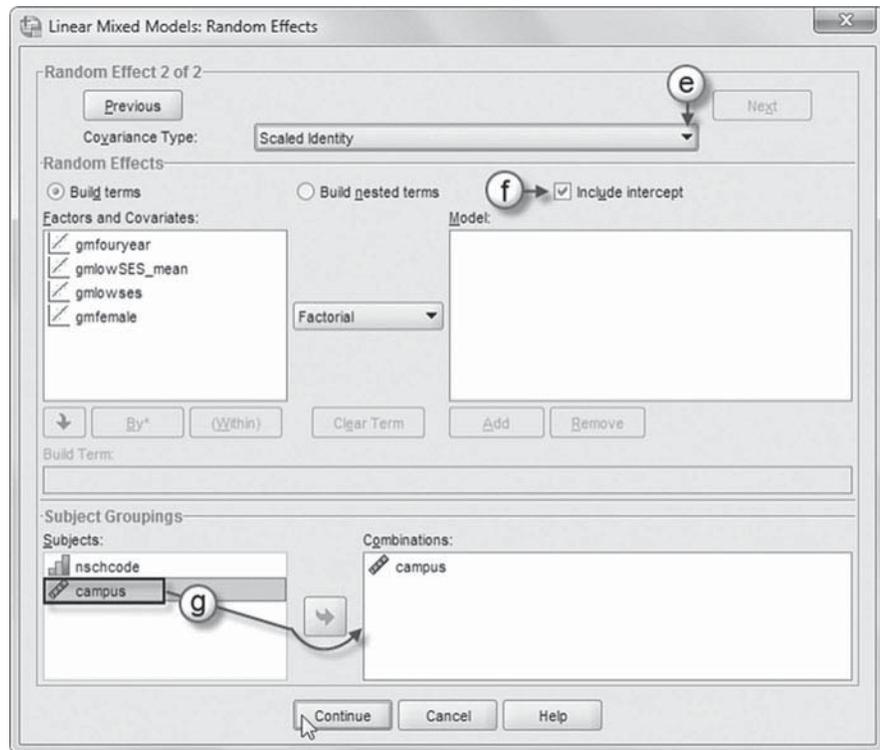
The *Scaled Identity* structure has constant variance and assumes that no correlation occurs between elements (IBM Corporation, 2012).

- b. We want the intercept to be included in the model, so click *Include intercept*.
- c. The *Subject Groupings* box displays the *nschcode* and *campus* variables that were specified as a subject variable in the *Specify Subjects and Repeated* dialog box show in step 2. We will specify *nschcode* as the subject for the random-effects Level 1 part of this model. Click to select *nschcode*, and then click the right-arrow button to move the variable into the *Combinations* box.
- d. At the top-right section of the window, click the NEXT button to access the *Random Effect 2 of 2* screen.

**Note:** The NEXT button may not work in earlier or unpatched versions of IBM SPSS when creating multilevel models with random intercepts. An update issued by IBM SPSS for software Version 19 addressed the problem, and Version 20 appears to have resolved the issue. A workaround to activating the NEXT button is to either (a) add or reenter a subject variable into the *Combinations* box or (b) add a variable from the *Factors and Covariates* column to the *Model* box and then remove it before proceeding to the *Random Effect 2 of 2* screen.

The *Random Effect 2 of 2* screen display is similar to the first screen and requires the following changes.

- e. Change the covariance type by clicking on the pull-down menu and selecting *Scaled Identity*.
- f. Click to select the *Include intercept* option.
- g. We will specify *campus* as the subject for the random-effects Level 2 part of this model. Click to select *campus*, and then click the right-arrow button to move the variable into the *Combinations* box.

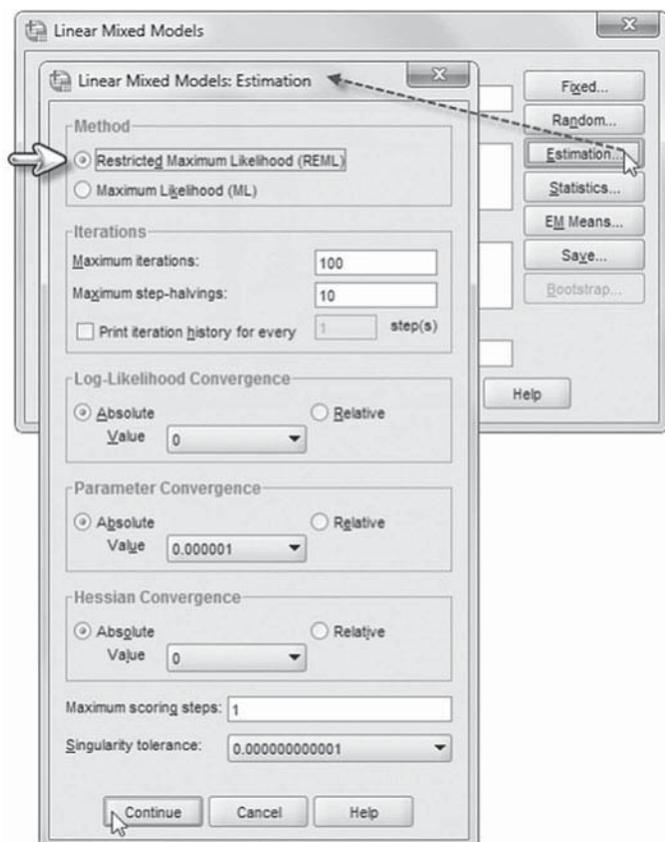


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

6. Click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: maximum likelihood (ML) or restricted maximum likelihood (REML).

In this chapter, we will use the default setting of REML to estimate the models.

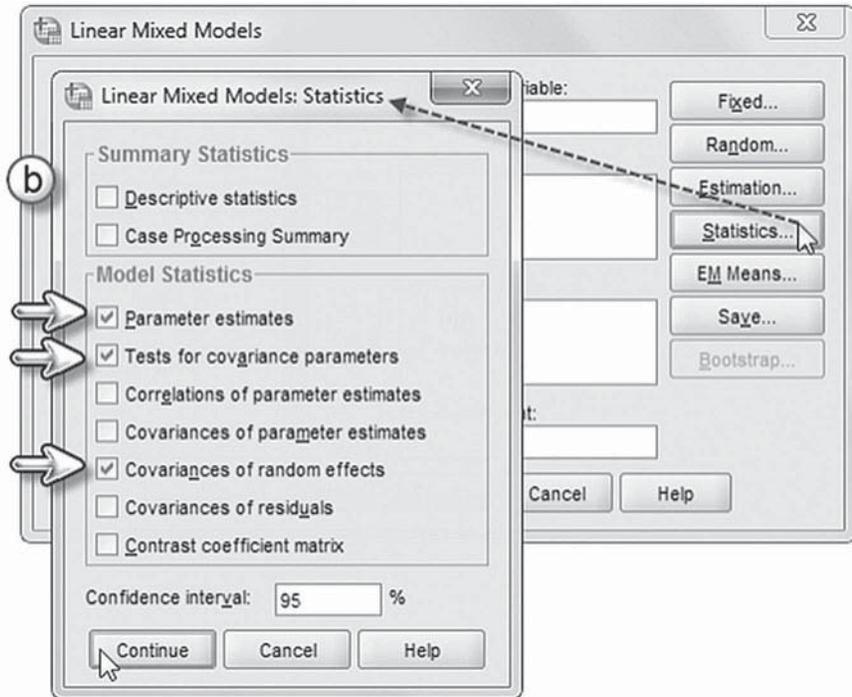
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



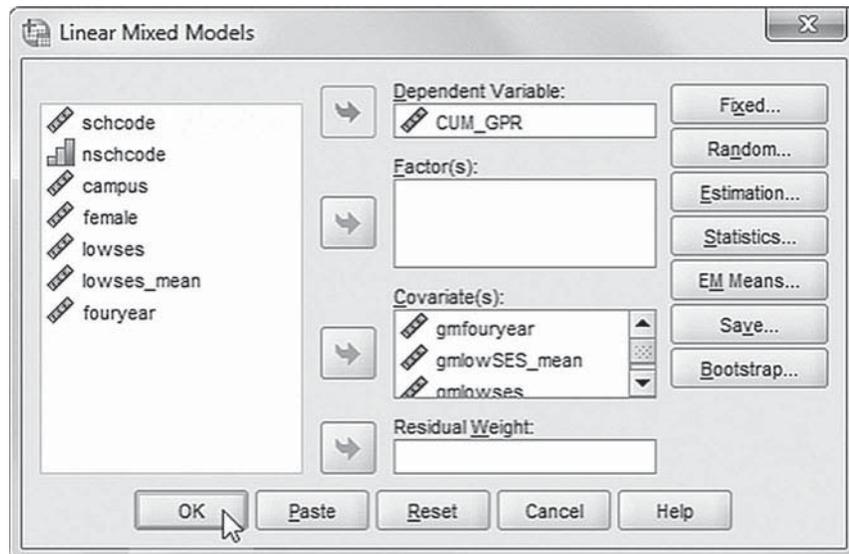
7. In the *Linear Mixed Models* dialog box, click the **STATISTICS** button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the **CONTINUE** button to return to the *Linear Mixed Models* dialog box.



8. Finally, in the *Linear Mixed Models* dialog box, click the **OK** button to run the model.



### Interpreting the Output From Model 1.1

From the following fixed-effects table (Table 8.8), we can see that of the Level 1 effects, females have significantly higher cumulative GPAs ( $\gamma = 0.332, p < .01$ ) than males; however, individual SES status does not affect cumulative GPA. At Level 2, aggregate high school social composition is significantly and negatively related to cumulative GPA ( $\gamma = -1.220, p < .01$ ). Institutional type, however, is not related to cumulative GPA ( $\gamma = 0.138, p = .655$ ).

TABLE 8.8 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	2.682	0.142	9.070	18.943	.000	2.363	3.002
gmfouryear	0.138	0.298	7.933	0.465	.655	-0.549	0.825
gmlowses_mean	-1.220	0.371	32.478	-3.286	.002	-1.977	-0.464
gmlowses	0.017	0.037	1,728.687	0.472	.637	0.055	0.090
gmfemale	0.332	0.028	1,735.674	11.905	.000	0.277	0.386

<sup>a</sup> Dependent variable: CUM\_GPR.

TABLE 8.9 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		0.305	0.010	29.300	.000	0.285	0.326
Intercept [subject = nschcode]	Variance	0.031	0.012	2.602	.009	0.015	0.066
Intercept [subject = campus]	Variance	0.177	0.093	1.905	.057	0.063	0.496

<sup>a</sup> Dependent variable: CUM\_GPR.

The variance component table (Table 8.9) suggests that there is still significant variance in intercepts across high schools ( $p < .01$ ) left to explain after the set of predictors was entered into the model. There is also still variance across colleges (Wald  $Z = 1.905$ , one-tailed  $p = .0285$ ).

### Model 1.2: Investigating a Random Slope

Level 2 high school or college variables can also be used to explain variation in Level 1 slopes. The Level 2 model can be expanded to include the random effect of gender on cumulative college GPA across high schools ( $u_{1j}$ ) and colleges ( $v_{1k}$ ). We first allow the female-GPA slope to vary randomly across both high schools and colleges as follows:

$$\beta_{1(jk)} = \gamma_{10} + u_{1j} + v_{1k}, \quad (8.10)$$

where  $\gamma_{10}$  is the intercept,  $u_{1j}$  is the residual high school effect, and  $v_{1k}$  is the residual college effect. In this model, however, we do not attempt to explain this variation. The combined model (with 10 parameters) will then be the following:

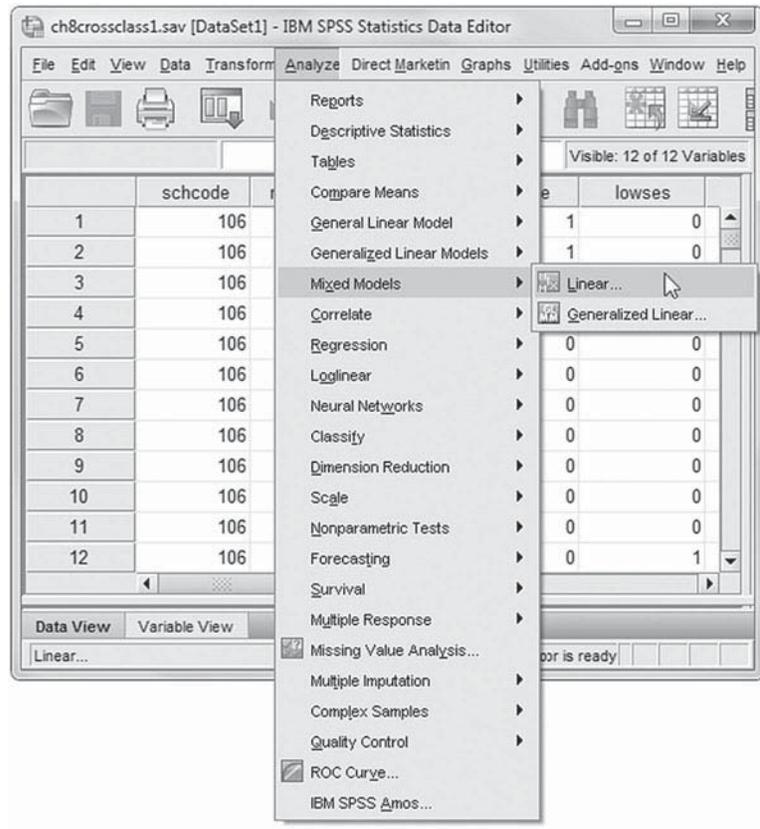
$$Y_{i(jk)} = \gamma_{00(jk)} + \gamma_{01}gmlowSES\_mean_j + \gamma_{02}gmfouryear_k + \gamma_{10}gmfemale_{i(jk)} + \gamma_{20}gmlowSES_{i(jk)} + u_{1j}gmfemale_{i(jk)} + v_{1k}female_{i(jk)} + u_{0j} + v_{0k} + \epsilon_{i(jk)}. \quad (8.11)$$

### Defining Model 1.2 with IBM SPSS Menu Commands

**Note:** Settings will default to those used in Model 1.1.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

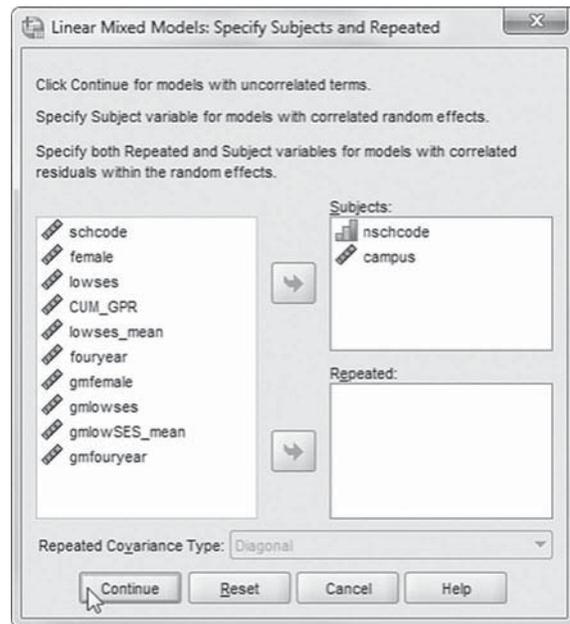
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

We will change the random-effects covariance type, so from the *Linear Mixed Models* main screen, click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

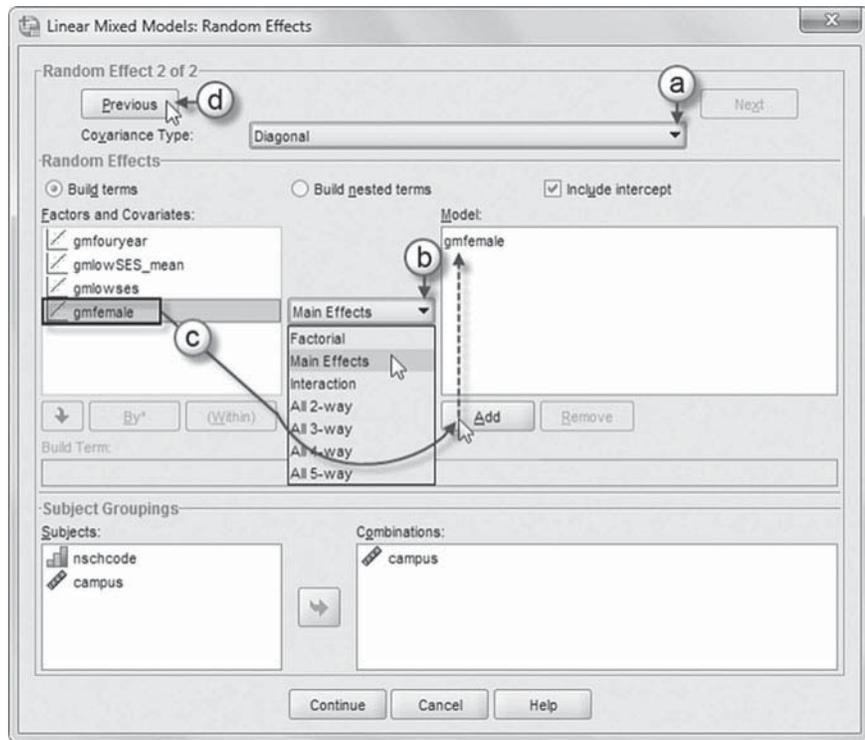


3. The *Random Effect 2 of 2* screen is displayed first as it was the last dialog box used in the prior model.

- a. Change the covariance type by clicking the pull-down menu and selecting: *Diagonal*.

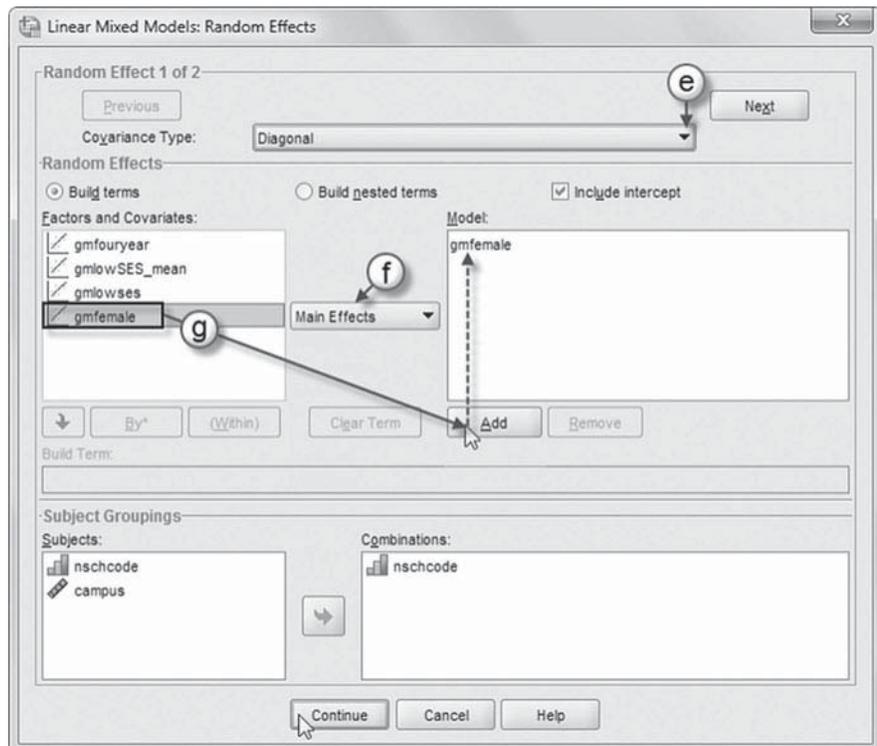
The *Diagonal* covariance type has heterogeneous variances and zero correlation between the elements (IBM Corporation, 2012).

- b. Change the default *Factorial* setting by clicking the pull-down menu to select *Main Effects*.
- c. Click to select *gmfemale*, and then click the ADD button to move the variable into the *Model* box.
- d. Click the PREVIOUS button to access the *Random Effect 1 of 1* screen.



- e. From the *Random Effect 1 of 2* screen, change the covariance type by clicking the pull-down menu and selecting *Diagonal*.
- f. Note the setting is *Main Effects*.
- g. Click to select *gmfemale*, and then click the ADD button to move the variable into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



4. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

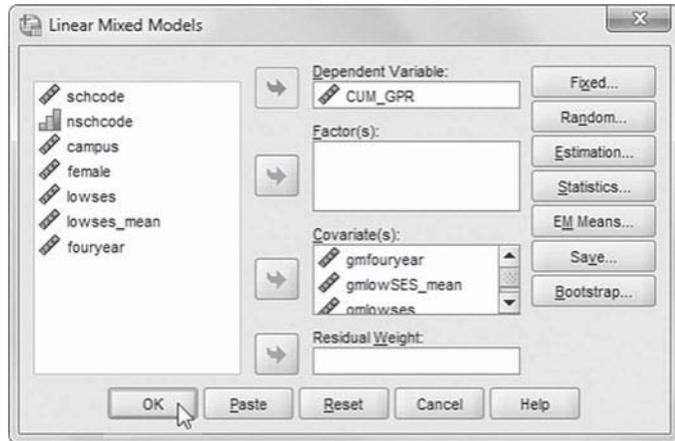


TABLE 8.10 Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Residual	0.293	0.010	28.863	.000	0.274	0.314
Intercept + gmfemale [subject = nschcode]	Var: Intercept 0.036	0.013	2.734	.006	0.017	0.073
	Var: gmfemale 0.040	0.020	2.052	.040	0.016	0.105
Intercept + gmfemale [subject = campus]	Var: Intercept 0.194	0.102	1.908	.056	0.069	0.542
	Var: gmfemale 0.038	0.033	1.155	.248	0.007	0.208

<sup>a</sup> Dependent variable: CUM\_GPR.

**Interpreting the Output From Model 1.2**

The variance components table (Table 8.10) suggests that the random slope (*gmfemale*) varies across high schools (Wald  $Z = 2.052$ , one-tailed  $p = .02$ ). However, the female–cumulative GPA slope does not vary across campuses (Wald  $Z = 1.16$ , one-tailed  $p = .124$ ). One explanation for the observed lack of variation across colleges may be the small sample size.

**Model 1.3: Explaining Variation Between Variables**

Since the Level 1 slope varies across high school cells, for demonstration purposes, we will propose that a Level 2 variable explains variation in the relationship between female and cumulative GPA. We will propose that the social composition of students’ high schools moderates (enhances or diminishes) the relationship between gender and cumulative GPA. The proposed cross-level interaction would look like this:

$$\beta_{1(jk)} = \gamma_{10} + \gamma_{11}gmloWSES\_mean_j * gmfemale_{i(jk)} + u_{1j}. \tag{8.12}$$

Equation 8.12 assumes that the random slope varies across high schools ( $u_{1j}$ ) but not colleges (i.e.,  $v_{1k}$  from Eq. 8.10 has been removed from the equation). This cross-level interaction can be interpreted as the effect of gender on attainment when the effect of high school social composition is controlled. For this model, in the intercept equation (see Eq. 8.8) we will remove the

*gmfouryear* variable since it was not significant in the previous model. The combined model with nine estimated parameters will then be as follows:

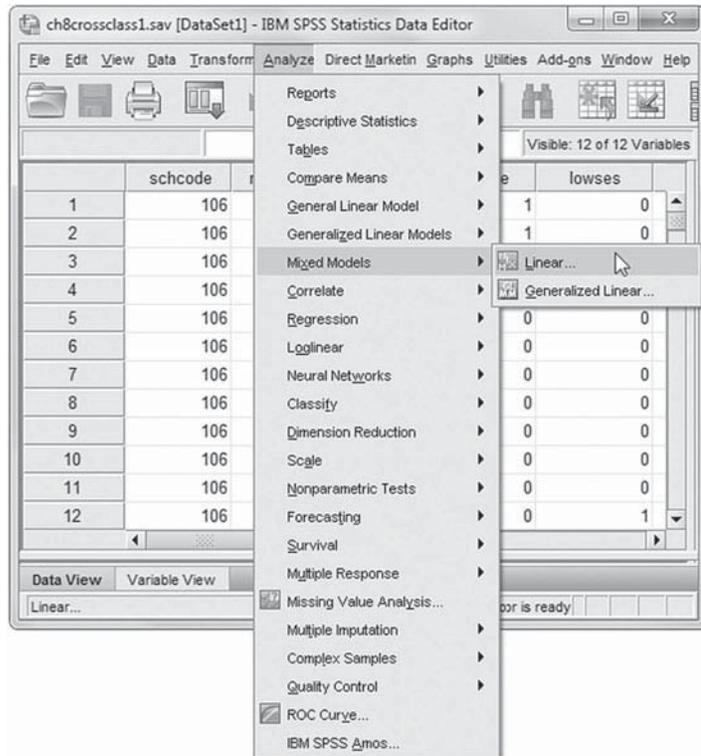
$$Y_{i(jk)} = \gamma_{00(ijk)} + \gamma_{01}gmlowSES\_mean_j + gmfemale_{i(jk)} + \gamma_{11}gmlowSES\_mean_j * gmfemale_{i(jk)} + \gamma_{20}gmlowSES_{i(jk)} + u_{1j}gmfemale + u_{0j} + v_{0k} + \varepsilon_{i(jk)}. \quad (8.13)$$

### Defining Model 1.3 with IBM SPSS Menu Commands

**Note:** Settings will default to those used in Model 1.2.

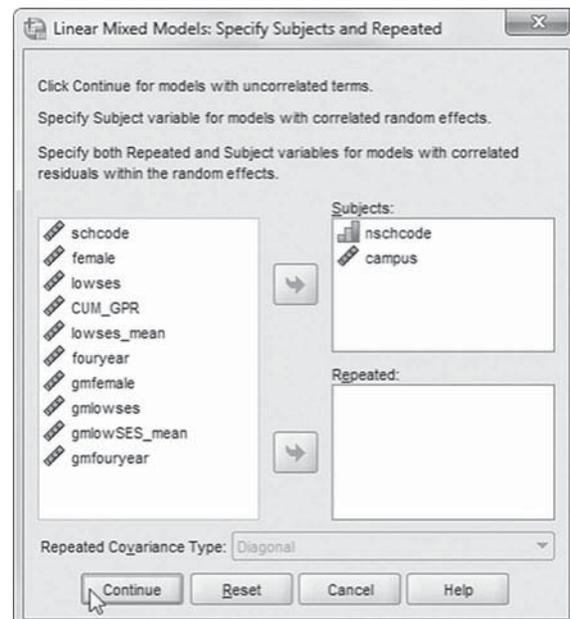
1. Go to the toolbar and select **ANALYZE, MIXED MODELS, LINEAR.**

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

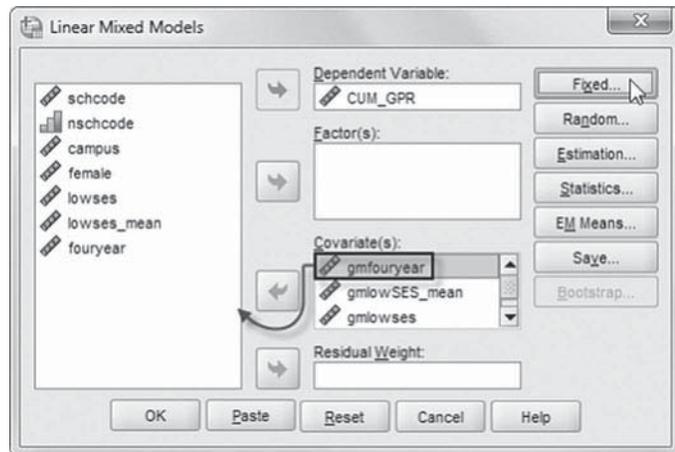
Click the **CONTINUE** button to display the *Linear Mixed Models* dialog box.



- From the *Linear Mixed Models* main screen we will remove a variable from the analysis. Click to select *gmfour-year*, and then click the left-arrow button to remove the variable.

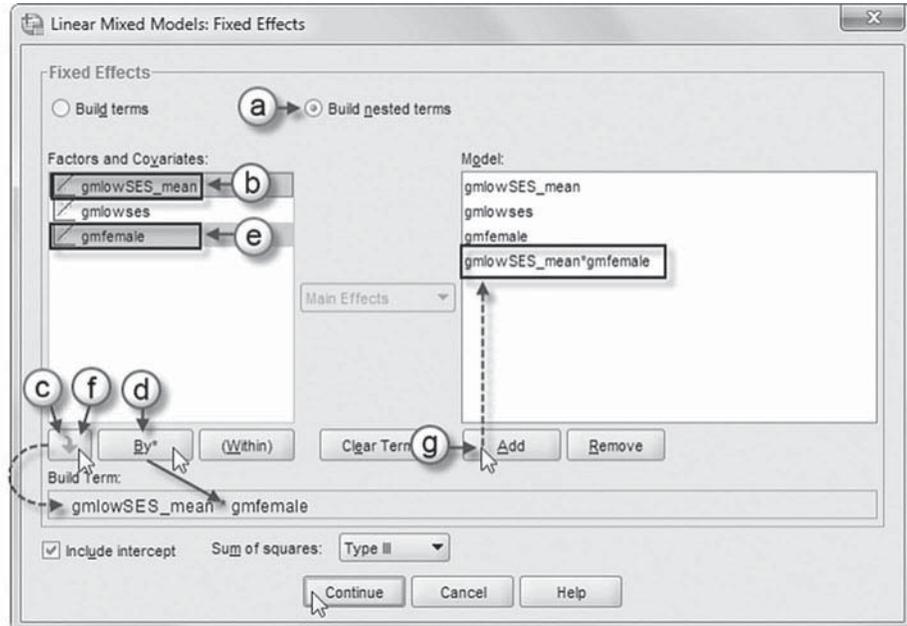
We will now add a fixed effect to the model, so click the FIXED button to access the *Fixed Effects* main screen.

One cross-level interaction (or nested terms) will be created and added to the model: *gmloWSES\_mean* and *gmfemale*. This interaction will tell us if social composition of students' high schools moderates (enhances or diminishes) the relationship between gender and cumulative GPA.



Add Interaction to Model 1.3: *gmloWSES\_mean\*gmfemale*

- Click to select *Build nested terms*.
- Now click to select the variable *gmloWSES\_mean* from the *Factors and Covariates* box.
- Click the arrow button below the *Factors and Covariates* box. This moves *gmloWSES\_mean*



- This moves *gmloWSES\_mean* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- Next, click the BY\* button, which will insert the computation command symbol: *gmloWSES\_mean\**.
- Click to select *gmfemale* from the *Factors and Covariates* box.
- Click the arrow button below the *Factors and Covariates* box to move *gmfemale* into the *Build Term* box and complete the interaction term: *gmloWSES\_mean\*gmfemale*.
- Click the ADD button to transfer the interaction into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click the RANDOM button to access the random-effects main screen.

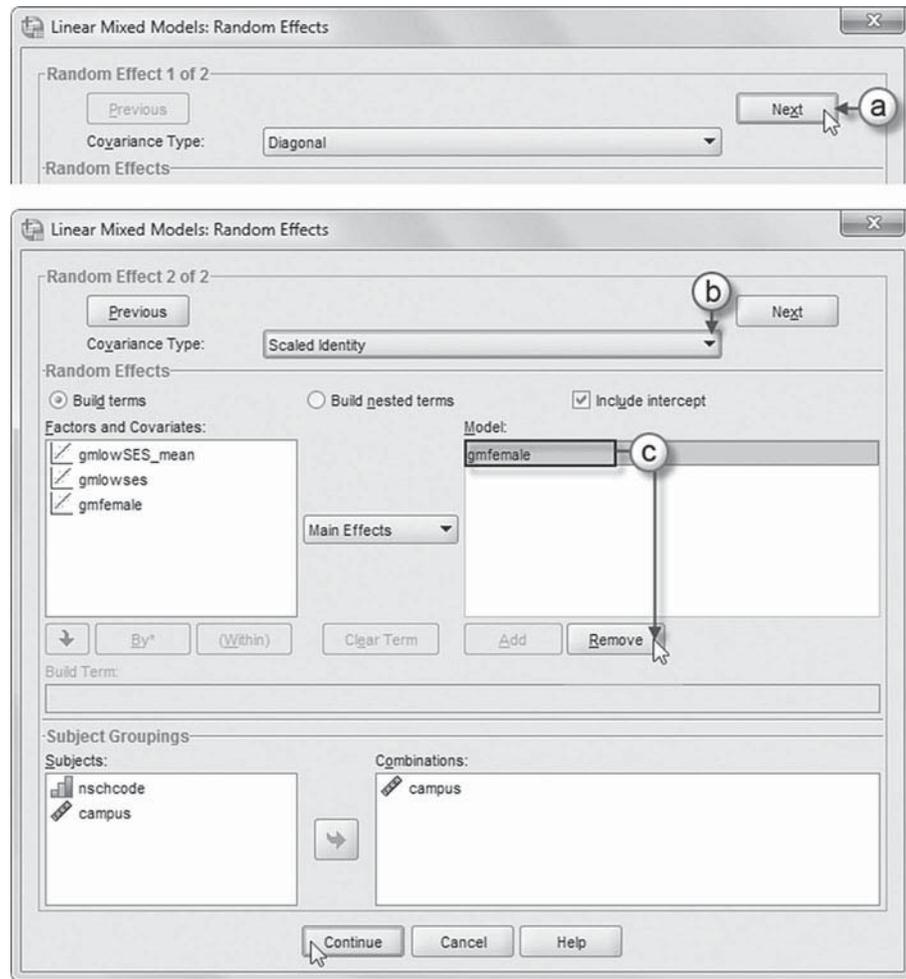
5. The *Random Effect 1 of 2* screen is displayed first, as it was the last dialog box used in the prior model.

- a. Click the NEXT button to access the *Random Effect 2 of 2* screen.
- b. Change the covariance type by clicking the pull-down menu and selecting *Scaled Identity*.

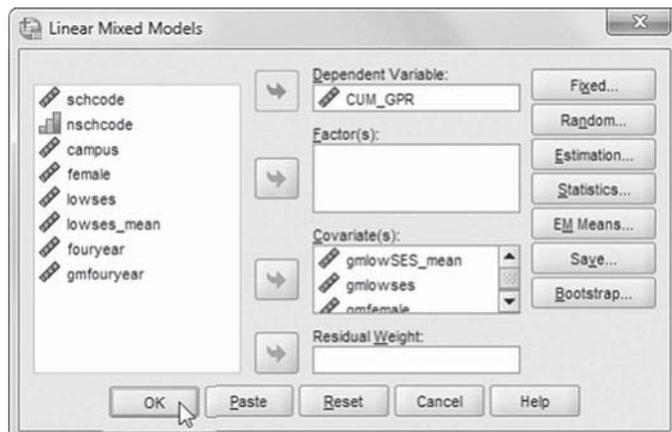
The *Scaled Identity* structure has constant variance and assumes that no correlation occurs between elements (IBM Corporation, 2012).

- c. We will remove *gmfemale* from the model by clicking to select the variable and then clicking the REMOVE button.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



6. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 1.3

The fixed-effects table (Table 8.11) suggests that the intercept model remains about the same as before. Regarding the model to explain random slopes, the positive Level 1 female–cumulative GPA slope is moderated by high school social composition ( $\gamma = -1.400, p < .01$ ). This finding is consistent with factor\*covariate interactions, suggesting that slopes differ across levels of the factor. In this case, we can interpret the result as the impact of gender on cumulative GPA when the social composition in students' high school is held constant. This indicates that the impact of the female slope ( $\gamma = 0.28, p < .05$ ) on cumulative GPA also depends on high school composition ( $\gamma = -1.40, p < .05$ ). Combining the coefficients suggests that the female coefficient is  $-1.12$  ( $-1.40 + 0.28 = -1.12$ ), when low social composition is high. Therefore, when high school social composition is increased by a unit (i.e., from the grand mean of 0), the advantage in cumulative GPA for females reported in the table (i.e., 0.28) actually disappears. Of course, this type of social composition variable is likely a proxy for more complex relationships in students' academic preparation for doing postsecondary work that might exist across high school settings within the state.

The variance component table (Table 8.12) suggests that there is still intercept variance (average GPA) to be explained between high schools (Wald  $Z = 2.663$ , one-tailed  $p < .01$ ) and colleges (Wald  $Z = 2.006$ , one-tailed  $p < .05$ ). After adding the random slope, there is still significant variance to be explained in the gender-GPA slopes across high schools (Wald  $Z = 1.601$ , one-tailed  $p = .0495$ ).

**TABLE 8.11** Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	2.677	0.140	10.201	19.125	.000	2.366	2.989
gmlowses_mean	-1.201	0.384	32.258	-3.131	.004	-1.982	-0.420
gmlowses	0.014	0.037	1,718.681	.394	.693	-0.058	0.086
gmfemale	0.275	0.042	25.535	6.496	.000	0.188	0.362
gmlowses_mean * gmfemale	-1.400	0.428	26.800	-3.270	.003	-2.279	-0.521

<sup>a</sup> Dependent variable: CUM\_GPR.

**TABLE 8.12** Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Residual	0.297	0.010	29.017	.000	0.278	0.318	
Intercept + gmfemale	Var: Intercept	0.034	0.013	2.663	.008	0.016	0.071
[subject = nschcode]	Var: gmfemale	0.020	0.013	1.601	.109	0.006	0.070
Intercept [subject = campus]	Variance	0.172	0.086	2.006	.045	0.065	0.456

<sup>a</sup> Dependent variable: CUM\_GPR.

## Developing a Cross-Classified Teacher Effectiveness Model

Our second example considers the effects of teacher effectiveness on students' outcomes in math. It is similar to our example in Chapter 4, except this time we examine the effects of successive teachers' effectiveness on students' current math scores. Multilevel models hold promise for examining the contributions of successive teachers and schools to student learning because they make better use of the available information than more simplified analyses (Thum, 2003). More specifically, over time, students are clustered in classrooms with teachers of differing skills and effectiveness, and in schools with differing community expectations, teacher skills, and academic processes. School factors can moderate learning conditions within classrooms (e.g., prolonged grouping assignments, differential access to curriculum, and inconsistent academic experiences for students within a particular grade level). Students, therefore, share similarities with peers within these hierarchical social groupings. The sum of these varied learning contexts enhances or diminishes students' academic outcomes in direct and indirect ways.

### The Data Structure and Model

In this example, we use a subset of student data ( $N = 4,136$ ) cross-classified by two successive classrooms at Level 2 (i.e., identified by 324 Year 1 teachers and 259 Year 2 teachers). At Level 3, the students attend 81 elementary schools. The data structure is different from the three-level model in Chapter 4 (i.e., students nested within classrooms within schools) because at the classroom level the data are "cross-classified"; that is, any two students in the cohort can have different combinations of Year 1 and Year 2 teachers (i.e., both teachers in common, one teacher in common, or no teachers in common). The variables are summarized in Table 8.13.

**TABLE 8.13** Data Definition of *ch8crossclass2.sav* ( $N = 4,136$ )

Variable	Level <sup>a</sup>	Description	Values	Measurement
<i>schcode</i>	School	School identifier (40 schools).	Integer	Ordinal
<i>teach1id</i>	Class	Identifier for students' first-year classroom teacher (324 teachers).	Integer	Scale
<i>teach2id</i>	Class	Identifier for students' second-year classroom teacher (259 teachers).	Integer	Scale
<i>math1</i>	Individual	Student math achievement scores from first-year classroom.	499 to 775	Scale
<i>math2</i>	Individual	Student math achievement scores from second-year classroom.	523 to 770	Scale
<i>female</i>	Individual	Demographic predictor variable representing students gender.	0 = Male 1 = Female	Scale
<i>lowses</i>	Individual	Dichotomous variable representing student socioeconomic status.	0 = Did Not Participate, 1 = Participant in Federal Free/Reduced Lunch Program	Scale
<i>Zmath1</i>	Individual	Standardized variable measuring math achievement.	-2.41 to 4.33	Scale
<i>effmath1</i>	Class	Predictor variable measuring first-year teacher effectiveness in classroom1 on a scale of 0 to 10 in terms of their classroom effectiveness in facilitating student learning.	(0, 2, 4, 6, 8, 10)	Scale
<i>effmath2</i>	Class	Predictor variable measuring first-year teacher effectiveness in classroom 2 on a scale of 0 to 10 in terms of their classroom effectiveness in facilitating student learning.	(0, 2, 4, 5, 6, 7, 8, 10)	Scale
<i>schqual</i>	School	Variable measuring the quality of the schools educational processes (e.g., leadership, academic expectations, and climate).	-4.8 to 1.8	Scale

<sup>a</sup> Individual = Level 1; Class = Level 2; School = Level 3.

The cross-classification of students by their previous and current teachers facilitates the accuracy of estimating the effects of successive teachers on student outcomes within the same model. One limitation to keep in mind about cross-classified models in examining the effects of successive teachers, however, is that the model assumes that the effects of previous teachers do not diminish (i.e., since each classroom contributes independently) in explaining the current achievement level (McCaffrey, Lockwood, Koretz, Louis, & Hamilton, 2004). The data structure in this example is similar to that in Figure 8.1.

Figure 8.1 indicates Student 1 and Student 2 have the same first-year teacher and second-year teacher. Students 3 and 4 have the same first-year teacher as the previous two students but different second-year teachers. Students 5 and 6 have different first-year and second-year teachers from each other, but Student 5 has the same second-year teacher as Student 4. All six of the students (with different configurations of first and second teachers) attend the same school (1). Student 7 begins a different school (2) and combination of teachers. Because we are focusing on two measurement occasions, this approach is similar to a type of gain score achievement model or a model that uses previous achievement as a control variable.

We reiterate that for cross-classified models, each RANDOM command identifies a separate cell or level. Their order does not matter. With cross-classified data structures, we need to use the unique Teacher 1 and Teacher 2 identifiers at Level 2 and not recode (i.e., rank) them within each school (1, ...,  $n_k$ ). The use of unique teacher identifiers generally will result in a longer computing time needed for estimating cross-classified models compared with similar nested multilevel models. For this example, we reduced the student part of the data set substantially (i.e., from more than 9,000 students to 4,136 students) and also reduced the number of Year 1 and Year 2 teachers and schools in order to decrease the time it takes to run the model from well over 1 hour to the present 4 to 5 minutes (on the final models). We note that the cost of reducing the data for demonstrating the approach, however, is a model that is a bit more challenging to fit optimally in terms of the our substantive goals (e.g., the variability in intercepts and slopes across units is reduced).

## Research Questions

In this cross-classified example, we can address a number of different types of research questions. For example, we might first consider: How much variance in students' math achievement is due to their previous and current classroom settings? In examining this question, we can consider whether the intercepts describing levels of Year 2 student achievement vary across classrooms and schools. A second question we might investigate is the following: Does the effectiveness of

	id	teacher1	teacher2	schcode	math1	math2
1	1	1	1	1	525	600
2	2	1	1	1	610	625
3	3	1	3	1	510	530
4	4	1	4	1	520	555
5	5	2	4	1	591	610
6	6	3	5	1	600	610
7	7	4	6	2	523	527

FIGURE 8.1 Horizontal data matrix for cross-classification analysis in IBM SPSS.

successive teachers have a measurable effect on students' Year 2 achievement levels? Subsequently, we could also investigate whether teacher effectiveness varies across schools. Third, we might ask: Does a particular variable (or set) explain possible variation in teacher effectiveness?

### Model 2.1: Intercept-Only Model (Null)

We will begin with an intercept-only model. This model represents the average achievement score across all individuals and occasions. It provides a preliminary estimate of the repeated measures variance, the classroom-level variance, and the school-level variance. Within individuals (Level 1), the model is the following:

$$Y_{i(j_1 j_2)k} = \pi_{0(j_1 j_2)k} + \varepsilon_{i(j_1 j_2)k}, \quad (8.14)$$

where  $Y_{i(j_1 j_2)k}$  is the math outcome for individual  $i$  cross-classified in classrooms  $j_1$  and  $j_2$  in school  $k$ ,  $\pi_{0(j_1 j_2)k}$  is the intercept, and  $\varepsilon_{i(j_1 j_2)k}$  is the residual term for individual-level model.

At Level 2, the intercept model is

$$\pi_{0(j_1 j_2)k} = \beta_{00k} + u_{j_1k} + u_{j_2k}, \quad (8.15)$$

where  $\beta_{00k}$  is the intercept,  $u_{j_1k}$  and  $u_{j_2k}$  are residuals associated with Classroom 1 and Classroom 2 in school  $k$ .

At Level 3, the basic model is

$$\beta_{00k} = \gamma_{000} + v_{00k}. \quad (8.16)$$

Through substitution, the basic random-intercept model can be specified as follows:

$$Y_{i(j_1 j_2)k} = \gamma_{000} + v_{00k} + u_{j_1k} + u_{j_2k} + \varepsilon_{i(j_1 j_2)k}, \quad (8.17)$$

where  $\gamma_{000}$  is the grand mean of achievement for schools, and  $v$ ,  $u$ , and  $\varepsilon$  represent residuals (i.e., normally and independently distributed in the population with an expected mean equal to 0 and some variance) associated with schools, classrooms, and students, respectively. For demonstration purposes, we note that we assume a scaled identity covariance structure at Level 2, with no covariance between random effects at that level. Equation 8.17 therefore implies five parameters that must be estimated (i.e., four variance components and one fixed effect). We can confirm the five estimated effects from the printout from Model 2.1 (Table 8.14).

**TABLE 8.14 Model Dimension<sup>a</sup>**

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
Random Effects	Intercept	1	Identity	1	schcode
	Intercept	1	Identity	1	teach2id
	Intercept	1	Identity	1	teach1id
Residual				1	
Total		4		5	

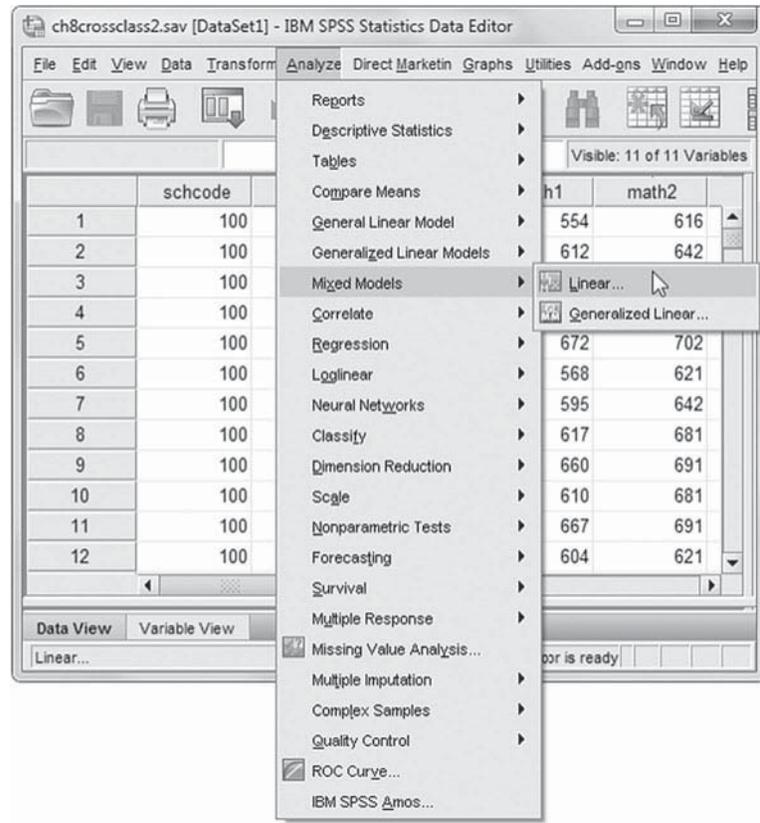
<sup>a</sup> Dependent variable: math2.

### Defining Model 2.1 (Null) with IBM SPSS Menu Commands

Launch the IBM SPSS program application, and select the *ch8cross-class2.sav* data file.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

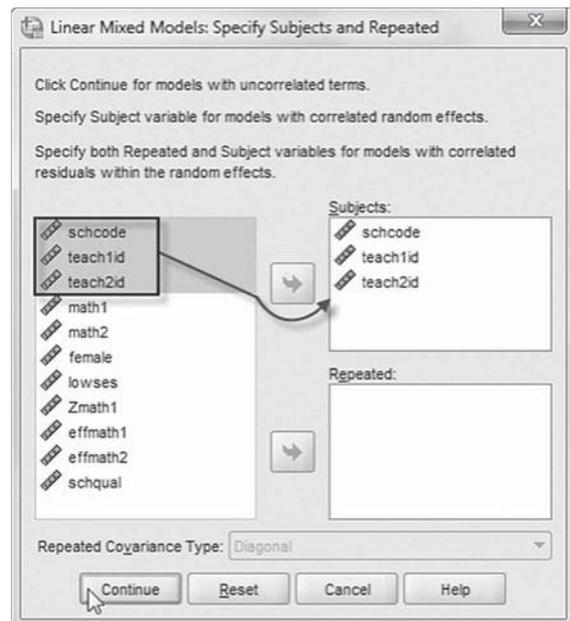
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.

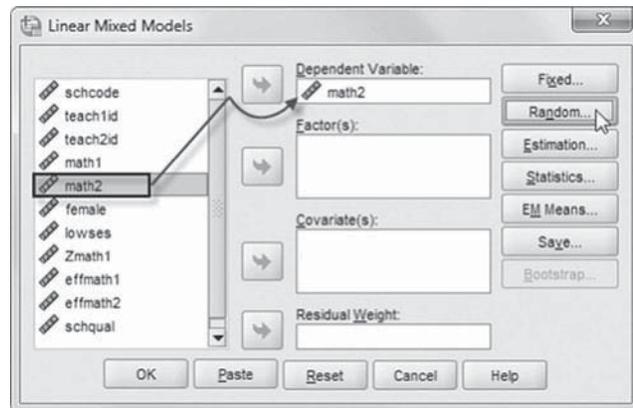
A subject is an observational unit that may be independent of other subjects. For this model, we will designate three subject identifiers for the model (*schcode*, *teach1id*, *teach2id*). Click to select *schcode*, *teach1id*, and *teach2id*, and then click the right-arrow button to move the variables into the *Subjects* box.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, and covariates, as well as accessing dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save*.

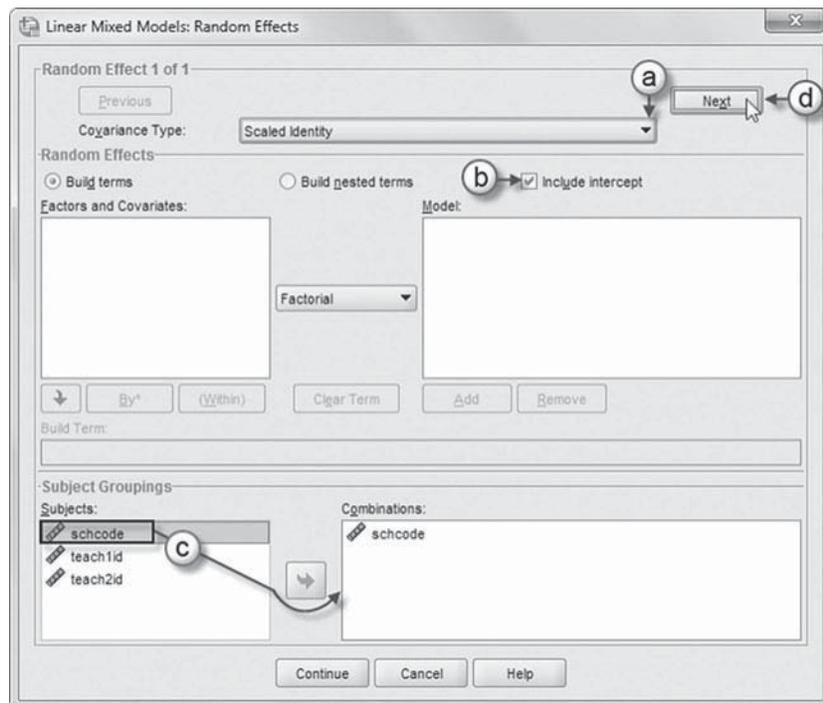
For this model, we will use *math2* as the dependent variable. Click to select *math2* from the left column listing, and then click the right-arrow button to transfer the variable into the *Dependent Variable* box.



We will now add random effects to this model.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 1* screen, which is the default when creating a model for the first time. The random-effects screen allows specifying random effects, interactions, intercept terms, and subject groupings.
- Begin by specifying the covariance structure from the default (VC) to scaled identity. Click the pull-down menu and select *Scaled Identity* (ID).



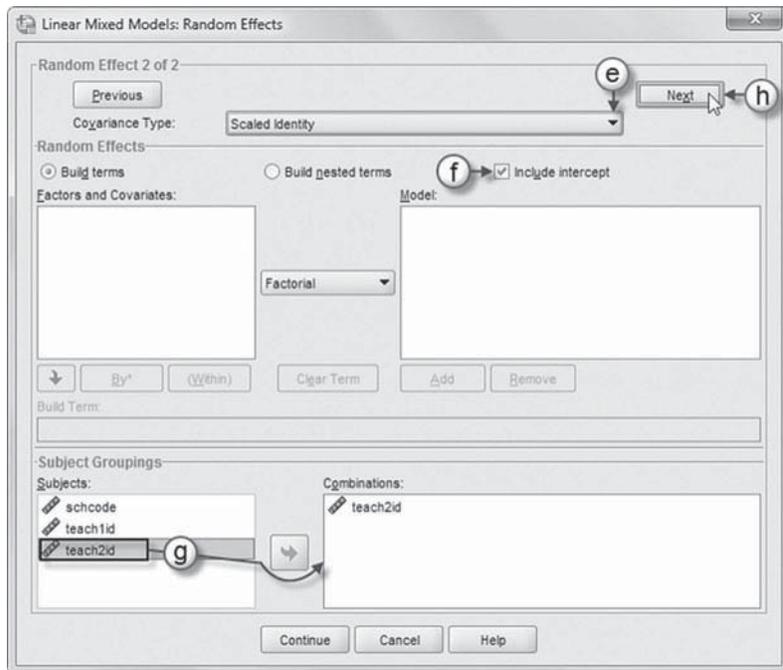
The scaled identity structure has constant variance and assumes that no correlation occurs between elements (IBM Corporation, 2012).

- We want the intercept to be included in the model, so click *Include intercept*.
- The *Subject Groupings* box displays the *schcode*, *teach1id*, and *teach2id* variables that were specified as a subject variable in the *Specify Subjects and Repeated* dialog box shown in step 2. We will specify *schcode* as the subject for the random-effects Level 3 part of this model. Click to select *schcode*, and then click the right-arrow button to move the variable into the *Combinations* box.
- At the top-right section of the window, click the NEXT button to access the *Random Effect 2 of 2* screen.

**Note:** The NEXT button may not work in earlier or unpatched versions of IBM SPSS when creating multilevel models with random intercepts. An update issued by IBM SPSS for software Version 19 addressed the problem, and Version 20 appears to have resolved the issue. A workaround to activating the NEXT button is to either (a) add or reenter a subject variable into the *Combinations* box or (b) add a variable from the *Factors and Covariates* column to the *Model* box and then remove it before proceeding to the *Random Effect 2 of 2* screen.

The *Random Effect 2 of 2* screen display is similar to the first screen and requires the following changes.

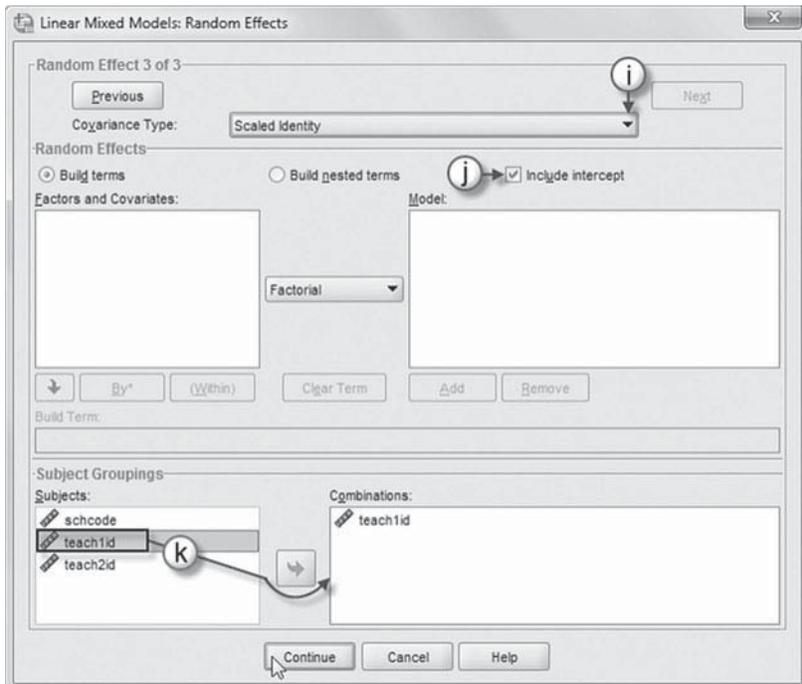
- e. Change the covariance type by clicking on the pull-down menu and selecting *Scaled Identity*.
- f. Click to select the *Include intercept* option.
- g. We will specify *teach2id* as the subject for the random-effects Level 2 part of this model. Click to select *teach2id*, and then click the right-arrow button to move the variable into the *Combinations* box.
- h. At the top-right section of the window, click the NEXT button to access the *Random Effect 3 of 3* screen.



**Note:** The NEXT button may not work in earlier or unpatched versions of IBM SPSS when creating multilevel models with random intercepts. An update issued by IBM SPSS for software Version 19 addressed the problem, and Version 20 appears to have resolved the issue. A workaround to activating the NEXT button is to either (a) add or reenter a subject variable into the *Combinations* box or (b) add a variable from the *Factors and Covariates* column to the *Model* box and then remove it before proceeding to the *Random Effect 3 of 3* screen.

The *Random Effect 3 of 3* screen display is similar to the prior screen and requires the following changes.

- i. Change the covariance type by clicking on the pull-down menu and selecting *Scaled Identity*.
- j. Click to select the *Include intercept* option.
- k. We will specify *teach1id* as the subject for the random-effects Level 3 part of this model. Click to select *teach1id*, and then click the right-arrow button to move the variable into the *Combinations* box.

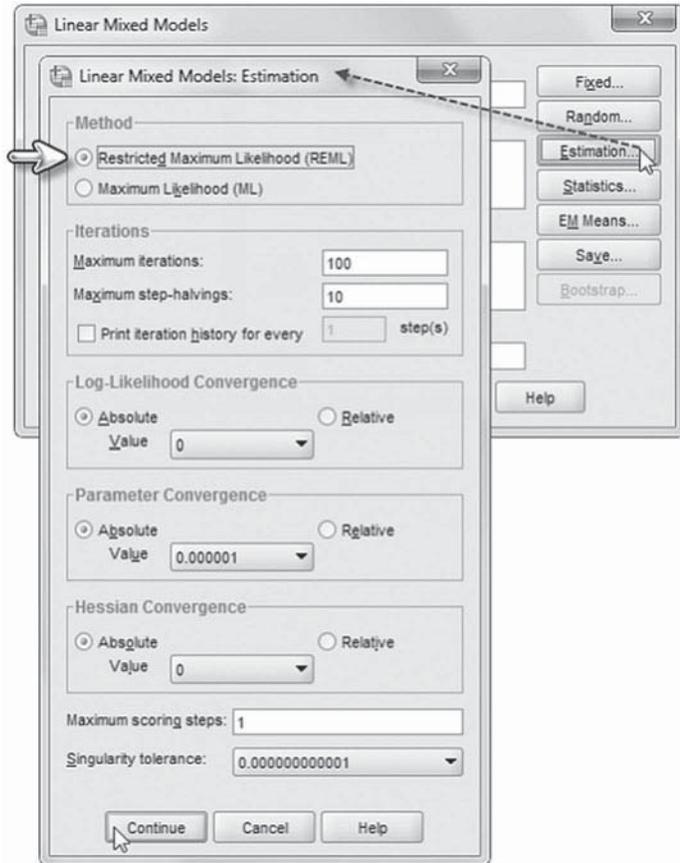


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Click the ESTIMATION button to access the *Linear Mixed Models: Estimation* dialog box, which displays two estimation method choices: ML or REML.

In this chapter, we will use the default setting of REML to estimate the models.

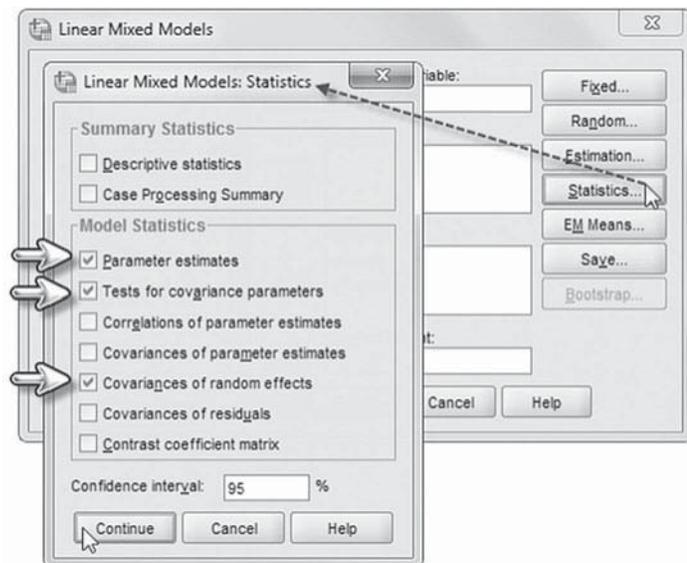
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



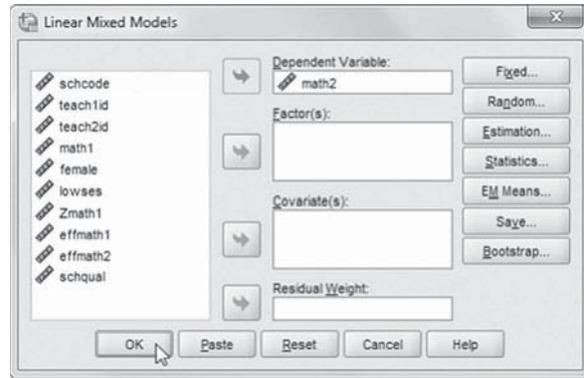
6. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting Output From Model 2.1 (Null)**

Our first model facilitates examining the distribution of math achievement in the data file with 4,136 students. The fixed-effect part of the model (Table 8.15) includes only the Level 3 intercept (or grand mean) for Year 2 math. The grand mean is 654.131 (*SE* = 2.047).

The variance components table (Table 8.16) suggests that most of the variance is due to differences in individuals (residual = 1,482.261), with about 14.7% of the total variation due to schools (i.e., 266.734/1,810.801 = 0.147), and trivial amounts of variance (about 2%) due to Teacher 2 and Teacher 1 in this subset of the data. As a comparison, in the full data set, the initial estimates for Classroom 1 variance was 4%, and for Classroom 2 variance, it was 7%.

**TABLE 8.15 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	654.131	2.047	77.947	319.509	.000	650.055	658.207

<sup>a</sup> Dependent variable: math2.

**TABLE 8.16 Estimates of Covariance Parameters<sup>a</sup>**

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		1,482.261	34.592	42.850	.000	1,415.989	1,551.635
Intercept	Variance	266.734	52.720	5.059	.000	181.066	392.934
	[subject = schcode]						
Intercept	Variance	35.184	15.158	2.321	.020	15.123	81.858
	[subject = teach2id]						
Intercept	Variance	26.622	12.915	2.061	.039	10.288	68.893
	[subject = teach1id]						

<sup>a</sup> Dependent variable: math2.

**Model 2.2: Defining the Cross-Classified Model with Previous Achievement**

For Model 2.2, we will add a covariate for previous achievement with a fixed (i.e., the same) coefficient for all persons; that is, we will first treat the effect as fixed across classrooms and schools. In defining the Level 1 model, we now account for previous achievement at Level 1. The Level 1 model is now

$$Y_{i(j)2k} = \pi_{0(j)2k} + \pi_{1(j)2k}Zmath1_{i(j)2k} + \epsilon_{i(j)2k}, \tag{8.18}$$

where, in addition to the parameters previously described,  $\pi_{1(j|2)k}$  represents the effect of beginning math achievement on Year 2 achievement. Entering previous achievement as a covariate is similar to creating a gain score model since there are only two achievement measurements in the model (see McCaffrey et al., 2004, for further discussion of teacher effectiveness models).

At the classroom level, we will consider beginning math achievement as fixed across classrooms:

$$\pi_{1(j|2)k} = \beta_{10k} \quad (8.19)$$

At Level 3 (school level), the equation with a fixed slope for previous achievement is the following:

$$\beta_{10k} = \gamma_{100} \quad (8.20)$$

From Equation 8.18, after substitution, the basic random-intercept model can be specified as follows:

$$Y_{i(j|2)k} = \gamma_{000} + \gamma_{100} Z_{math1}_{i(j|2)k} + v_{00k} + u_{j1k} + u_{j2k} + \varepsilon_{i(j|2)k}, \quad (8.21)$$

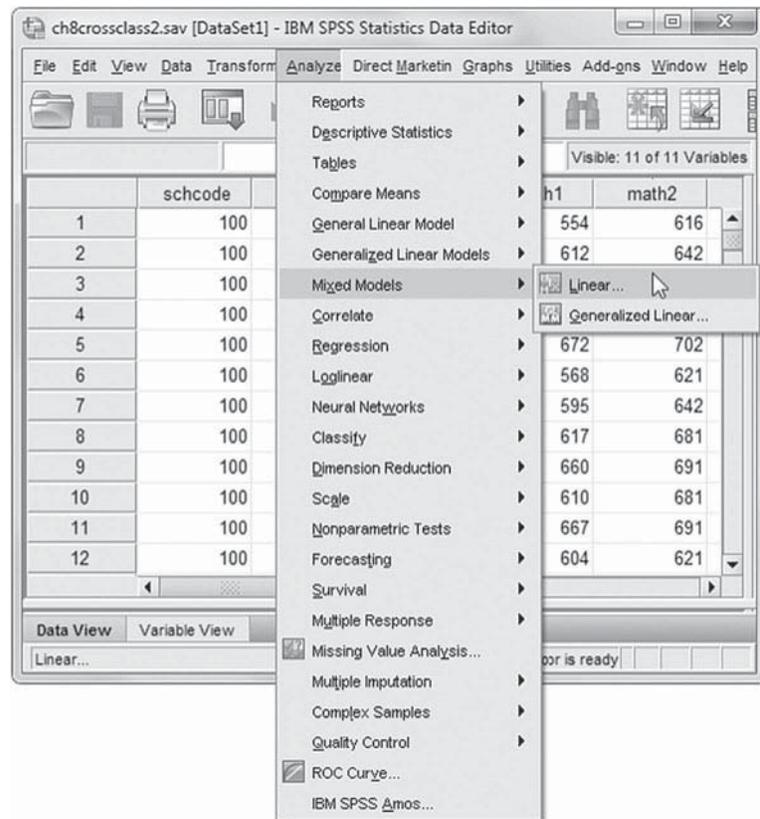
where  $\gamma_{000}$  is the grand mean of achievement for schools adjusted for initial scores;  $\gamma_{100}$  is the effect for previous achievement for schools; and  $v$ ,  $u$ , and  $\varepsilon$  represent residuals (i.e., normally and independently distributed in the population with expected mean values equal to 0 and some variance) associated with schools, classrooms, and students, respectively. This model specifies six estimated parameters (i.e., two fixed effects, three random variance components, and one residual variance).

### Defining Model 2.2 with IBM SPSS Menu Commands

Settings will default to those used in Model 2.1.

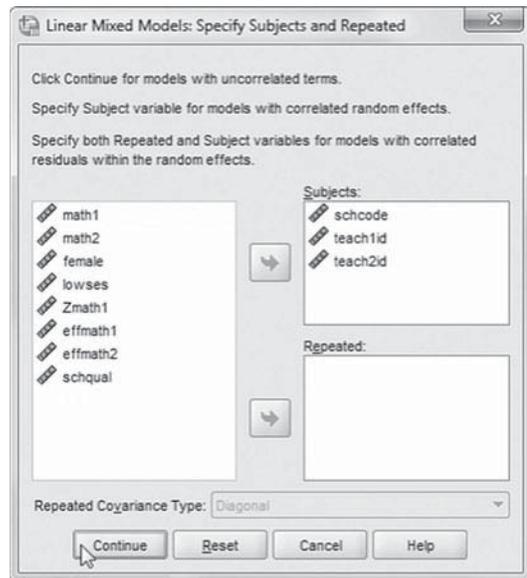
1. Go to the toolbar and select **ANALYZE, MIXED MODELS, LINEAR**.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



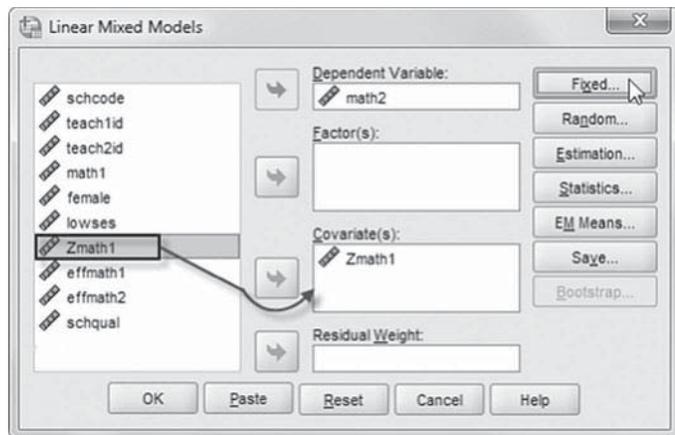
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

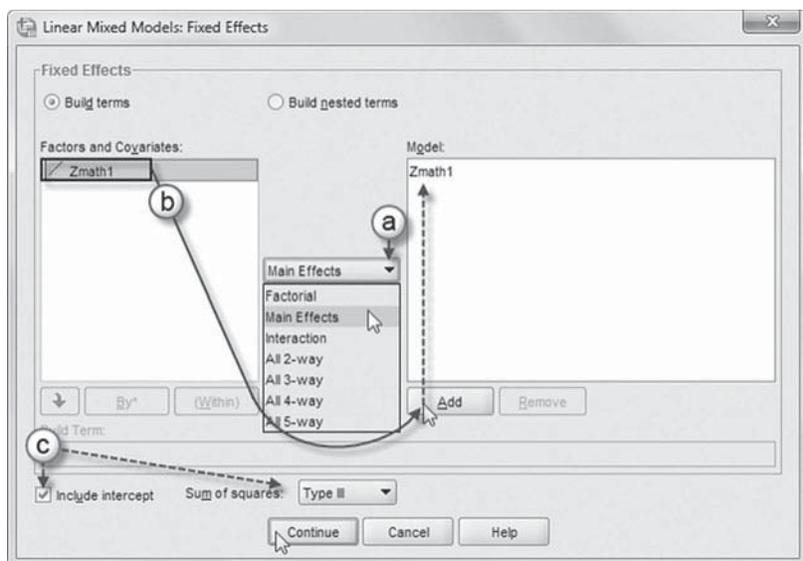


3. From the *Linear Mixed Models* main screen, we will add a variable to the analysis. Click to select *Zmath1*, and then click the right-arrow button to move the variable to the *Covariate(s)* box.

We will now add a fixed effect to the model, so click the FIXED button to access the *Fixed Effects* main screen.

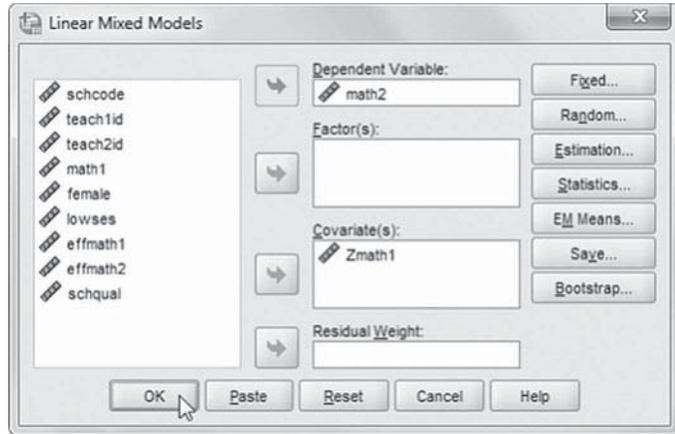


- 4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
- b. Click to select *Zmath1* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
- c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 2.2**

In Model 2.2, we added a standardized (mean  $[M] = 0$ , standard deviation  $[SD] = 1$ ) previous achievement variable, which changes the intercept (which now represents achievement at Time 2 adjusted for beginning achievement) to 653.48. Table 8.17 suggests that previous math achievement affects subsequent achievement ( $\gamma_{100} = 28.62, p < .01$ ). This coefficient suggests that a 1-SD increase in *Zmath1* would produce a 28.62-point increase in the Math 2 score.

If we compare the variance components of Model 2.1 (Table 8.16) and Model 2.2 (Table 8.18), we see that entering the previous achievement variable into the model decreases the Level 1 variance considerably (i.e., from 1,482.26 to 767.06). This represents a reduction in variance (or  $R^2$ ) of about 48.3% ( $715.2/1,482.26 = 0.483$ ). Notice also, however, that the size of the Classroom 1 variance ( $\sigma^2_{class1} = 54.16$ ) and Classroom 2 variance ( $\sigma^2_{class2} = 38.20$ ) components are somewhat larger than in Model 2.1.

**TABLE 8.17 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	653.482	1.313	82.082	497.571	.000	650.869	656.094
Zmath1	28.620	0.473	4,066.184	60.542	.000	27.693	29.547

<sup>a</sup> Dependent variable: math2.

**TABLE 8.18 Estimates of Covariance Parameters<sup>a</sup>**

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		767.060	17.907	42.835	.000	732.753	802.973
Intercept [subject = schcode]	Variance	83.983	20.879	4.022	.000	51.591	136.711
Intercept [subject = teach2id]	Variance	38.202	9.855	3.876	.000	23.040	63.339
Intercept [subject = teach1id]	Variance	54.160	11.229	4.823	.000	36.074	81.314

<sup>a</sup> Dependent variable: math2.

As Hox (2010) notes, this often occurs in multilevel analyses, but makes it unclear which variance components to use in establishing a baseline model. As we noted in Chapter 1, this result can occur because the amount of variance explained is not a simple concept in multilevel modeling (Snijders & Bosker, 1994). Hox explains that the sampling process in multilevel designs creates some between-group variability in all Level 1 variables, even if there are, in fact, no real group differences in the population. The variability at successive levels (e.g., Level 2 or Level 3) may be much greater than the model assumes in calculating initial variance components, such that the intercept-only model (Model 1.1) can overestimate the variance at Level 1 and may underestimate the variance at Level 2 (Hox, 2010).

Model 2.2, with the previous achievement variable added, estimates the variance accounted for in the dependent variable, and, conditional on this effect, the variance components estimated for Level 2 may be a bit more accurately estimated. Hox (2010) suggests using the classroom variance components from Model 2.2 as the baseline from which to make comparisons in variance reduction. We note in passing that after accounting for previous achievement in our example, the variance in math achievement at the classroom level is about 6% for Class 1 and 4% for Class 2.

**Model 2.3: Adding Teacher Effectiveness and a Student Background Control**

We can next add the teacher effectiveness variables to the model. Individual teachers were evaluated from 0 to 10 in terms of their classroom effectiveness in facilitating student learning (after adjustment for student composition) based on their previous student cohort. The effectiveness scores were then transferred into the current student cohort. We display the variation in effectiveness scores in the following tables. Tables 8.19 and 8.20 suggest that teacher effectiveness was distributed similarly across both sets of classrooms in this sample data set.

In Model 2.2, at Level 1 we will add a control for student SES:

$$Y_{i(j1j2)k} = \pi_{0(j1j2)k} + \pi_{1(j1j2)k}Zmath1_{i(j1j2)k} + \pi_{2(j1j2)k}lowses_{i(j1j2)k} + \epsilon_{i(j1j2)k} \tag{8.22}$$

At Level 2, the model to explain intercepts can now be defined as

$$\pi_{0(j1j2)k} = \beta_{00k} + \beta_{01k}teacheff1_{(j1j2)k} + \beta_{02k}teacheff2_{(j1j2)k} + u_{j1k} + u_{j2k} \tag{8.23}$$

**TABLE 8.19 Descriptive Statistics**

	<b>N</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Deviation</b>
effmath1	324	0	10	5.15	2.959
Valid N (Listwise)	324				

**TABLE 8.20 Descriptive Statistics**

	<b>N</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Deviation</b>
effmath2	259	0	10	5.01	2.293
Valid N (Listwise)	259				

In this model, we will first assume that teacher effects do not vary randomly across schools:

$$\beta_{01k} = \gamma_{010}, \quad (8.24)$$

$$\beta_{02k} = \gamma_{020}. \quad (8.25)$$

We might also consider any number of classroom controls at this level (e.g., aggregated socioeconomic status of students in the classroom, aggregated academic background of students, and information about teachers' experience and other background characteristics). Because the teacher effectiveness scores were previously adjusted for student composition, we note that it is generally redundant to add similar background controls (e.g., SES) at both the student and classroom levels or to add similar controls (e.g., SES) for both classrooms since it would not be expected that there would be much difference in the classroom demographics within schools (or over time). This could change if schools grouped students within classrooms in some particular manner (e.g., by previous math ability).

The combined model with nine estimated parameters will now be the following:

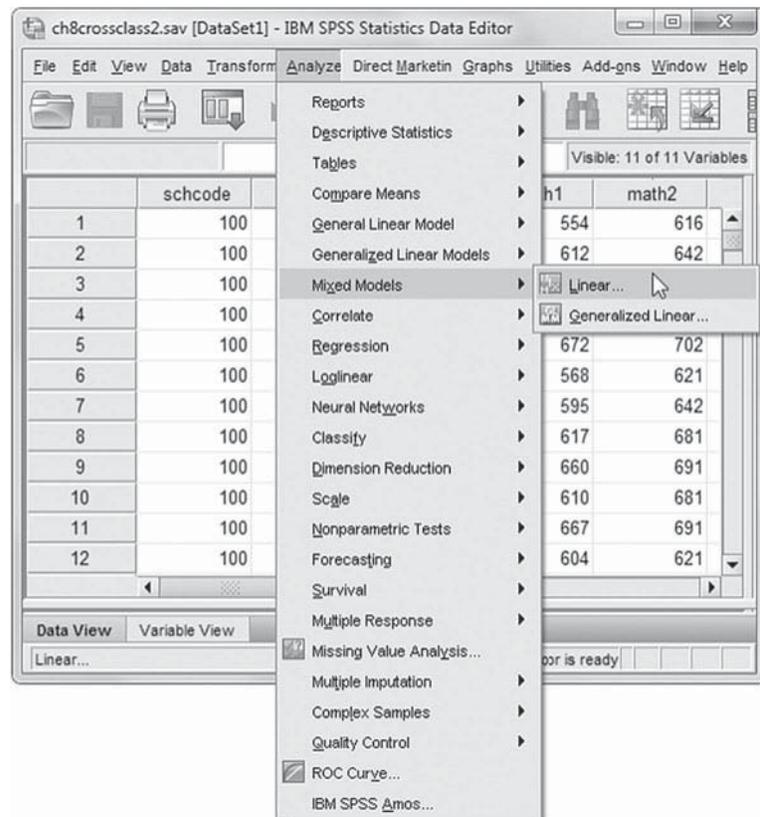
$$Y_{i(j_1j_2)k} = \gamma_{000} + \gamma_{010}teacheff1_{(j_1j_2)k} + \gamma_{020k}teacheff2_{(j_1j_2)k} + \gamma_{100}Zmath1_{i(j_1j_2)k} + \gamma_{200}loruses_{i(j_1j_2)k} + \nu_{00k} + u_{j_1k} + u_{j_2k} + \varepsilon_{i(j_1j_2)k}. \quad (8.26)$$

### Defining Model 2.3 with IBM SPSS Menu Commands

**Note:** Settings will default to those used in Model 2.2.

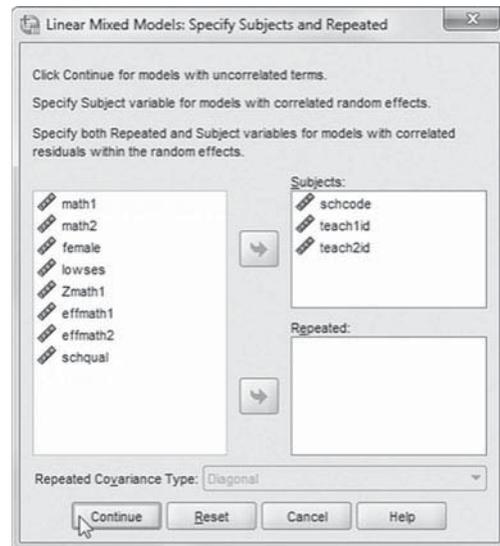
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

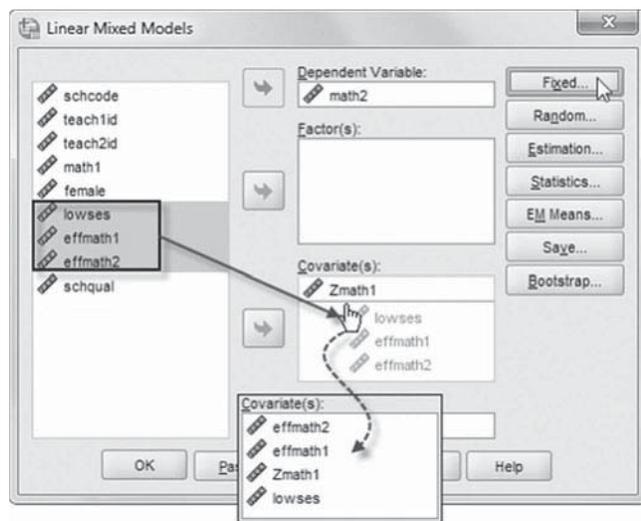


3. The *Linear Mixed Models* dialog box displays the default settings for Model 2.2.

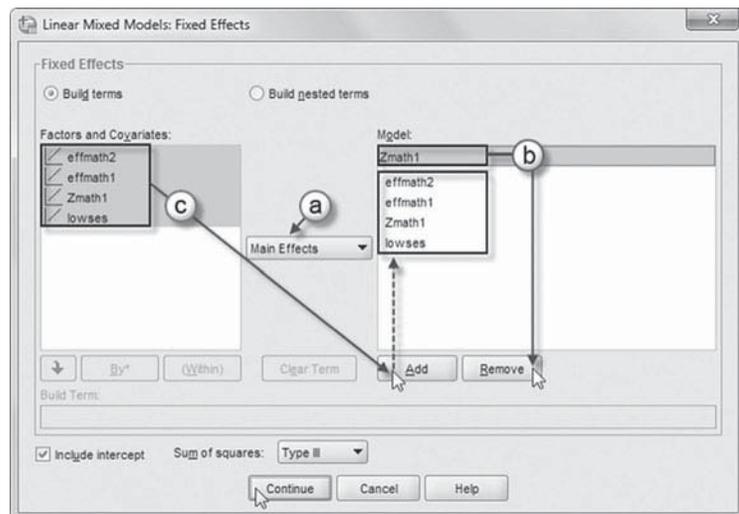
We will add three predictors to the model (*lowses*, *effmath1*, and *effmath2*). Click to select the three variables, and then “drag” them to *Covariate(s)* box. To facilitate reading of the output tables we will change the sequence of the variables by dragging individual variables into the following order (see insert): *effmath2*, *effmath1*, *Zmath1*, and *lowses*.

We may now proceed to define fixed effects for the variable.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

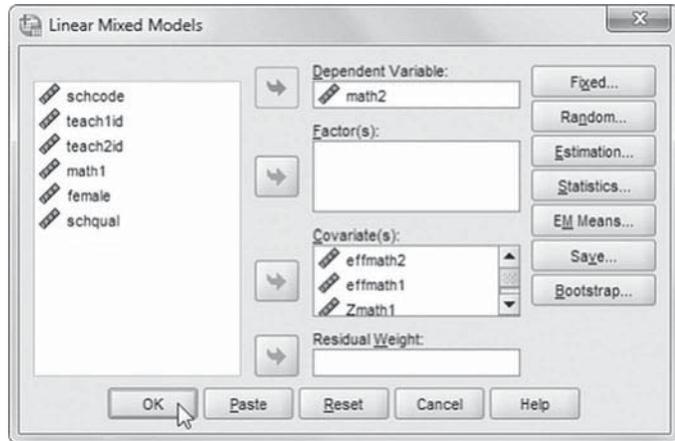


- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default *Main Effects* setting used in the prior model.
- b. We will rearrange the sequence order of the variables, so click to select *Zmath1*, and then click the REMOVE button to clear the *Model* box.
- c. Now click to select the four variables (*effmath2*, *effmath1*, *Zmath1*, and *lowses*) from the *Factors and Covariates* box, and then click the ADD button to move the variables into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



**Interpreting the Output From Model 2.3**

The fixed-effect estimates (Table 8.21) suggest that in this subset of the data, the effectiveness of the first teacher ( $\gamma = 1.09, p < .01$ ) contributed slightly more to ending math outcomes than the effectiveness of the second teacher ( $\gamma = 0.73, p = .01$ ). Both effects, however, would be considered relatively small.

The variance components table (Table 8.22) suggests that after adding the teacher effects for Classroom 1 and Classroom 2 to the model, there is still variance in Year 2 achievement levels to be explained at the classroom and school levels. Adding the Level 2 effectiveness variables reduces the variance for each classroom, especially in the first classroom (from about 54.16 to 44.60 or  $R^2 = 0.177$ ). The school-level variance is also reduced slightly from adding the teacher effectiveness variables to the model (from 83.98 to 73.81, or  $R^2 = 0.121$ ).

**TABLE 8.21 Estimates of Fixed Effects<sup>a</sup>**

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	646.389	2.261	361.609	285.919	.000	641.943	650.835
effmath2	0.733	0.283	175.141	2.590	.010	0.174	1.292
effmath1	1.093	0.225	225.841	4.856	.000	0.649	1.536
Zmath1	28.426	0.487	4,077.812	58.317	.000	27.470	29.381
lowses	-4.498	0.974	4,090.252	-4.619	.000	-6.408	-2.589

<sup>a</sup> Dependent variable: math2.

**TABLE 8.22 Estimates of Covariance Parameters<sup>a</sup>**

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Residual	764.150	17.823	42.875	.000	730.005	799.893
Intercept [subject = schcode] Variance	73.806	18.827	3.920	.000	44.767	121.681
Intercept [subject = teach2id] Variance	35.451	9.493	3.734	.000	20.974	59.918
Intercept [subject = teach1id] Variance	44.595	10.065	4.431	.000	28.653	69.407

<sup>a</sup> Dependent variable: math2.

**Model 2.4: Adding a School-Level Predictor and a Random Slope**

Next, we might investigate whether teacher effectiveness varies across schools. At the school level, the model with random effectiveness slopes would be the following:

$$\beta_{01k} = \gamma_{010} + v_{01k}, \tag{8.27}$$

$$\beta_{02k} = \gamma_{020} + v_{02k}, \tag{8.28}$$

where  $\gamma_{010}$  and  $\gamma_{020}$  are the average teacher effectiveness means for Classroom 1 and Classroom 2, respectively, and we will use  $v_{01k}$  and  $v_{02k}$  as the respective school residuals for the slope effects. At Level 3, we will define a diagonal covariance matrix for the intercept and slope (i.e., because we determined that the unstructured covariance matrix did not converge in this model). The combined model with 11 estimated parameters will then be the following:

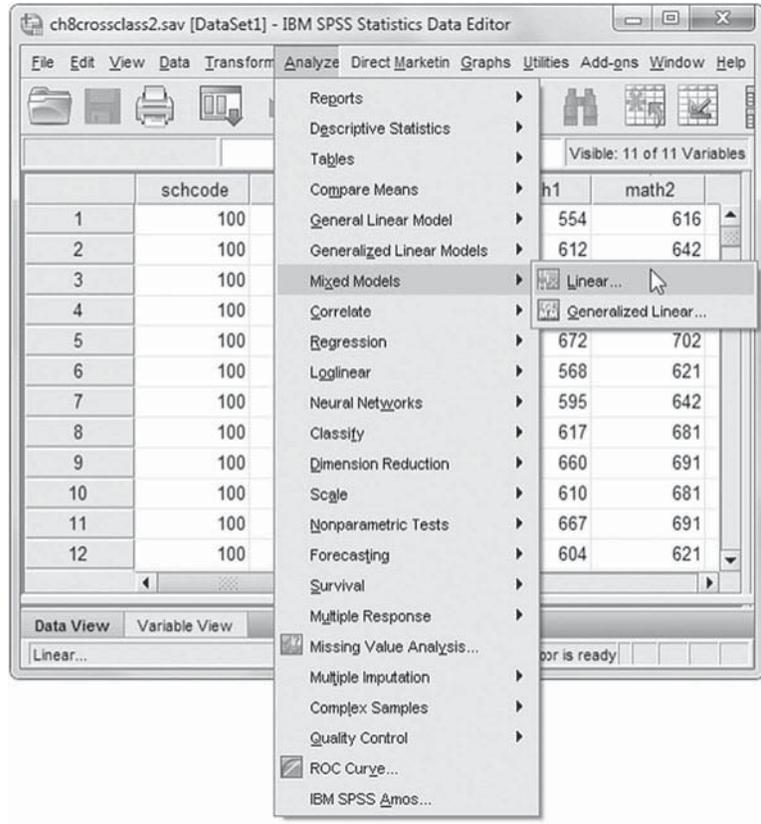
$$Y_{i(j_1,j_2)k} = \gamma_{000} + \gamma_{010}teacheff1_{(j_1,j_2)k} + \gamma_{020}teacheff2_{(j_1,j_2)k} + \gamma_{100}Zmath1_{(j_1,j_2)k} + \gamma_{200}lowwes_{i(j_1,j_2)k} + v_{00k} + v_{01k}teacheff1_{(j_1,j_2)k} + v_{02k}teacheff2_{(j_1,j_2)k} + u_{j_1k} + u_{j_2k} + \epsilon_{i(j_1,j_2)k}. \tag{8.29}$$

**Defining Model 2.4 with IBM SPSS Menu Commands**

**Note:** Settings will default to those used in Model 2.3.

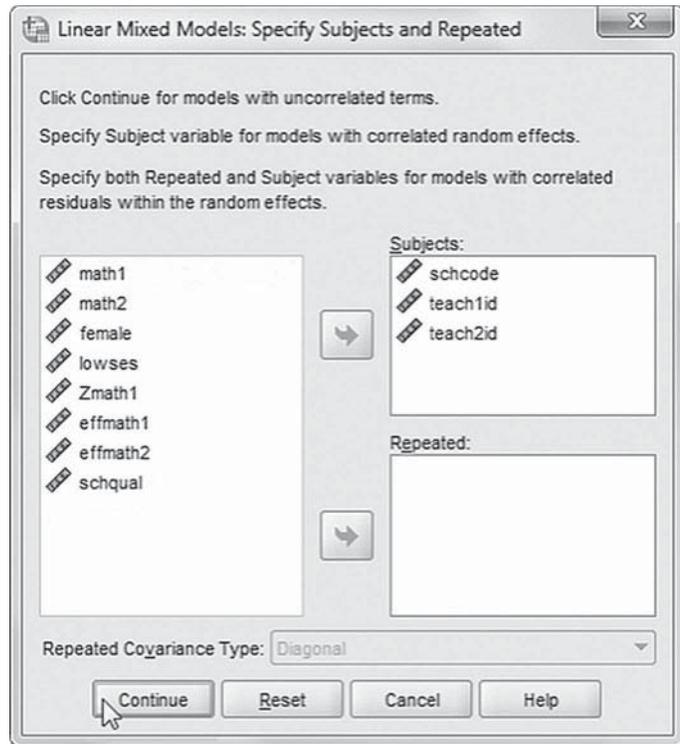
1. Go to the toolbar and select **ANALYZE, MIXED MODELS, LINEAR**.

This command enables access to the *Linear Mixed Models: Specified Subjects and Repeated* dialog box.



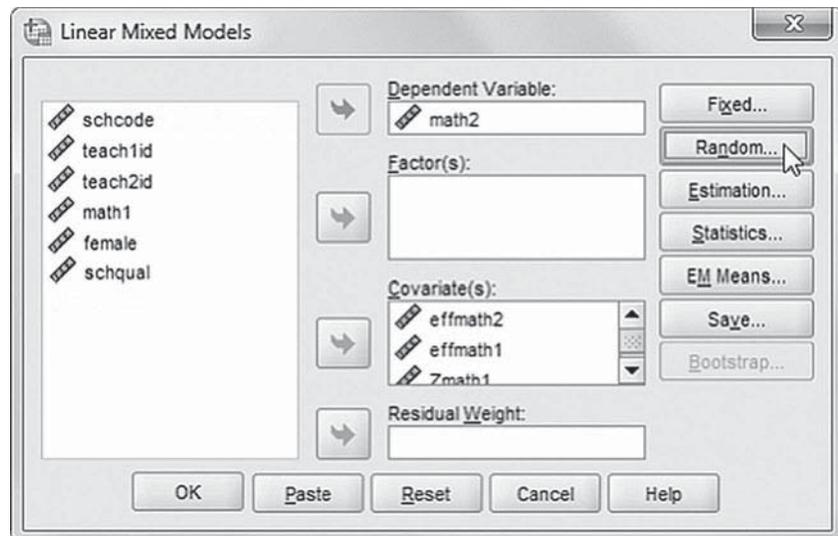
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

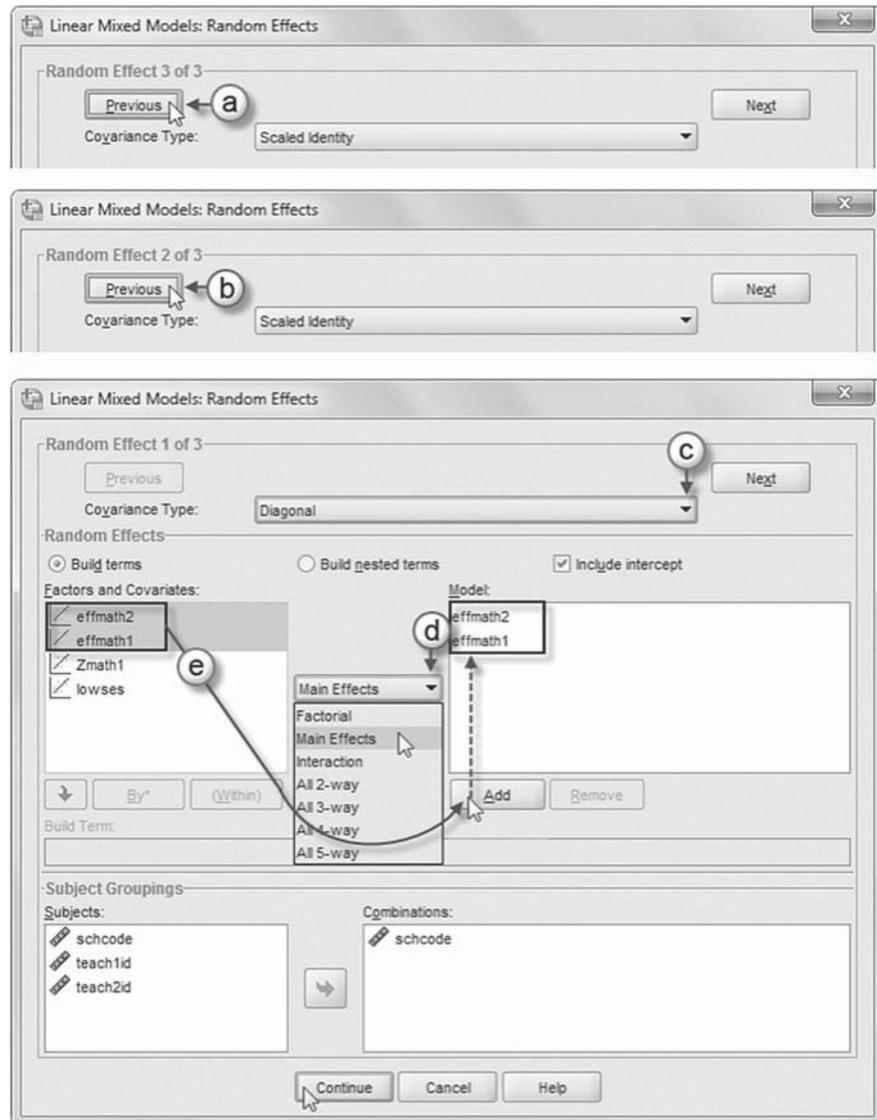


3. The *Linear Mixed Models* dialog box displays default settings from the prior model.

We will now modify the random effects to this model. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



- 4a. The *Linear Mixed Models: Random Effects* displays the *Random Effect 3 of 3* screen, which is the default from the prior model. We will change the Level 3 random effects but need to access the *Random Effect 1 of 3* dialog screen. First, click the PREVIOUS button to access the *Random Effect 2 of 3* main screen.
- b. From the *Random Effect 2 of 3* screen, click the PREVIOUS button to access the *Random Effect 1 of 3* main screen.
- c. From the *Random Effect 1 of 3* screen, change the covariance type by clicking the pull-down menu and selecting *Diagonal*.

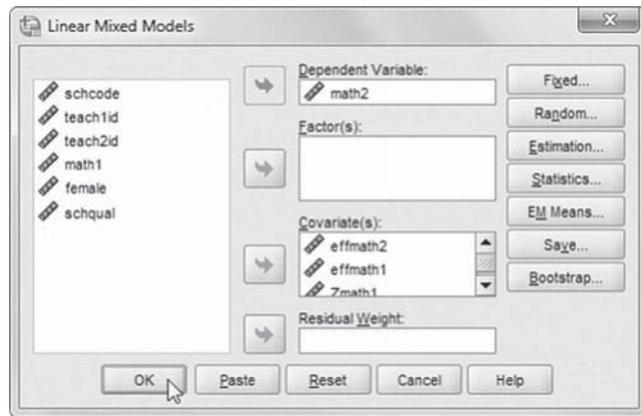


The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

- d. Change the default factorial setting by clicking the pull-down menu and selecting *Main Effects*.
- e. We will add two variables to the model (*effmath2* and *effmath1*). Click to select both variables, and then click the ADD button to move them to the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

- Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



### Interpreting the Output From Model 2.4

We first examine whether effectiveness varies across schools. We provide the variance components table first (Table 8.23). The table suggests that the effectiveness of Teacher 1 did not seem to vary across schools (one-tailed  $p = .205$ ). It may be, however, that the effectiveness of Teacher 2 does vary across schools (one-tailed  $p = .055$ ). Given these results, we can reformulate the Level 2 model by removing the random effect for Teacher 1 effectiveness ( $v_{01k}$ ) in Equation 8.27.

### Model 2.5: Examining Level 3 Differences Between Institutions

We will then add a school-level variable that may explain differences in math achievement between schools. In this example, we will use the quality of the school’s educational processes (e.g., the school’s leadership, academic expectations, and climate) as a possible predictor of differences in school math outcomes. At Level 3, the intercept model is the following:

$$\beta_{00k} = \gamma_{000} + \gamma_{001}schqual_k + v_{00k}, \tag{8.30}$$

where  $\gamma_{000}$  is the adjusted school-level mean,  $\gamma_{001}$  is the coefficient representing the impact of the school-level predictor (school quality) on math outcomes, and  $v_{00k}$  is the school-level residual. We could, of course, add other relevant school predictors to the model (e.g., school size, staff stability, and wstudent composition). The combined model with 11 estimated parameters will then be as follows:

$$Y_{i(j,2)k} = \gamma_{000} + \gamma_{001}schqual_k + \gamma_{010}teacheff1_{(j,2)k} + \gamma_{020k}teacheff2_{(j,2)k} + \gamma_{100}Zmath1_{i(j,2)k} + \gamma_{200}lowuses_{i(j,2)k} + v_{00k} + v_{02k}teacheff2_{(j,2)k} + u_{j1k} + u_{j2k} + \epsilon_{i(j,2)k}. \tag{8.31}$$

TABLE 8.23 Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Residual	764.311	17.834	42.857	.000	730.144	800.076	
Intercept + effmath2 + effmath1 Var: Intercept [subject = schcode]	46.838	21.193	2.210	0.027	19.295	113.695	
	Var: effmath2	0.788	0.493	0.493	0.110	0.231	2.687
	Var: effmath1	0.343	0.427	0.804	0.421	0.030	3.929
Intercept [subject = teach2id] Variance	29.796	9.257	3.219	0.001	16.207	54.780	
Intercept [subject = teach1id] Variance	41.053	10.449	3.929	0.000	24.928	67.608	

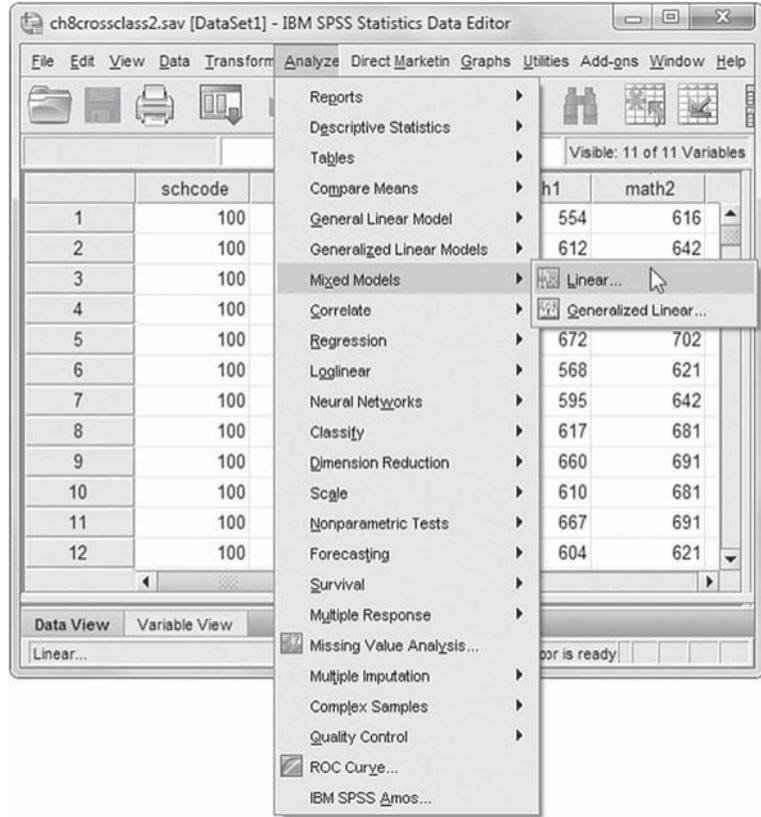
<sup>a</sup> Dependent variable: math2.

### Defining Model 2.5 with IBM SPSS Menu Commands

**Note:** Settings will default to those used in Model 2.4.

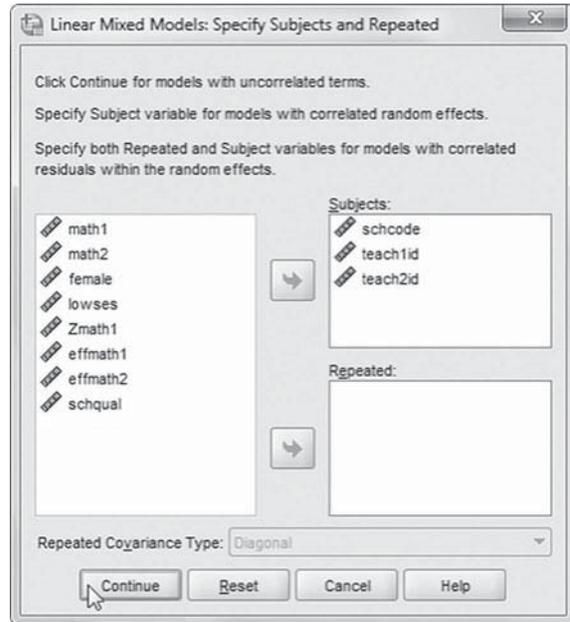
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

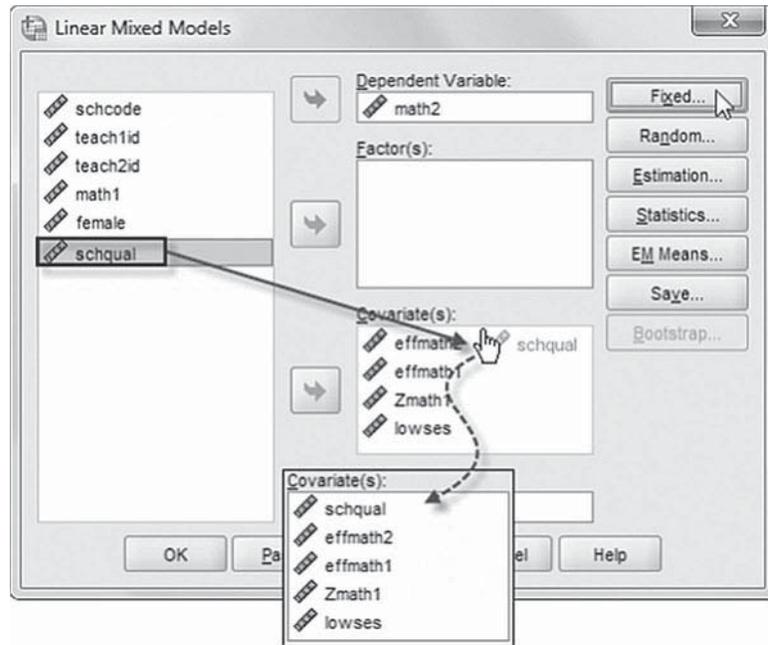
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box displays the default settings for Model 2.4.

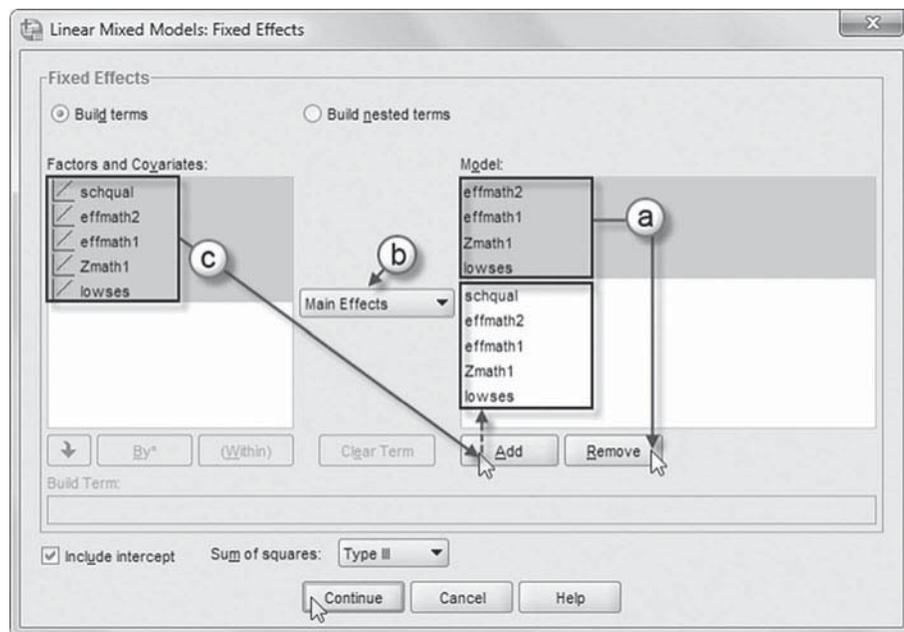
We will add one predictor (*schqual*) to the model. Click to select *schqual*, and then “drag” the variable to *Covariate(s)* box above *effmath*. This changes the sequence order of the variables (see insert) to the following: *schqual*, *effmath2*, *effmath1*, *Zmath1*, and *lowses*. We may now proceed to define fixed effects for the variable.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



- 4a. The *Linear Mixed Models: Fixed Effects* dialog box displays the default *Main Effects* setting used in the prior model.

We will rearrange the sequence order of the variables, so click to select *effmath2*, *effmath1*, *Zmath1*, and *lowses*. Then click the REMOVE button to clear the *Model* box.



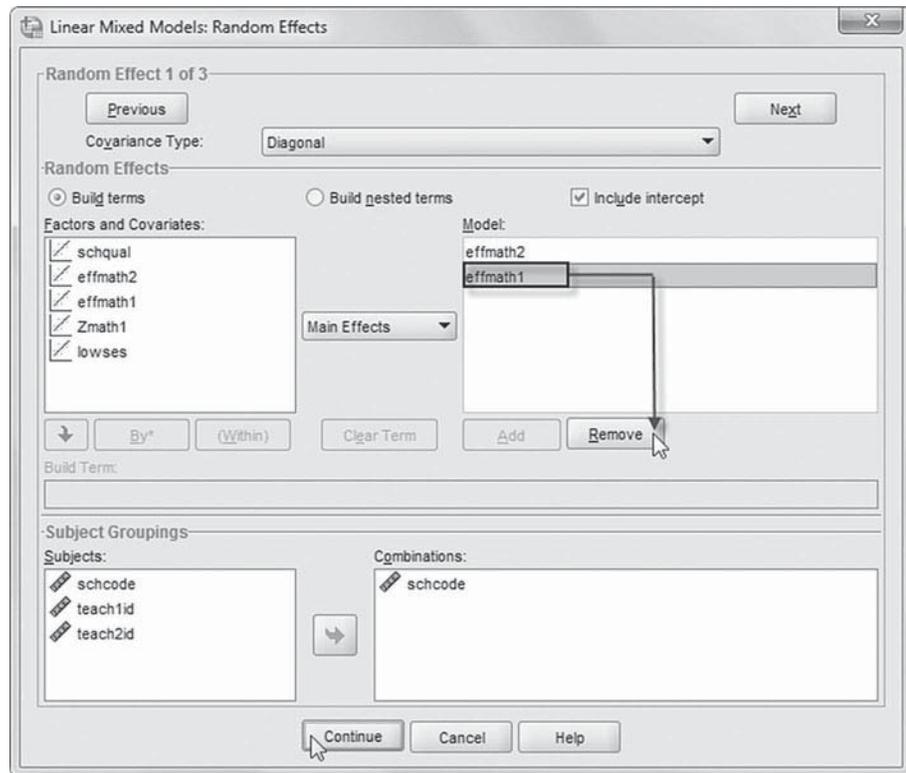
- b. The default setting is *Main Effects*.
- c. Now click to select the five variables (*schqual*, *effmath2*, *effmath1*, *Zmath1*, and *lowses*) from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. We will now modify the Level 1 random effects to this model. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. The *Linear Mixed Models: Random Effects* displays the *Random Effect 1 of 3 of 3* screen, which is the default from the prior model. We will change the Level 3 random effects by removing a variable from the model.

Click to select *effmath1*, and then click the REMOVE button.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



6. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

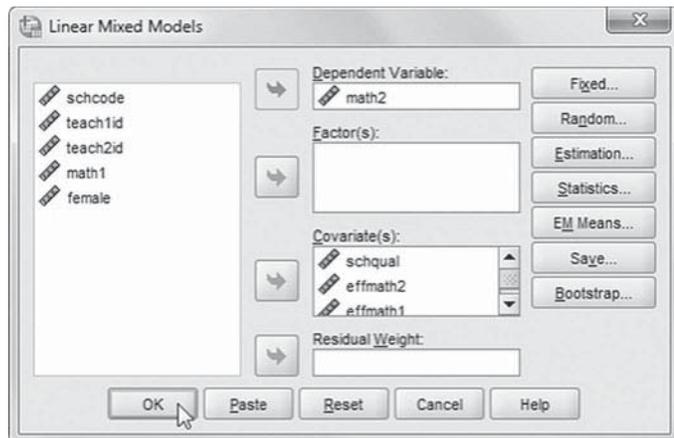


TABLE 8.24 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	646.029	2.195	306.310	294.294	.000	641.709	650.348
schqual	2.278	1.303	89.300	1.748	.084	-0.311	4.867
effmath2	0.735	0.309	120.918	2.377	.019	0.123	1.347
effmath1	1.109	0.225	224.520	4.926	.000	0.665	1.552
Zmath1	28.386	0.488	4,075.644	58.202	.000	27.430	29.343
lowses	-4.426	0.974	4,087.879	-4.543	.000	-6.337	-2.516

<sup>a</sup> Dependent variable: math2.

TABLE 8.25 Estimates of Covariance Parameters<sup>a</sup>

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval		
					Lower Bound	Upper Bound	
Residual	763.940	17.818	42.875	.000	729.804	799.672	
Intercept + effmath2	Var: Intercept	49.916	19.584	2.549	.011	23.136	107.694
[subject = schcode]	Var: effmath2	0.911	0.516	1.765	.078	0.300	2.764
Intercept [subject = teach2id]	Variance	29.650	9.231	3.212	.001	16.108	54.578
Intercept [subject = teach1id]	Variance	44.841	10.111	4.435	.000	28.823	69.762

<sup>a</sup> Dependent variable: math2.

### Interpreting the Output From Model 2.5

Table 8.24 provides some preliminary evidence that school quality is related to ending math achievement ( $\gamma = 2.28, p < .10$ ), remembering that we have a relatively small subset of 81 schools. The variance component table (Table 8.25) suggests that Teacher 2 effectiveness varies across schools (Wald  $Z = 1.765$ , one-tailed  $p = .039$ ).

### Model 2.6: Adding a Level 3 Cross-Level Interaction

Finally, we will add a school-level variable that might moderate the relationship between Teacher 2 effectiveness and Year 2 math achievement. In this case, we will use quality of the school's educational processes (e.g., the school's leadership, academic expectations, and climate). The Level 3 model for explaining the slope is now the following:

$$\beta_{02k} = \gamma_{020} + \gamma_{021}schqual_k + v_{02k}. \quad (8.32)$$

The combined model with 12 estimated parameters will then be

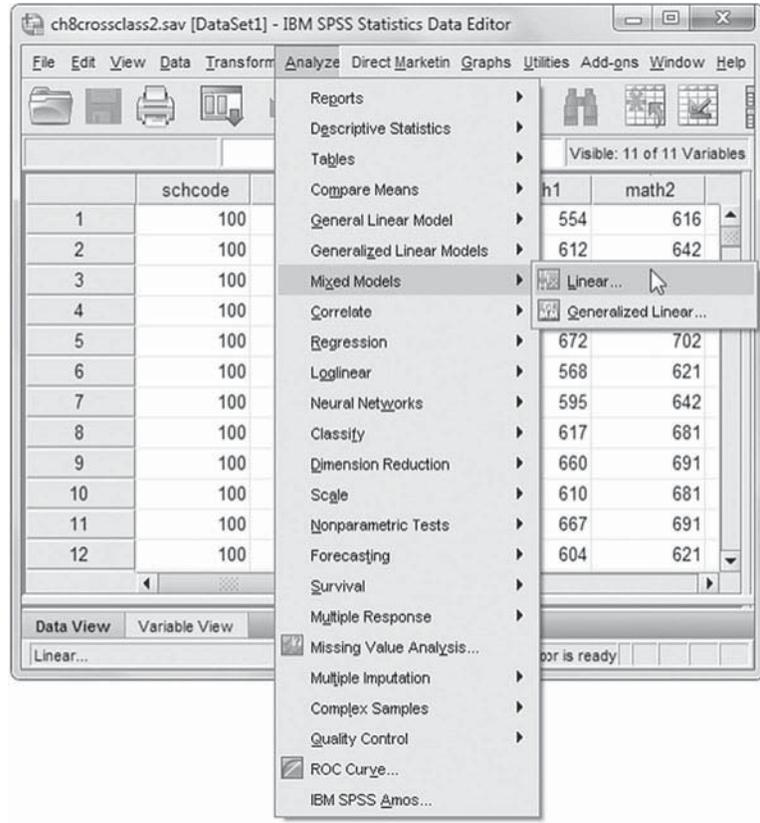
$$Y_{i(j,2)k} = \gamma_{000} + \gamma_{001}schqual_k + \gamma_{010}teacheff1_{(j,2)k} + \gamma_{020}teacheff2_{(j,2)k} + \gamma_{021}schqual_k * teacheff2_{(j,2)k} + \gamma_{100}Zmath1_{i(j,2)k} + \gamma_{200}lowses_{i(j,2)k} + v_{00k} + v_{02k}teacheff2_{(j,2)k} + u_{j1k} + u_{j2k} + \epsilon_{i(j,2)k}. \quad (8.33)$$

**Defining Model 2.6 with IBM SPSS Menu Commands**

**Note:** Settings will default to those used in Model 2.5.

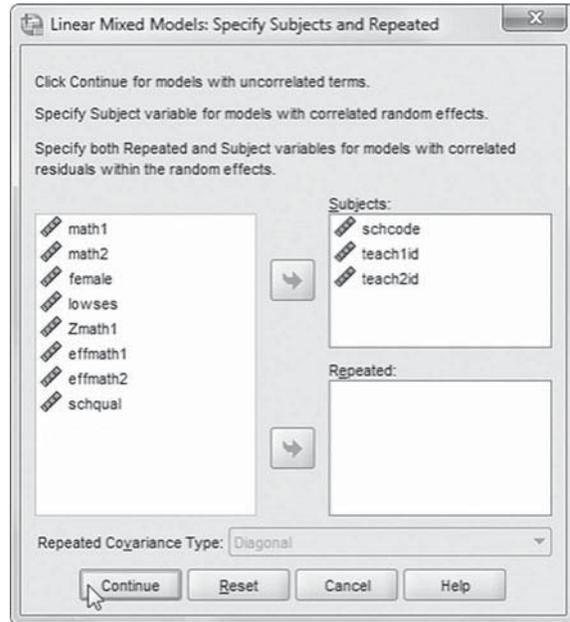
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays the default settings from the prior model.

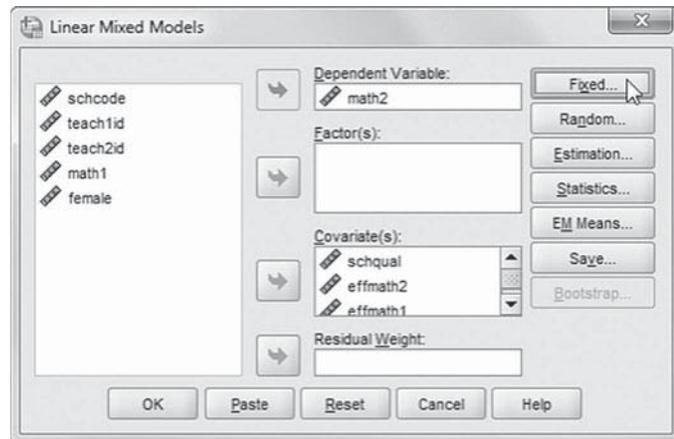
Click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* dialog box displays the default settings for Model 2.5.

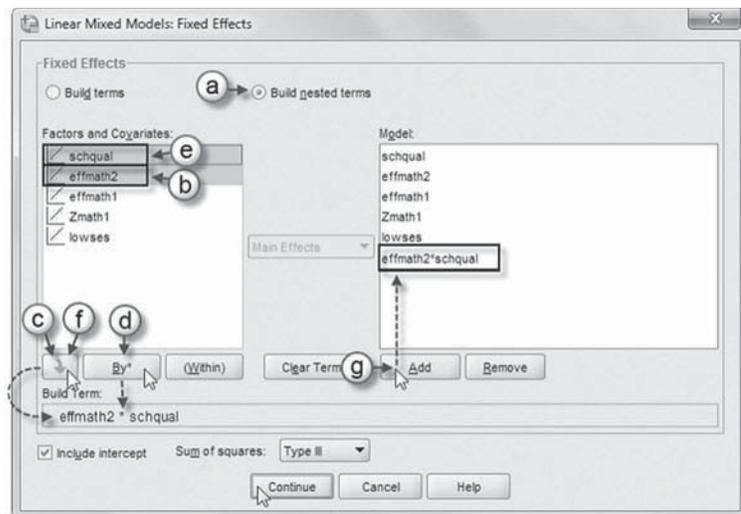
We will now add a fixed effect to the model, so click the FIXED button to access the *Fixed Effects* main screen.

One cross-level interaction (or nested terms) will be created and added to the model:  $effmath2 * schqual$ . This interaction will tell us the relationship between quality of the school's educational processes ( $schqual$ ) and math achievement ( $effmath2$ ).



Add Interaction to Model 2.6:  
 $effmath2 * schqual$

- Click to select *Build nested terms*.
- Now click to select the variable  $effmath2$  from the *Factors and Covariates* box.
- Click the arrow button below the *Factors and Covariates* box. This moves  $effmath2$  into the *Build Term* box to create a cross-level interaction by linking variables and terms.
- Next, click the BY\* button, which will insert the computation command symbol:  $effmath2*$ .
- Click to select  $schqual$  from the *Factors and Covariates* box.
- Click the arrow button below the *Factors and Covariates* box to move  $schqual$  into the *Build Term* box and complete the interaction term:  $effmath2 * schqual$ .
- Click the ADD button to transfer the interaction into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

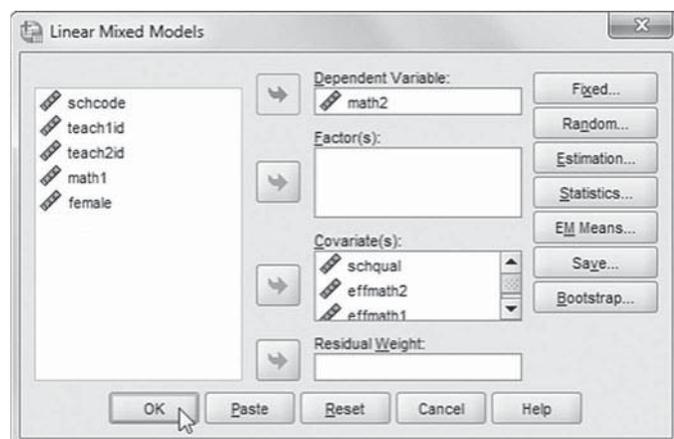


TABLE 8.26 Estimates of Fixed Effects<sup>a</sup>

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	645.718	2.201	307.461	293.367	.000	641.387	650.049
schqual	4.221	1.905	183.148	2.215	.028	0.462	7.980
effmath2	0.814	0.314	120.484	2.591	.011	0.192	1.437
effmath1	1.110	0.225	224.367	4.933	.000	0.666	1.553
Zmath1	28.378	0.488	4,074.882	58.186	.000	27.422	29.334
lowses	-4.433	0.974	4,087.802	-4.550	.000	-6.344	-2.523
schqual * effmath2	-0.458	0.330	90.252	-1.389	.168	-1.113	0.197

<sup>a</sup> Dependent variable: math2.

TABLE 8.27 Estimates of Covariance Parameters<sup>a</sup>

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		763.956	17.819	42.874	.000	729.818	799.690
Intercept + effmath2 [subject = schcode]	Var: Intercept	48.876	19.509	2.505	.012	22.353	106.871
	Var: effmath2	0.925	0.524	1.765	.078	0.305	2.808
Intercept [subject = teach2id]	Variance	29.528	9.218	3.203	.001	16.015	54.445
Intercept [subject = teach1id]	Variance	44.688	10.099	4.425	.000	28.696	69.591

<sup>a</sup> Dependent variable: math2.

### Interpreting the Output From Model 2.6

The fixed-effects table (Table 8.26) suggests that the quality of the school's key educational processes does not moderate the relationship between Teacher 2 effectiveness and achievement levels in math ( $\gamma = -0.46, p > .10$ ). The effects of the other variables on Year 2 achievement remain about the same as the previous model. Given that the cross-level interaction is not significant, for parsimony we should remove it from the final model.

The variance component table (Table 8.27) suggests that at Level 3, after adding the cross-level interaction, there may still be some slope variance left to explain at the school level (Wald  $Z = 1.765$ , one-tailed  $p = .039$ ). There is also still significant intercept variance in ending achievement to be explained across schools and classrooms. We could continue to add predictors to the model that might explain this variation in school and classroom intercepts.

## Summary

IBM SPSS MIXED proves very flexible in handling a variety of multilevel cross-classified data sets. They extend the types of data structures and problems that can be investigated with multilevel modeling. Cross-classified models also open up additional opportunities to investigate individuals' growth over time within their organizational settings. It is important to keep in mind, however, that their complexity and size will add to the challenge of trying to estimate these types of partly hierarchical data structures.

## CHAPTER 9

# Concluding Thoughts

In our 2009 book, *An Introduction to Multilevel Modeling Techniques* (2nd ed.), we sought to expand the application of multilevel-modeling techniques to a new range of research questions. In that book, we argued that most of the new variants of the simple multilevel model could, in fact, be subsumed under a general modeling framework of latent variables and simultaneous equations. The modeling we present in that book uses two popular multilevel statistical programs: HLM and Mplus. Although we are very satisfied with those excellent programs, their cost and the learning curve associated with their use present barriers to many colleagues who wanted to use the book in their graduate research courses. Unfortunately, relatively few social science computing labs make these programs available for student and classroom use. The problem makes it challenging to take full advantage of these software programs in introducing multilevel-modeling techniques to students.

Our own initial solution to this problem was to generate sets of handouts we could use with our students that made use of the familiar IBM SPSS modeling framework to introduce multilevel modeling. As we began to refine some of these materials, we realized there might be wider interest in having a set of hands-on activities that would illustrate major concepts in multilevel and longitudinal models. In this resulting workbook, we have tried to provide an applied introduction to multilevel modeling along with instruction for managing multilevel data, specifying a range of multilevel models, and interpreting output generated through the SPSS MIXED procedure. After “driving it around the block” a few times, we feel that it has definite utility for investigating a variety of multilevel and longitudinal models with continuous outcomes. We also noted a few cautions along the way. In compiling the workbook, we triangulated many of our results with other software and sometimes found small differences in individual parameters but, overall, a high level of substantive convergence. Our goal has been to widen exposure to and understanding of some general multilevel-modeling principles and applications. The workbook was designed as a complement to our more in-depth treatment of the statistical and conceptual issues surrounding multilevel modeling provided in our introductory multilevel book.

With an eye toward the how-to aspect of setting up a study to investigate a research problem, in the first chapter of the second edition of the workbook we introduced a number of key conceptual and methodological issues. Several elements have to be brought together including knowledge of previous research, the definition and development of the specific research problem, the goals of the research, the consideration of various appropriate methods of investigation (including their advantages and shortcomings) in light of the structure of the available data, and the means of communication with potential users of the study’s results. Our presentation there was designed to set the stage for the subsequent development of several different multilevel models that are the focus of the following chapters. We also briefly updated readers regarding issues of potential importance for model building including options for categorical outcomes, working with missing data, and sample weights. We devoted Chapter 2 to discussing a number

of conceptual and practical issues associated with the management of multilevel data. We believe readers will find the material useful in setting up their own data analyses since each study seems to bring its own unique challenges in preparing the data for the intended analyses. Our objective there was to highlight the different ways that hierarchical data can be structured, while offering a general introduction to essential data management procedures available within IBM SPSS.

Each of the remaining chapters dealt with various types of multilevel models for different types of data structures (cross-sectional, growth, and cross-classified) to cover a broad number of research possibilities. In particular, in this second edition we expanded the types of multilevel models that readers might find useful in various situations (e.g., piecewise and parallel growth models, latent variable outcomes, and regression discontinuity design). Although the traditional two- or three-level multilevel cross-sectional model has become a more common feature in social science research over the past decade, the other formulations that we introduced here are rapidly growing in their popularity and use. It is our hope that making these modeling capabilities available through SPSS will make them more accessible to a wider audience, even though we only scratched the surface regarding the use of these techniques with longitudinal and cross-classified data structures and more complex research designs that can be specified in SPSS MIXED, albeit with a bit more creativity required in setting up the data set.

As the reader can tell from the treatment we provide in this workbook, multilevel models can get quite complex. It is one thing to dump a set of variables into a single-level ordinary least squares (OLS) regression model but quite another to adopt a recklessly exploratory approach in a multilevel framework. Although exploratory analysis is an important and necessary feature of empirical research, the models we covered in this workbook demand a more disciplined approach to model conceptualization and specification. With multiple levels of analysis and the possibility for numerous cross-level interactions, choices about fixed or random slopes and intercepts, centering, weighting, and estimation algorithms, as well as the interpretation of the results from these models, can quickly become bogged down and rendered useless or, worse, misleading.

In this workbook, we began with the principle that quantitative analysis represents the translation of abstract theories into concrete models and that conceptual frameworks are essential guides to empirical investigation. The statistical model is therefore designed to represent a set of proposed theoretical relationships that are thought to exist in a population from which we have a sample of data. With this as a guiding principle, the researcher makes conscious decisions about the analysis that define research questions, design, data structures, and methods of analysis. The potential complexity of the multilevel model thus demands the careful attention of the analyst.

As we pointed out in Chapter 1, this does not ignore the importance of exploratory work. We recognize its place and suggest only that such work be conducted with careful thought about the possible relationships one might expect to see. The myriad combinations of model specification rule out the multilevel model as an effective “data-mining” tool. In fact, the models are very data demanding and will perform poorly without adequate data at each level of interest. But with an ample framework for organizing the analysis, adequate data, and careful thought to sensible model specification, the multilevel framework opens up dramatic new possibilities for exploratory work.

This workbook is written at a time when multilevel statistical models are better able to capture the complex contextual or environmental factors that condition the behaviors and attitudes of lower level units operating within those contexts or environments. Coincidentally, we also find ourselves at a moment where large-scale hierarchical data are more readily available than they were even 10 years ago. Although it would seem to be an incredibly exciting time for social science researchers (and indeed it is), we still find ourselves in want of data to drive empirical models suggested by our conceptual frameworks. Perhaps satisfaction on this front will be ever elusive; that is, as our modeling ability and statistical knowledge expand, so will our need for data to drive ever more demanding models.

One issue we call attention to through the activities we provided in this workbook is that multilevel models are quite demanding of data, and the analyst should be very aware of the limitations that exist as model specification becomes more complicated. As an example, our initial data sets (e.g., 100 to 200 individuals in 25 or 30 units) that we selected were fine for illustrating fixed effects; however, the moment we wished to illustrate random slopes, we needed much larger data sets to identify significant differences across groups. They often required 20 to 30 individuals within groups and 200 or more groups to be able to detect variability in the slopes. This gave us pause to consider that many published multilevel studies may report nonsignificant results due primarily to making Type II errors (i.e., failure to reject the null hypothesis) related to not having adequate data to detect the effects to begin with; that is, the available data are not up to the task of sufficiently supporting the complexity of the desired analyses. This suggests that the failure may not always be the proposed conceptual model but, rather, the shortcomings of the data and, perhaps, their measurement qualities. Hence, we emphasize that just because a model can be complex does not mean it should be complex. Parsimony is one indicator of a good empirical application of theory.

Another issue we touched on in Chapter 1 concerns the sampling scheme for the data used in multilevel studies. Researchers have for some time now been concerned about so-called design effects resulting from complex sampling schemes—that is, where a combination of multistage cluster sampling and disproportionate stratified sampling is used. The multilevel model actually capitalizes on such sample designs, incorporating the nesting of data that result from multistage clustering. Although the multilevel model incorporates this important sample design feature, it does not address the disproportionate stratified sampling that generally occurs with such sampling. This aspect of the design is usually dealt with through the application of sampling weights during the model specification stage. Weighting is pretty routine and straightforward in single-level analyses but becomes more challenging in the multilevel framework. Although some programs such as Mplus and HLM can now accommodate weights at multiple levels of analysis (e.g., student-level weights and school-level weights), SPSS uses a global weight and, therefore, forces the researcher to choose between weights that may exist across levels.

The efficacy of various weighting schemes used in different software programs is an issue that needs further research. Applying or not applying sample weights can change model estimates of standard errors considerably. Failure to take sample design into account by applying the appropriate weights can bias estimates of standard errors downward. Because standard errors are used in determining hypothesis tests about individual parameters, downward-biased estimates can lead to more findings of significance than should be the case. For the time being, if sample weighting is essential to the analysis, it will likely be better currently to use another of the available programs or to revert to a single-level formulation within SPSS through its COMPLEX SAMPLES module (see Asparouhov, 2006; and Rabe-Hesketh & Skrondal, 2006).

A third issue we ask readers to consider is missing data. Missing data can present serious challenges for the researcher and should be dealt with as forthrightly as possible. Rarely are data missing completely at random (MCAR). If one were to go with the IBM SPSS defaults regarding missing data, any case with even one piece of missing data would be eliminated from the analysis (i.e., where data are not vertically organized). We emphasize that researchers need to be aware of this and, where possible, seek viable solutions for data sets that contain considerable amounts of missing data. We favor situations where the software will actually estimate model parameters in the presence of the missing data (e.g., where variables are arranged vertically in IBM SPSS) since, in truth, there is no way of completely compensating for the fact that data have been irreversibly lost (Raykov & Marcoulides, 2008). We can try to obtain further information about the patterns of missing data and use this information to help devise a manner for dealing with the missing data. We do not recommend regression-based missing data substitution, which is an option within SPSS. For SPSS MIXED, we instead encourage the analyst to employ multiple imputation strategies, which produce multiple imputed data sets by repeatedly drawing values for

each missing datum from a distribution (Peugh & Enders, 2004). Unfortunately, this procedure is not available in the IBM SPSS Base or Advanced program module, but it can be added separately. At a minimum, we encourage the analyst to become very familiar with the data being used, to identify where data are missing, and to develop a strategy for testing the effects of missing data on the results generated by the model.

Finally, as should be very clear from the models we have developed throughout the workbook, there is a logical process underlying the development of the models and a series of steps involved for moving from the partitioning of variance between levels to the specification of models that may have random intercepts and slopes. We strongly encourage readers to devise their own naming system to keep track of the various models used in any given analysis. Without a clear history of model specification, it is sometimes very difficult to understand how one arrived at a final model. At the risk of twisting an old saw, it is often the journey that is more telling than the final destination.

Multilevel modeling provides us with another powerful means for investigating the types of processes referred to by our theories in more refined ways. Although the models presented in this workbook were simplified for purposes of demonstrating the techniques, we hope that our step-by-step guide serves as a foundation for the more thorough models that can be formulated and tested. We encourage the reader searching for more detail to consult the many other excellent resources available and referenced throughout this workbook.

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# Appendix A: Syntax Statements

Please note that syntax statements may be obtained through the “Paste” feature in the IBM SPSS menu interface. The syntax statements give information about the model such as the routine and the dependent variable (MIXED math) and the grouping variable [SUBJECT(schcode)]. The /FIXED command specifies the fixed effects in the model (i.e., achievement and the intercept), the random effects (/RANDOM = INTERCEPT), the type of covariance matrix (COVTYPE = VARIANCE COMPONENTS), and the estimation method (METHOD = MAXIMUM LIKELIHOOD). When using the “Paste” command, the estimation method will display a /CRITERIA command line<sup>1</sup> detailing settings for the method, iterations, log-likelihood convergence, parameter convergence, Hessian convergence, maximum scoring steps, and singularity tolerance. This line has been omitted for the models in this section but may be viewed in the syntax files (.sps) accompanying the data sets for each chapter and is available for downloading from the publisher’s Web site.

Using the syntax can be useful in making quick changes to the model as well as helping to keep a record of what models have been developed. They should be saved in a manner that allows easy referencing for future use (e.g., RprofMod1.sps and RprofMod2.sps).

---

## Chapter 3: Defining a Basic Two-Level Multilevel Regression Model (*ch3multilevel.sav*)

### Tables 3.2 and 3.3 (*ch3multilevel.sav*)

```
REGRESSION
/MISSING LISTWISE
/STATISTICS COEFF OUTS R ANOVA
/CRITERIA = PIN(.05) POUT(.10)
/NOORIGIN
/DEPENDENT math
/METHOD = ENTER ses.
```

#### Step 1: Examining Variance Components Using the Null Model

#### Model 1 (Null): Tables 3.4, 3.5, 3.6, and 3.7 (*ch3multilevel.sav*)

```
MIXED math
/FIXED = | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(VC).
```

#### Step 2: Building the Individual-Level (or Level 1) Random Intercept Model

#### Model 2: Tables 3.8, 3.9, 3.10, 3.11, and 3.12 (*ch3multilevel.sav*)

```
MIXED math WITH ses
/FIXED = ses | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(VC).
```

---

1. CRITERIA = CIN(95) MXITER(100) MXSTEP(10) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)

**Step 3: Building the Group-Level (or Level 2) Random Intercept Model**

## Model 3: Tables 3.13 and 3.14 (ch3multilevel.sav)

```
MIXED math BY public WITH ses_mean pro4yrc ses
  /FIXED = public ses_mean pro4yrc ses | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(VC).
```

**Treat “Public” Variable as a Covariate**

## Model 3A: Tables 3.15 and 3.16 (ch3multilevel.sav)

```
MIXED math WITH public ses_mean pro4yrc ses
  /FIXED = public ses_mean pro4yrc ses | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(VC).
```

**Step 4: Adding a Randomly Varying Slope (the Random Slope and Intercept Model)**

## Model 4: Tables 3.17, 3.18, and 3.19 (ch3multilevel.sav)

```
MIXED math WITH public ses_mean pro4yrc ses
  /CRITERIA = CIN(95) MXITER(100) MXSTEP(10) SCORING(1) SINGULAR(0.000000000001)
  HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)
  /FIXED = public ses_mean pro4yrc ses | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT ses | SUBJECT(schcode) COVTYPE(VC).
```

**Step 5: Explaining Variability in the Random Slope (More Complex Random Slopes and Intercept Models)**

## Table 3.20 (ch3multilevel.sav)

```
MIXED math WITH ses_mean ses
  /FIXED = ses_mean ses ses*ses_mean | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID).
```

**Step 5: Explaining Variability in the Random Slope (More Complex Random Slopes and Intercept Models)**

## Model 5: Tables 3.21, 3.22, and 3.23 (ch3multilevel.sav)

```
MIXED math WITH public ses_mean pro4yrc ses
  /FIXED = public ses_mean pro4yrc ses ses_mean*ses pro4yrc*ses public*ses |
  SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT ses | SUBJECT(schcode) COVTYPE(VC).
```

**Removed Nonsignificant Interactions (ses\_mean\*ses, pro4yrc\*ses)****Model 5A: Table 3.24 (ch3multilevel.sav)**

```
MIXED math WITH public ses_mean pro4yrc ses
  /FIXED = public ses_mean pro4yrc ses public*ses | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT ses | SUBJECT(schcode) COVTYPE(VC) .
```

**Chapter 4: Three-Level Univariate Regression Models (ch4threelevelURM.sav)****Model 1 (Null): Table 4.2 (ch4threelevelURM.sav)**

```
MIXED math
  /FIXED = | SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
  /RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID) .
```

**Defining Predictors at Each Level****Model 2: Table 4.4 (ch4threelevelURM.sav)**

```
MIXED math WITH gmschlowSES_mean gmaggtcheffect gmteacheffect gmclasslowses_
mean gmlowses
  /FIXED = gmschlowSES_mean gmaggtcheffect gmteacheffect gmclasslowses_mean gm-
lowses | SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
  /RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID) .
```

**Group-Mean Centering****Model 3: Tables 4.5 and 4.6 (ch4threelevelURM.sav)**

```
MIXED math WITH gmschlowSES_mean gmaggtcheffect groupteacheffect groupclass-
lowses_mean grouplowses
  /FIXED = gmschlowSES_mean gmaggtcheffect groupteacheffect groupclasslowses_
mean grouplowses | SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
  /RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID) .
```

**Does the Slope Vary Randomly Across Schools?****Model 4: Tables 4.7, 4.8, and 4.13 (ch4threelevelURM.sav)**

```
MIXED math WITH gmschlowSES_mean gmaggtcheffect gmteacheffect gmclasslowses_
mean gmlowses
```

```

/FIXED = gmschlowSES_mean gmaggtcheffect gmteacheffect gmclasslowses_mean gm-
lowses | SSTYPE(3)
/METHOD = ML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT gmteacheffect | SUBJECT(schcode) COVTYPE(UN)
/RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID).

```

### **Preliminary Investigation of the Interaction**

#### **Test Interaction Model A: Table 4.9 (ch4threelevelURM.sav)**

```

MIXED math WITH teacheffect classlowses_mean
/FIXED = teacheffect classlowses_mean teacheffect*classlowses_mean | SSTYPE(3)
/METHOD = ML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID).

```

#### **Table 4.10: Test Interaction Model B (Grand-Mean-Centered Variables) (ch4threelevelURM.sav)**

```

MIXED math WITH gmteacheffect gmclasslowses_mean
/FIXED = gmteacheffect gmclasslowses_mean gmteacheffect*gmclasslowses_mean |
SSTYPE(3)
/METHOD = ML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID).

```

### **Examining a Level 2 Interaction**

#### **Model 5: Tables 4.11, 4.12, and 4.14 (ch4threelevelURM.sav)**

```

MIXED math WITH gmschlowSES_mean gmaggtcheffect gmteacheffect gmclasslowses_
mean gmlowses
/FIXED = gmschlowSES_mean gmaggtcheffect gmteacheffect gmclasslowses_mean gm-
classlowses_mean*gmteacheffect gmlowses | SSTYPE(3)
/METHOD = ML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT gmteacheffect | SUBJECT(schcode) COVTYPE(UN)
/RANDOM = INTERCEPT | SUBJECT(schcode*Rteachid) COVTYPE(ID).

```

---

## **Chapter 5: Examining Individual Change with Repeated Measures Data (ch5growthdata-vertical.sav, ch5growthdata-horizontal.sav, and ch5experimentaldesigndata.sav)**

### **Graphing the Linear and Nonlinear Growth Trajectories**

**Figures 5.2, 5.3, and 5.4 (Select Cases and Generate Graph) (ch5growthdata-vertical.sav)**

#### **Select Subset of Individuals**

```

USE ALL.
COMPUTE filter_$ = (id < 18).

```

```
VARIABLE LABELS filter_$ 'id < 18 (FILTER)'.
VALUE LABELS filter_$ 0 'Not Selected' 1 'Selected'.
FORMATS filter_$ (f1.0).
FILTER BY filter_$.
EXECUTE.
```

### **Figures 5.2, 5.3, and 5.4** (*ch5growthdata-vertical.sav*)

(Variations are achieved by changing the Chart Editor's Properties options.)

```
GRAPH
  /SCATTERPLOT(BIVAR) = time WITH test BY id
  /MISSING = LISTWISE.
```

### **Coding Time Interval Variables (time to quadtime)** (*ch5growthdata-vertical.sav*)

```
RECODE time (0 = 0) (1 = 1) (2 = 4) INTO quadtime.
EXECUTE.
```

### **Coding Time Interval Variables (time to orthtime and orthquad)** (*ch5growthdata-vertical.sav*)

#### **orthtime**

```
RECODE time (0 = 1) (1 = 0) (2 = 1) INTO orthtime.
EXECUTE.
```

#### **orthquad**

```
RECODE time (0 = 1) (1 = -2) (2 = 1) INTO orthquad.
EXECUTE.
```

### **Model with No Predictors**

#### Model 1.1: Table 5.4 (*ch5growthdata-vertical.sav*)

```
MIXED test WITH time quadtime
  /FIXED = | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(id) COVTYPE(ID)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(ID).
```

### **What Is the Shape of the Trajectory?**

#### Model 1.1A: Tables 5.5 and 5.6 (*ch5growthdata-vertical.sav*)

```
MIXED test WITH time quadtime
  /FIXED = time quadtime | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT | SUBJECT(id) COVTYPE(ID)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(ID).
```

### **Does the Time-Related Slope Vary Across Groups?**

#### Model 1.1B: Tables 5.7, 5.8, and 5.9 (*ch5growthdata-vertical.sav*)

```
MIXED test WITH time quadtime
  /FIXED = time quadtime | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT time | SUBJECT(id) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(ID).
```

### **Examining Orthogonal Components**

#### Model 1.2: Tables 5.10 and 5.12 (ch5growthdata-vertical.sav)

```
MIXED test WITH orthtime orthquad
  /FIXED = orthtime orthquad | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT orthtime | SUBJECT(id) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(ID).
```

### **Investigating Other Level 1 Covariance Structures**

#### Repeated Measures ANOVA Tests of Within-Subjects Contrasts

#### Table 5.11 (ch5growthdata-horizontal.sav)

```
GLM test1 test2 test3
  /WSFACTOR = time 3 Polynomial
  /MEASURE = test
  /METHOD = SSTYPE(3)
  /EMMEANS = TABLES(OVERALL)
  /EMMEANS = TABLES(time)
  /PRINT = OPOWER HOMOGENEITY
  /CRITERIA = ALPHA(.05)
  /WSDESIGN = time.
```

### **Investigating Other Level 1 Covariance Structures**

#### Model 1: Table 5.13 Identity Covariance Matrix, Level 1; Unstructured Covariance Matrix, Level 2 (ch5growthdata-vertical.sav)

```
MIXED test WITH orthtime orthquad
  /FIXED = orthtime orthquad | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT orthtime | SUBJECT(id) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(ID).
```

### **Investigating Other Level 1 Covariance Structures**

#### Model 2: Table 5.13 Diagonal Covariance Matrix, Level 1; Diagonal Covariance Matrix, Level 2 (ch5growthdata-vertical.sav)

```
MIXED test WITH orthtime orthquad
  /FIXED = orthtime orthquad | SSTYPE(3)
  /METHOD = REML
```

```

/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT orthtime | SUBJECT(id) COVTYPE(DIAG)
/REPEATED = Index1 | SUBJECT(id) COVTYPE(DIAG) .

```

### ***Investigating Other Level 1 Covariance Structures***

Model 3: Table 5.13 Diagonal Covariance Matrix, Level 1; Unstructured Covariance Matrix, Level 2 (ch5growthdata-vertical.sav)

```

MIXED test WITH orthtime orthquad
/FIXED = orthtime orthquad | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT orthtime | SUBJECT(id) COVTYPE(UN)
/REPEATED = Index1 | SUBJECT(id) COVTYPE(DIAG) .

```

### ***Investigating Other Level 1 Covariance Structures***

Model 4: Table 5.13 Autoregressive Covariance Matrix, Level 1; Diagonal Covariance Matrix, Level 2 (ch5growthdata-vertical.sav)

```

MIXED test WITH orthtime orthquad
/FIXED = orthtime orthquad | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT orthtime | SUBJECT(id) COVTYPE(DIAG)
/REPEATED = Index1 | SUBJECT(id) COVTYPE(AR1) .

```

### ***Adding the Between-Subjects Predictors***

Model 1.3: Tables 5.15, 5.16, and 5.17 (ch5growthdata-vertical.sav)

```

MIXED test WITH orthtime orthquad ses effective
/FIXED = ses effective orthtime ses*orthtime effective*orthtime orthquad |
SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT orthtime | SUBJECT(id) COVTYPE(UN)
/REPEATED = Index1 | SUBJECT(id) COVTYPE(DIAG) .

```

### ***Tests of within-Subjects Contrasts (Repeated Measures ANOVA, Tests of within-Subject 1` Contrasts)***

Table 5.18 (ch5growthdata-horizontal.sav)

```

GLM test1 test2 test3 BY effective WITH ses
/WSFACTOR = time 3 Polynomial
/MEASURE = test
/METHOD = SSTYPE(3)
/EMMEANS = TABLES(OVERALL) WITH(ses = MEAN)
/EMMEANS = TABLES(time) WITH(ses = MEAN)
/PRINT = OPOWER HOMOGENEITY
/CRITERIA = ALPHA(.05)
/WSDESIGN = time
/DESIGN = ses effective.

```

**Graphing the Growth Rate Trajectories**

Figure 5.8: Growth Rate Trajectories by Teacher Effectiveness (ch5growthdata-vertical.sav)

```
GRAPH
  /LINE(MULTIPLE) = MEAN(test) BY time BY effective.
  Recoding time to timenonlin1
```

**Examining Growth Using an Alternative Specification of the Time-Related Variable****Coding Time Interval Variables**

## Recoding time to timenonlin1 (ch5growthdata-vertical.sav)

```
RECODE time (0 = 0) (1 = 0.5) (2 = 1) INTO timenonlin1.
EXECUTE.
```

## Recoding time to timenonlin2 (ch5growthdata-vertical.sav)

```
RECODE time (0 = 0) (1 = 0.6) (2 = 1) INTO timenonlin2.
EXECUTE.
```

## Recoding time to timenonlin3 (ch5growthdata-vertical.sav)

```
RECODE time (0 = 0) (1 = 0.7) (2 = 1) INTO timenonlin3.
EXECUTE.
```

## Recoding time to timenonlin (ch5growthdata-vertical.sav)

```
RECODE time (0 = 0) (1 = .53) (2 = 1) INTO timenonlin.
EXECUTE.
```

## Table 5.20 (ch5growthdata-vertical.sav)

```
MIXED test WITH timenonlin1
  /FIXED = timenonlin1 | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT timenonlin1 | SUBJECT(id) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(DIAG).
```

**Estimating the Final Time-Related Model**

## Model 2.1: (ch5growthdata-vertical.sav)

```
MIXED test WITH timenonlin
  /FIXED = timenonlin | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT timenonlin | SUBJECT(id) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(id) COVTYPE(DIAG).
```

**Adding the Two Predictors**

## Model 2.2: Tables 5.21 and 5.22 (ch5growthdata-vertical.sav)

```
MIXED test WITH timenonlin ses effective
```

```

/FIXED = ses effective timenonlin ses*timenonlin effective*timenonlin |
SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT timenonlin | SUBJECT(id) COVTYPE(UN)
/REPEATED = Index1 | SUBJECT(id) COVTYPE(DIAG).

```

### **An Example Experimental Design**

Tables 5.24 and 5.25 (ch5experimentaldesigndata.sav)

```

MIXED math WITH time treatment
/FIXED = treatment time time*treatment | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT time | SUBJECT(id) COVTYPE(UNR)
/REPEATED = time | SUBJECT(id) COVTYPE(DIAG).

```

Table 5.26 (ch5experimentaldesigndata.sav)

```

MIXED math By time WITH treatment
/FIXED = treatment time time*treatment | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(id) COVTYPE(UNR)
/REPEATED = time | SUBJECT(id) COVTYPE(DIAG).

```

Table 5.27 (ch5experimentaldesigndata.sav)

```

MIXED math WITH timenonlin treatment
/FIXED = treatment timenonlin timenonlin*treatment | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT timenonlin | SUBJECT(id) COVTYPE(VC)
/REPEATED = time | SUBJECT(id) COVTYPE(DIAG).

```

---

## **Chapter 6: Methods for Examining Organizational-Level Change (ch6graduationdata.sav and ch6RD-1data.sav)**

Model 1.1 (Null): Tables 6.2 and 6.3 (ch6graduationdata.sav)

```

MIXED gradproportion
/FIXED = | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(stateid) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(rid*stateid) COVTYPE(ID)
/REPEATED = time | SUBJECT(rid*stateid) COVTYPE(AR1).

```

### **Adding Growth Rates**

Model 1.2: Tables 6.6 and 6.7 (ch6graduationdata.sav)

```

MIXED gradproportion WITH time1
/FIXED = time1 | SSTYPE(3)
/METHOD = REML

```

```

/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT time1 | SUBJECT(stateid) COVTYPE(DIAG)
/RANDOM = INTERCEPT time1 | SUBJECT(rid*stateid) COVTYPE(DIAG)
/REPEATED = time | SUBJECT(rid*stateid) COVTYPE(AR1).

```

## Adding Time-Varying Covariates

### Model 1.3: Tables 6.8 and 6.9 (ch6graduationdata.sav)

```

MIXED gradproportion WITH percentFinAid tuition time1
/FIXED = percentFinAid tuition time1 | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT time1 | SUBJECT(stateid) COVTYPE(DIAG)
/RANDOM = INTERCEPT time1 | SUBJECT(rid*stateid) COVTYPE(DIAG)
/REPEATED = time | SUBJECT(rid*stateid) COVTYPE(AR1).

```

## Explaining Differences in Growth Trajectories Between Institutions

### Model 1.4: Tables 6.10 and 6.11 (ch6graduationdata.sav)

```

MIXED gradproportion WITH aveFamilyshare aveRetention mathselect percentFTfac-
ulty percentFinAid tuition time1
/FIXED = aveFamilyshare aveRetention mathselect percentFTfaculty percentFi-
nAid tuition time1 time1*mathselect time1*percentFTfaculty | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT time1 | SUBJECT(stateid) COVTYPE(DIAG)
/RANDOM = INTERCEPT time1 | SUBJECT(rid*stateid) COVTYPE(DIAG)
/REPEATED = time | SUBJECT(rid*stateid) COVTYPE(AR1).

```

## Adding a Model to Examine Growth Rates at Level 3

### Model 1.5: Tables 6.12 and 6.13 (ch6graduationdata.sav)

```

MIXED gradproportion WITH aveFamilyshare aveRetention mathselect percentFTfac-
ulty percentFinAid tuition time1
/FIXED = aveFamilyshare aveRetention mathselect percentFTfaculty percentFi-
nAid tuition time1 time1*aveFamilyshare time1*aveRetention time1*mathselect
time1*percentFTfaculty | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT time1 | SUBJECT(stateid) COVTYPE(DIAG)
/RANDOM = INTERCEPT time1 | SUBJECT(rid*stateid) COVTYPE(DIAG)
/REPEATED = time | SUBJECT(rid*stateid) COVTYPE(AR1).

```

## Regression Discontinuity Models to Explain Learning Differences

### Model 2.1: Tables 6.16 and 6.17 (ch6RD-1data.sav)

```

MIXED nmath WITH npretest treatment
/FIXED = npretest treatment | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT treatment | SUBJECT(teachcode) COVTYPE(UN).

```

## Adding Explanatory Variables at Level 2

### Model 2.2: Table 6.18 (ch6RD-1data.sav)

```
MIXED nmath WITH teachqual classcomp npretest treatment
/FIXED = teachqual classcomp npretest treatment teachqual*treatment
classcomp*treatment | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT treatment | SUBJECT(teachcode) COVTYPE(UN).
```

## Establishing the Prepolicy and Policy Trends

### Model 3.1: Tables 6.20, 6.21, and 6.22 (ch6RD-2data.sav)

```
MIXED freshadmit WITH implement0 implement1
/FIXED = implement0 implement1 | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schid) COVTYPE(ID)
/REPEATED = Index1 | SUBJECT(schid) COVTYPE(DIAG).
```

## Final Model with Covariates Added

### Model 3.2: Tables 6.23 and 6.24 (ch6RD-2data.sav)

```
MIXED freshadmit WITH private prestige implement0 implement1
/FIXED = private prestige implement0 implement1 implement0*private
implement0*prestige implement1*private implement1*prestige | SSTYPE(3)
/METHOD = REML
/PRINT = COVB SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schid) COVTYPE(ID)
/REPEATED = Index1 | SUBJECT(schid) COVTYPE(DIAG).
```

---

## Chapter 7: Multivariate Multilevel Models (*ch7latentconstructs.sav*, *ch7worklifeorg.sav*, *ch7achievement.sav*, and *ch7PGachievement.sav*)

### Table 7.2 (ch7latentconstructs.sav)

```
DESCRIPTIVES VARIABLES = W1varied W2value W3team Plassess P2progress P3evstand
/STATISTICS = MEAN STDDEV KURTOSIS SKEWNESS.
```

### Table 7.3 (ch7latentconstructs.sav)

```
FACTOR
/VARIABLES W1varied W2value W3team Plassess P2progress P3evstand
/MISSING LISTWISE
/ANALYSIS W1varied W2value W3team Plassess P2progress P3evstand
/PRINT INITIAL EXTRACTION
/CRITERIA FACTORS(2) ITERATE(25)
/EXTRACTION PAF
/ROTATION NOROTATE
/METHOD = CORRELATION.
```

Table 7.4 (ch7latentconstructs.sav)

```

FACTOR
/VARIABLES W1varied W2value W3team P1assess P2progress P3evstand
/MISSING LISTWISE
/ANALYSIS W1varied W2value W3team P1assess P2progress P3evstand
/PRINT INITIAL EXTRACTION ROTATION
/CRITERIA FACTORS(2) ITERATE(25)
/EXTRACTION PAF
/CRITERIA ITERATE(25) DELTA(0)
/ROTATION OBLIMIN
/METHOD = CORRELATION.

```

Model 1.1 (Null): Tables 7.5, 7.6, and 7.7 (ch7worklifeorg.sav)

```

MIXED work by assessjob
/FIXED = assessjob | noint SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/Random = assessjob | Subject (orgcode) COVTYPE(DIAG)
/Random = assessjob | Subject(Rid*orgcode) COVTYPE(DIAG)
/REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(ID).

```

Table 7.8 (ch7worklifeorg.sav)

```

MIXED work by assessjob
/FIXED = assessjob | noint SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/Random = assessjob | Subject (orgcode) COVTYPE(UNR)
/Random = assessjob | Subject(Rid*orgcode) COVTYPE(UNR)
/REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(ID).

```

Model 1.2 (Final Null Model): Table 7.10 (ch7worklifeorg.sav)

```

MIXED work by assessjob
/FIXED = assessjob | noint SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/Random = assessjob | Subject (orgcode) COVTYPE(UNR)
/Random = assessjob | Subject(Rid*orgcode) COVTYPE(UNR)
/REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(DIAG).

```

## Adding Level 2 Predictors

Model 1.3: Tables 7.11, 7.12, and 7.13 (ch7worklifeorg.sav)

```

MIXED work by assessjob with stability female
/FIXED = assessjob stability*assessjob female*assessjob | noint SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/Random = assessjob | Subject (orgcode) COVTYPE(UNR)
/Random = assessjob | Subject(Rid*orgcode) COVTYPE(UNR)
/REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(DIAG).

```

## Adding the Organizational Predictors

### Model 1.4: Tables 7.14 and 7.15 (ch7worklifeorg.sav)

```
MIXED work by assessjob with female stability gmresources gmorgprod
  /FIXED=assessjob gmorgprod*assessjob gmresources*assessjob stability*assessjob
  female*assessjob| noint SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /Random = assessjob | Subject (orgcode) COVTYPE(UN)
  /Random = assessjob | Subject(orgcode*Rid) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(DIAG).
```

## Examining Equality Constraints

### Table 7.16 (ch7worklifeorg.sav)

```
MIXED work by assessjob with female stability gmresources gmorgprod
  /FIXED = assessjob female stability gmorgprod gmresources| noint SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /Random = assessjob | Subject (orgcode) COVTYPE(UN)
  /Random = assessjob | Subject(orgcode*Rid) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(DIAG).
```

## Investigating a Random Level 2 Slope

### Defining Models 1.6 and 1.7 with IBM SPSS Menu Commands

#### Model 1.6 (ch7worklifeorg.sav)

```
MIXED work by assessjob with female stability gmresources gmorgprod
  /FIXED = assessjob female stability gmorgprod gmresources| noint SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /Random = assessjob stability | Subject (orgcode) COVTYPE(DIAG)
  /Random = assessjob | Subject(orgcode*Rid) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(DIAG).
```

#### Model 1.7 (ch7worklifeorg.sav)

```
MIXED work by assessjob with female stability gmresources gmorgprod
  /FIXED=assessjob gmorgprpd*assessjob gmresources*assessjob stability*assessjob
  female*assessjob| noint SSTYPE(3)
  /METHOD = ML
  /PRINT = G SOLUTION TESTCOV
  /Random = assessjob stability | Subject (orgcode) COVTYPE(DIAG)
  /Random = assessjob | Subject(orgcode*Rid) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(orgcode*Rid) COVTYPE(DIAG).
```

## Multivariate Multilevel Model for Correlated Observed Outcomes

### Model 2.1 (Null): Tables 7.18 and 7.19 (ch7achievement.sav)

```
MIXED achieve BY Index1
  /FIXED = Index1 | NOINT SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = Index1 | SUBJECT(schcode) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(UN) .
```

## Building a Complete Model (Predictors and Cross-Level Interactions)

### Model 2.2: Table 7.20 (ch7achievement.sav)

```
MIXED achieve BY female Index1 WITH gmacadpress
  /FIXED = Index1 Index1*gmacadpress Index1*female | NOINT SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = Index1 | SUBJECT(schcode) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(UN) .
```

### Table 7.21: Treating Female as a Covariate (ch7achievement.sav)

```
MIXED achieve BY Index1 WITH gmacadpress female
  /FIXED = Index1 Index1*gmacadpress Index1*female | NOINT SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = Index1 | SUBJECT(schcode) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(UNR) .
```

### Table 7.22: Treating Female as Categorical (Factor) (ch7achievement.sav)

```
MIXED achieve BY female Index1 WITH gmacadpress
  /FIXED = Index1 Index1*gmacadpress Index1*female | NOINT SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = Index1 | SUBJECT(schcode) COVTYPE(UN)
  /REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(UNR) .
```

## Correlations Between Tests at Each Level

### Model 2.3: Table 7.23 (ch7achievement.sav)

```
MIXED achieve BY Index1 with gmacadpress female
  /FIXED = Index1 gmacadpress*Index1 index1*female | NOINT SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = Index1 | SUBJECT(schcode) COVTYPE(UNR)
  /REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(UNR) .
```

## Defining a Parallel Growth Process Specifying the Time Model

### Model 3.1: Tables 7.25, 7.26, and 7.27 (ch7PGachievement.sav)

```
MIXED achieve by math with orthtime orthquadtime
  /FIXED = math math*orthtime math*orthquadtime | noint SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
```

```

/Random = math | Subject (schcode) COVTYPE(UN)
/REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(AR1) .

```

## Adding the Predictors

### Model 3.2: Tables 7.29, 7.30, and 7.31 (ch7PGachievement.sav)

```

MIXED achieve by math with orthtime orthquadtime female schcontext
/FIXED = math math*schcontext math*female math*orthtime math*orthquadtime
schcontext*math*orthtime female*math*orthtime | noint SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/Random = math | Subject (schcode) COVTYPE(UN)
/REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(AR1) .

```

### Table 7.32 (ch7PGachievement.sav)

```

MIXED achieve BY math WITH orthtime orthquadtime female schcontext
/FIXED = math math*schcontext math*female math*orthtime math*orthquadtime
schcontext*math*orthtime female*math*orthtime | NOINT SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = math math*orthtime | SUBJECT(schcode) COVTYPE(CSH)
/REPEATED = Index1 | SUBJECT(schcode*Rid) COVTYPE(AR1) .

```

---

## Chapter 8: Cross-Classified Multilevel Models (ch8crossclass1.sav and ch8crossclass2.sav)

### Table 8.7 (ch8crossclass1.sav)

```

MIXED CUM_GPR
/FIXED = | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(nschcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(campus) COVTYPE(ID) .

```

## Adding a Set of Level 1 and Level 2 Predictors

### Model 1.1: Tables 8.8 and 8.9 (ch8crossclass1.sav)

```

MIXED CUM_GPR WITH gmfouryear gmlowSES_mean gmlowSES gmfemale
/FIXED = gmfouryear gmlowSES_mean gmlowSES gmfemale | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(nschcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(campus) COVTYPE(ID) .

```

## Investigating a Random Slope

### Model 1.2: Table 8.10 (ch8crossclass1.sav)

```

MIXED CUM_GPR WITH gmfouryear gmlowSES_mean gmlowSES gmfemale
/FIXED = gmfouryear gmlowSES_mean gmlowSES gmfemale | SSTYPE(3)

```

```

/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT gmfemale | SUBJECT(nschcode) COVTYPE(DIAG)
/RANDOM = INTERCEPT gmfemale | SUBJECT(campus) COVTYPE(DIAG).

```

## Explaining Variation Between Variables

### Model 1.3: Tables 8.11 and 8.12 (ch8crossclass1.sav)

```

MIXED CUM_GPR WITH gmlowSES_mean gmlowSES gmfemale
/FIXED = gmlowSES_mean gmlowSES gmfemale gmlowSES_mean*gmfemale | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT gmfemale | SUBJECT(nschcode) COVTYPE(DIAG)
/RANDOM = INTERCEPT | SUBJECT(campus) COVTYPE(ID).

```

## Intercept-Only Model

### Model 2.1: Tables 8.14, 8.15, and 8.16 (ch8crossclass2.sav)

```

MIXED math2
/FIXED = | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(teach2id) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(teach1id) COVTYPE(ID).

```

## Defining the Cross-Classified Model with Previous Achievement

### Model 2.2: Tables 8.17 and 8.18 (ch8crossclass2.sav)

```

MIXED math2 WITH Zmath1
/FIXED = Zmath1 | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(teach2id) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(teach1id) COVTYPE(ID).

```

## Adding Teacher Effectiveness and a Student Background Control

### Model 2.3: Tables 8.21 and 8.22 (ch8crossclass2.sav)

```

MIXED math2 WITH effmath2 effmath1 Zmath1 lowSES
/FIXED = effmath2 effmath1 Zmath1 lowSES | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = INTERCEPT | SUBJECT(schcode) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(teach2id) COVTYPE(ID)
/RANDOM = INTERCEPT | SUBJECT(teach1id) COVTYPE(ID).

```

## School-Level Predictor and Random Slope

### Model 2.4: Table 8.23 (ch8crossclass2.sav)

```
MIXED math2 WITH effmath2 effmath1 Zmath1 lowses
  /FIXED = effmath2 effmath1 Zmath1 lowses | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT effmath2 effmath1 | SUBJECT(schcode) COVTYPE(DIAG)
  /RANDOM = INTERCEPT | SUBJECT(teach2id) COVTYPE(ID)
  /RANDOM = INTERCEPT | SUBJECT(teach1id) COVTYPE(ID).
```

## Level 3 Differences Between Institutions

### Model 2.5: Tables 8.24 and 8.25 (ch8crossclass2.sav)

```
MIXED math2 WITH schqual effmath2 effmath1 Zmath1 lowses
  /FIXED = schqual effmath2 effmath1 Zmath1 lowses | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT effmath2 | SUBJECT(schcode) COVTYPE(DIAG)
  /RANDOM = INTERCEPT | SUBJECT(teach2id) COVTYPE(ID)
  /RANDOM = INTERCEPT | SUBJECT(teach1id) COVTYPE(ID).
```

## Adding a Level 3 Cross-Level Interaction

### Model 2.6: Tables 8.26 and 8.27 (ch8crossclass2.sav)

```
MIXED math2 WITH schqual effmath2 effmath1 Zmath1 lowses
  /FIXED = schqual effmath2 effmath1 Zmath1 lowses effmath2*schqual | SSTYPE(3)
  /METHOD = REML
  /PRINT = G SOLUTION TESTCOV
  /RANDOM = INTERCEPT effmath2 | SUBJECT(schcode) COVTYPE(DIAG)
  /RANDOM = INTERCEPT | SUBJECT(teach2id) COVTYPE(ID)
  /RANDOM = INTERCEPT | SUBJECT(teach1id) COVTYPE(ID).
```

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# Appendix B: Model Comparisons Across Software Applications

For comparative purposes, as mentioned in Chapters 5 and 7, we provide the results generated by IBM SPSS and Mplus. The estimation procedures provide substantial agreement of fixed effects, robust standard errors, and variance components in the model. These Mplus estimates (Table B.1) are with the middle interval “freely estimated” as 0.529. This provided the optimal estimate for the SPSS MIXED results (Table B.2) and provides an estimate of the initial status the same as in Table 5.2. If we add those, we obtain the intercept estimate for time 3 in Table 5.2 (57.094).

Table B.3 is a comparison of the estimates produced with Mplus (using a two-level latent variable analysis) and the MIXED specification in chapter 7, Table 7.15.

**TABLE B.1 Mplus Estimates**

Parameter	Estimate	Std. Error	Est./SE	Sig.
Model with T2 = 0.529 Intercept	48.632	0.104	466.258	.000
Time (Nonlinear)	8.462	0.122	69.294	.000
Level 2 Variance				
Intercept	30.703	2.853	10.763	.000
Slope	28.377	5.593	5.073	.000

**TABLE B.2 SPSS Estimates**

Parameter	Estimate	Std. Error	t	Sig.
Model with T2 = 0.529 Intercept	48.632	0.098	495.049	.000
Time (Nonlinear)	8.462	0.122	69.564	.000
Level 2 Variance				
Intercept	31.283	0.949	32.973*	.000
Slope	29.461	2.082	14.150*	.000

\*Wald Z coefficient.

Table B.3 is a comparison of the estimates with produced with Mplus (using a two-level latent variable analysis) and the MIXED specification in chapter 7, Table 7.15.

**TABLE B.3 Comparison of SPSS Unstandardized Estimates in Table 7.15 with Mplus Unstandardized Estimates**

Parameter	SPSS Estimates				Mplus Estimates	
	Estimate	Std. Error	t	Sig.	Estimate	Std. Error
[assessjob=0]	3.909	0.056	69.537	0.000	3.923	0.077
[assessjob=1]	3.738	0.065	57.681	0.000	3.869	0.083
[assessjob=0] * gmorgprod	0.344	0.119	2.878	0.005	0.306	0.113
[assessjob=1] * gmorgprod	0.383	0.144	2.666	0.009	0.379	0.140
[assessjob=0] * gmresources	0.101	0.044	2.273	0.025	0.086	0.040
[assessjob=1] * gmresources	0.060	0.053	1.124	0.263	0.051	0.052
[assessjob=0] * stability	0.129	0.062	2.071	0.039	0.155	0.067
[assessjob=1] * stability	0.171	0.069	2.493	0.013	0.193	0.072
[assessjob=0] * female	-0.038	0.057	-0.666	0.506	-0.029	0.057
[assessjob=1] * female	-0.001	0.062	-0.016	0.988	0.000	0.064

---

# Appendix C: Syntax Routine to Estimate Rho From Model's Variance Components

We developed a syntax routine that estimates a two-level intraclass correlation ( $\rho$ ) from the null model's variance components. The first part generates the variance components and saves the file to the "C:\Program Files" path noted in the syntax but may be changed to a different location on your computer's hard drive. The second part of the routine retrieves the variance component file estimates, then computes and displays  $\rho$  on-screen. In this example, we use *read* as the outcome and *schcode* as the grouping (Level 2) variable.

## **\*\*Part 1: Set Up the Variance Components Model**

```
VARCOMP read BY schcode
  /RANDOM = schcode
  /OUTFILE = VAREST ('C:\Program Files\IBM\VAREST.sav')
  /METHOD = REML
  /CRITERIA = ITERATE(50)
  /CRITERIA = CONVERGE(1.0E-8)
  /DESIGN
  /INTERCEPT = INCLUDE.
```

## **\*\*Part 2: Retrieve the Variance Component Estimates and Compute and Display Rho**

```
Get file = "C:\Program Files\IBM\VAREST.sav" /Drop = ROWTYPE_ VARNAME_.
Rename Var (VC1 VC2 = Between Within).
Compute rho = Between/(Between+Within).
SAVE OUTFILE = "C:\Program Files\IBM\VAREST.sav".
LIST.
```

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